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Research Paper

Variance estimation using linear combination of non-conventional measures, quartile average and deciles mean

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ABSTRACT : Use of auxiliary information in survey sampling plays important role in getting more precision for estimating population parameters. Thus it has now become indispensable to use auxiliary information, thus in this paper we propose new modified ratio estimators by using the auxiliary information of non conventional location measures, non conventional measures of dispersion, quartile average, decile mean and their linear combinations for estimating population variance. The properties associated with proposed estimators are assessed by mean square error, bias and compared with existing estimators. By this comparison we conclude that our proposed estimators are more efficient than the existing estimators. To support the theoretical results, numerical study is provided.

KEY WORDS : Simple random sampling, Bias, Mean square error, Downtown's method, Deciles, Efficiency

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INTRODUCTION :

Here we consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on ***Ui, i=1,2,3,…,N*** giving a vector $Y = \{y_1, y_2, ..., y_N\}$. The goal is to estimate the population means or $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ its variance $S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2$ on the basis of random sample selected from a population U. Sometimes in sample surveys along with the study variable Y, information on auxiliary variable X, which is positively correlated with Y, is also available. The information on auxiliary variable X, may be utilized to obtain a more efficient estimator of the population. In this paper, our aim is to estimate the population variance on the basis of a random sample of size n selected from the population U . The problem of constructing efficient estimators for the population variance has been widely discussed by various authors such as Isaki (1983), who proposed ratio and regression estimators. Latter various authors such as Kadilar and Cingi (2006) and Sumramani and Kumarapandiyan (2015) improved the already existing estimators.

Notations:

N = Population size. n = ample size. $y = \frac{1}{n}$, Y= study variable. X= Auxiliary variable. $\bar{X}, \bar{Y}$ = Population means.

$\bar{x}, \bar{y} =$ Sample means. $S_Y^2, S_x^2 =$ Population variances. $s_y^2, s_x^2 =$ sample variances. $C_x, C_y =$ Co-efficient of variation. $\rho =$ Correlation co-efficient. $\beta_{1(x)} =$ Skewness of the auxiliary variable. $\beta_{2(x)} =$ Kurtosis of the auxiliary variable. $\beta_{2(y)} =$ Kurtosis of the study variable. $M_d =$ Median of the auxiliary variable. B (.) =Bias of the estimator. MSE (.) = Mean square error. $\hat{S}_R^2 =$ Ratio type variance estimator. $\hat{S}_{Kc1}^2, \hat{S}_{jG}^2, =$ Existing modified ratio estimators. QD= quartile deviation, G = Gini's Mean Difference, D = Downtown's method and S_{pw} = probability Weighted moments, D_i, i =1,2,…10 = Deciles, Hodges lemma= HL= Median [(Yj+Yk)/2 ,1≤ j≤ k ≤ N], TM = Tri- Mean, MR = Mid range, $Q_{M.A}$ = quartile mean average, $D_{M.A}$ = Decile mean average.

MATERIALS AND METHODS :

In this paper, we discuss the existing estimators in the literature and the modified estimators proposed by the author using the linear combination of non conventional location measures, non conventional measures of dispersion, quartile average and decile mean and the results are compared with the existing estimators.

Ratio type variance estimator proposed by Isaki (1983):

Isaki suggested a ratio type variance estimator for the population variance S_y^2 when the population variance S_x^2 of the auxiliary variable X is known. Its bias and mean square error are given by :

$$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \qquad \text{......(1)}$$

$$\text{Bias } (\hat{S}_R^2) = \gamma S_y^2 \left[(\beta_{2(x)} - 1) - (\lambda_{22} - 1)\right]$$

$$\text{MSE } (\hat{S}_R^2) = \gamma S_y^4 \left[(\beta_{2(y)} - 1) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\right]$$

Kadilar and Cingi (2006) estimators:

The authors suggested four ratio type variance estimators using known values of C.V and co-efficient of kurtosis of an auxiliary variable X.

$$\hat{S}_{kc1}^2 = s_y^2 \left[\frac{S_x^2 + C_x}{s_x^2 + C_x}\right] \qquad \text{......(2)}$$

$$\text{Bias } (\hat{S}_{kc1}^2) = \gamma S_y^2 A_1 \left[A_1(\beta_{2(x)} - 1) - (\lambda_{22} - 1)\right]$$

$$\text{MSE } (\hat{S}_{kc1}^2) = \gamma S_y^4 \left[(\beta_{2(y)} - 1) + A_1^2 (\beta_{2(x)} - 1) - 2A_1(\lambda_{22} - 1)\right]$$

Recent developments:

Sumramani and Kumarapandiyan (2015) estimators where the authors used median, quartiles and Deciles of an auxiliary variable.

$$\hat{S}_{jG}^2 = s_y^2 \left[\frac{S_x^2 + \alpha w_i}{s_x^2 + \alpha w_i}\right] \qquad \text{....(3)}$$

$$\text{Bias } (\hat{S}_{jG}^2) = \gamma S_y^2 A_{jG} \left[A_{jG}(\beta_{2(x)} - 1) - (\lambda_{22} - 1)\right]$$

$$\text{MSE } (\hat{S}_{jG}^2) = \gamma S_y^4 \left[(\beta_{2(y)} - 1) + A_{jG}^2 (\beta_{2(x)} - 1) - 2A_{jG}(\lambda_{22} - 1)\right]$$

When α=0 in eq. (3), the above estimator reduces to Isaki (1983) estimator.

When α=1 in eq. (3), the above estimator reduces to Kadilar and Cingi (2006) estimator.

Proposed estimator :

We have proposed a new modified ratio type variance estimator of the auxiliary variable by using linear combination of non conventional and conventional measures.

$$\hat{S}_{MS1}^2 = s_y^2 \left[\frac{S_x^2 + Q_{M.A}}{s_x^2 + Q_{M.A}}\right], \quad \hat{S}_{MS2}^2 = s_y^2 \left[\frac{S_x^2 + (MR + Q_{M.A})}{s_x^2 + (MR + Q_{M.A})}\right],$$

$$\hat{S}_{MS3}^2 = s_y^2 \left[\frac{S_x^2 + (HL + Q_{M.A})}{s_x^2 + (HL + Q_{M.A})}\right], \quad \hat{S}_{MS4}^2 = s_y^2 \left[\frac{S_x^2 + (TM + Q_{M.A})}{s_x^2 + (TM + Q_{M.A})}\right],$$

$$\hat{S}_{MS5}^2 = s_y^2 \left[\frac{S_x^2 + (G + Q_{M.A})}{s_x^2 + (G + Q_{M.A})}\right], