Visit us - www.researchjournal.co.in ■ DOI : 10.15740/HAS/IRJAES/8.1/100-103

International Research Journal of Agricultural Economics and Statistics

Volume 8 | Issue 1 | March, 2017 | 100-103 ■ e ISSN-2231-6434

Research Paper

A new approach of ratio estimation in sample surveys

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Paper History :
Received : *05.01.2017;*
Revised : *26.01.2017;*
Accepted : *05.02.2017*

ABSTRACT : This article deals with the estimation of population mean under simple random sampling using a new form of ratio estimator. The expression for mean square error and bias has been obtained. An efficiency comparison is considered for proposed estimator with the classical ratio, product and exponential ratio estimator. Finally an empirical study is also carried out to judge the performance of proposed estimator.

KEY WORDS : Simple random sampling, Ratio estimator, Mean square error, Efficiency, **AMS Classification:** 62D05

HOW TO CITE THIS PAPER : Jeelani, M. Iqbal, Rizvi, S.E.H., Sharma, Manish Kr., Nazir, Nageena and Jeelani, Faheem (2017). A new approach of ratio estimation in sample surveys. *Internat. Res. J. Agric. Eco. & Stat.,* **8** (1) : 100-103, **DOI : 10.15740/HAS/IRJAES/8.1/100-103.**

INTRODUCTION :

In sampling it is a well known fact that the proper use of auxiliary information supplied by auxiliary variable improves the efficiency of the estimators of population parameters of the study variable. Ratio estimation is one such example where this criterion is fulfilled. Ratio estimators are used for estimation of population mean when study variable and auxiliary variable are highly correlated with each other. Early historical developments of the ratio method of estimation are being well presented by, Sen (1993). Some of the modified ratio estimators given by Cochran (1977); Murthy (1967); Prasad (1989); Sen (1993); Singh and Tailor (2003 and 2005); Upadhyaya and Singh (1999); Yan and Tian (2010) and Jeelani and Maqbool (2013) are available in literature.

The classical ratio estimator for population mean $\bar{Y}$ of the study variable is defined as $\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X}$ assuming that population mean of the auxiliary variable is known $\bar{y}$ and $\bar{x}$ are sample means of study variable and auxiliary variable, and the mean square of the classical ratio estimator is $MSE(\bar{y}_r) = \delta \bar{Y}^2 [C_y^2 + C_x^2 - 2\rho_{yx} C_y C_x]$. Where $\delta = 1 - \frac{f}{n}, f = \frac{1}{n}, \hat{R}$ population ratio, ρ_{yx} is correlation co-efficient between study variable and auxiliary variable, C_y and C_x are co-efficient of variations of study variable and auxiliary variable. Also $C_y = \frac{S_y}{\bar{Y}}, C_x = \frac{S_x}{\bar{X}}, \rho_{yx} = \frac{S_{yx}}{S_y S_x}$, respectively.

Suggested estimator:

In this section we suggest a new ratio estimator by adapting an exponential estimator given by, Bahl and Tuteja (1991). The suggested ratio estimator will take the form as given below;

$$\bar{y}_{E(MIJ)} = \psi\bar{y} + [\bar{y}_{E(BT)} - \psi(\bar{y}_{E(BT)})] = \psi\bar{y} + (1-\psi)\bar{y}_{E(BT)} \quad (1)$$

where, $\bar{y}_{E(BT)}$ is the exponential estimator proposed by, Bahl and Tuteja (1991) and is as follows;

$$\bar{y}_{E(BT)} = \bar{y}\exp\left(\frac{\bar{\bar{X}} - \bar{x}}{\bar{\bar{X}} + \bar{x}}\right) \quad (2)$$

Also in equ. (1) ψ is is a constant such that the mean square error of the proposed estimator $\bar{y}_{E(MIJ)}$ is minimized.

Estimation of bias and mean square error of the suggested estimator :

For calculation of bias and mean square error of the proposed estimator we have $\bar{y} = \bar{Y}(1+\eta_0)$ and $\bar{x} = \bar{X}(1++\eta_1)$, such that the expectation of η_0 and η_1 is equal to zero E (η_0) = E (η_1) = 0

Now we have

$$E(\eta_0)^2 = \delta\left(\frac{S_y}{\bar{\bar{Y}}}\right)^2 \quad (3)$$

$$E(\eta_1)^2 = \delta\left(\frac{S_x}{\bar{\bar{X}}}\right)^2 \quad (4)$$

$$E(\eta_0, \eta_1) = \delta\ \frac{S_{yx}}{S_y S_x}\left(\frac{S_y}{\bar{\bar{Y}}} x \frac{S_x}{\bar{\bar{X}}}\right) \quad (5)$$

where,

$$\delta = 1 - f/n,\ f = 1/n,\ S_y = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \bar{Y})^2}{N-1}},\ S_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{X})^2}{N-1}},$$

$$S_{yx} = \sqrt{\frac{\Sigma(y_i - \bar{Y})(x_i - \bar{X})}{N-1}}$$

Applying Taylor series expansion and expectations on equ. (1), we can have the Bias and MSE of the proposed estimator in the following steps;

$$\bar{y}_{E(MIJ)} - \bar{Y} = \bar{Y}\{\eta_0 - K - \psi x K\} \quad (6)$$

where, $K = \frac{\eta_1}{2} - \frac{3}{8}\eta_1{}^2 + \frac{\eta_0 x \eta_1}{2}$

Applying the value of K in equ. (6), the above equation changes to ;

$$\bar{y}_{E(MIJ)} - \bar{Y} = \bar{Y}\left\{\eta_0 - \left(\frac{\eta_1}{2} - \frac{3}{8}\eta_1{}^2 + \frac{\eta_0 x \eta_1}{2}\right) - \psi\left(\frac{\eta_1}{2} - \frac{3}{8}\eta_1{}^2 + \frac{\eta_0 x \eta_1}{2}\right)\right\} \quad (7)$$

Utilizing the equ. (7), the Bias of proposed estimator will be;

$$Bais(\bar{y}_{E(MIJ)}) = \delta\bar{Y}\left(\frac{S_x}{\bar{\bar{X}}}\right)^2 [L - \psi L] \quad (8)$$

where, $L = \left(\frac{3}{8} - \frac{Z_{yx}}{2}\right)$

Taking the value of L in equ. (8), Bias of $\bar{y}_{E(MIJ)}$ becomes;

$$Bais\ (\bar{y}_{E(MIJ)}) = \delta\bar{Y}\left(\frac{S_x}{\bar{\bar{X}}}\right)^2\left[\left(\frac{3}{8} - \frac{Z_{yx}}{2}\right) - \psi\left(\frac{3}{8} - \frac{Z_{yx}}{2}\right)\right] \quad (9)$$

where, $Z_{yx} = \frac{S_{yx}}{S_y S_x} / \left\{\frac{S_y}{\bar{\bar{Y}}} / \frac{S_x}{\bar{\bar{X}}}\right\}$

Now the MSE of $\bar{y}_{E(MIJ)}$ will take the following form;

$$MSE(\bar{y}_{E(MIJ)}) - \bar{Y} = \bar{Y}\{\eta_0 - \Gamma - \psi x \Gamma\} \quad (10)$$

where

$\Gamma = \frac{\eta_1}{2}$

Utilizing the value of Γ in equ. (10), we have;

$$MSE\ (\bar{y}_{E(MIJ)}) - \bar{Y} = \bar{Y}\left\{\eta_0 - \frac{\eta_1}{2} - \psi\frac{\eta_1}{2}\right\} \quad (11)$$

Again by using Taylor series expansion and taking expectations and squaring on both sides in equ. (11) then the MSE of $\bar{y}_{E(MIJ)}$ will be

$$MSE\ (\bar{y}_{E(MIJ)}) = \delta\bar{Y}\left\{\left(\frac{S_x}{\bar{\bar{X}}}\right)^2 + \frac{\{S_x/\bar{X})^2(1-\psi)^2}{4} - \frac{S_{yx}}{S_y S_x}\left(\frac{S_y}{\bar{\bar{Y}}}\right)\left(\frac{S_x}{\bar{\bar{X}}}\right)(1-\psi)\right\} \quad (12)$$

The optimum value of ψ to minimize the mean square error of equ. (1) can be easily found as;

$$\frac{\partial}{\partial\psi} MSE\ (\bar{y}_{E(MIJ)}) = 0 \quad (13)$$

Then

$$\psi^* = 1 - 2\frac{S_{yx}}{S_y S_x} / \left\{\frac{S_y}{\bar{\bar{Y}}} / \frac{S_x}{\bar{\bar{X}}}\right\} \quad (14)$$

where,

$$S_y = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \bar{Y})^2}{N-1}},\ S_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{X})^2}{N-1}},$$

$$S_{yx} = \sqrt{\frac{\Sigma(y_i - \bar{Y})(x_i - \bar{X})}{N-1}}$$

Now recalling the equ. (1) and applying ψ* in it in place of ψ such that the MSE of $\bar{y}_{E(MIJ)}$ is minimum in place of.

Then the MSE ($\bar{y}_{E(MIJ)}$) will be

$$MSE(\bar{y}_{E(MIJ)}) = \frac{1-f}{n}\bar{Y}^2\left(\frac{S_y}{\bar{Y}}\right)^2\left(1-\left(\frac{S_{yx}}{S_yS_x}\right)^2\right) \quad (15)$$

Now;

$$\bar{y}_{E(MIJ)}^* = \left(1-2\frac{S_{yx}}{S_yS_x}/\left\{\frac{S_y}{\bar{Y}}/\frac{S_x}{\bar{X}}\right\}\right)\bar{y}+2\frac{S_{yx}}{S_y S_x}/\left\{\frac{S_y}{\bar{Y}}/\frac{S_x}{\bar{X}}\right\}\bar{y}_{E(BT)} \quad (16)$$

where, $\bar{y}_{E(BT)}=\bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$

The mean square error of (15) will tend to mean square error of (16) by replacing ψ in (1) by ψ*, then the mean square error of (16) will be ;

$$MSE(\bar{y}_{E(MIJ)}^*) = \frac{1-f}{n}\bar{Y}^2\left(\frac{S_y}{\bar{Y}}\right)^2\left(1-\left(\frac{S_{yx}}{S_yS_x}\right)^2\right) \quad (17)$$

As we know the classical ratio, product and exponential estimator given by, Bahl and Tuteja (1991) for population mean $\bar{Y}$ and there mean square errors by applying Taylor's first degree expansion are as follows;

$$(\bar{y}_r)=\frac{\bar{y}}{\bar{x}}\bar{X} \text{ and } MSE(\bar{y}_r)=\frac{1-f}{n}\bar{Y}^2\left\{\left(\frac{S_y}{\bar{Y}}\right)^2+\left(\frac{S_x}{\bar{X}}\right)^2\left(1-2\frac{S_{yx}}{S_yS_x}/\left\{\frac{S_y}{\bar{Y}}/\frac{S_x}{\bar{X}}\right\}\right)\right\} \quad (18)$$

$$(\bar{y}_{pr})=\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) \text{ and } MSE(\bar{y}_{pr})=\frac{1-f}{n}\bar{Y}^2\left\{\left(\frac{S_y}{\bar{Y}}\right)^2+\left(\frac{S_x}{\bar{X}}\right)^2\left(1+2\frac{S_{yx}}{S_yS_x}/\left\{\frac{S_y}{\bar{Y}}/\frac{S_x}{\bar{X}}\right\}\right)\right\} \quad (19)$$

$$\bar{y}_{E(BT)}=\bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \text{ and } MSE(\bar{y}_{E(BT)}) = \frac{1-f}{n}\bar{Y}^2$$

$$\left\{\left(\frac{S_y}{\bar{Y}}\right)^2+\left(\frac{S_x}{\bar{X}}\right)^2/4--\frac{S_{yx}}{S_yS_x}\left[\left(\frac{S_y}{\bar{Y}}\right)\left(\frac{S_x}{\bar{X}}\right)\right]\right\} \quad (20)$$

For efficiency comparison we have;

The MSE of (17) will be less than MSE of (18), (19) and (20) if and only if

$$\psi<3-4\frac{S_{yx}}{S_yS_x}/\left\{\frac{S_y}{\bar{Y}}/\frac{S_x}{\bar{X}}\right\} \text{ then } MSE(\bar{y}_{E(MIJ)})<MSE(\bar{y}_r) \quad (21)$$

$$\psi<3+4\frac{S_{yx}}{S_yS_x}/\left\{\frac{S_y}{\bar{Y}}/\frac{S_x}{\bar{X}}\right\} \text{ then } MSE(\bar{y}_{E(MIJ)})<MSE(\bar{y}_{pr}) \quad (22)$$

$$\psi<2-4\frac{S_{yx}}{S_yS_x}/\left\{\frac{S_y}{\bar{Y}}/\frac{S_x}{\bar{X}}\right\} \text{ then } MSE(\bar{y}_{E(MIJ)})<\bar{y}_{E(BT)} \quad (23)$$

Empirical study :

For empirical study two natural populations where considered from Singh and Chaudhary (1986) and Cochran (1997). The population parameters and the constants computed from the above populations are given Table 1. The efficiency comparison is given in Table 2.

Conclusion :

From theoretical discussion and numerical example, we infer that the proposed estimator is more efficient than the classical ratio, product estimator and the exponential ratio estimator given by, Bahl and Tuteja (1991), as the efficiency of the proposed estimator is much higher than the efficiencies of the classical ratio,

Table 1 : Parameters of the population

Populations \ Parameters	N	n	$\bar{Y}$	$\bar{X}$	ρ_{yx}	S_y	C_y	S_x	C_x
Singh and Chaudhary (1986)	22	5	22.62	1467.54	0.92	33.04	1.46	2562.14	1.74
Cochran (1997)	49	20	116.16	98.67	0.69	98.82	0.85	102.97	1.04

Table 2 : Relative efficiency comparison

	Estimators			
	$(\bar{y}_r)$	$(\bar{y}_{pr})$	$\bar{y}_{E(BT)}$	$\bar{y}_{E(MIJ)}$
Singh and Chaudhary (1986)	180.63	167.13	210.13	286.41
Cochran (1997)	128.27	110.56	166.32	212.19

product estimator and the exponential ratio estimator given by, Bahl and Tuteja (1991). Hence, we strongly recommend that the proposed modified estimator may be preferred over the classical ratio, product estimator and the exponential ratio estimator given by, Bahl and Tuteja (1991)for the use of practical applications.

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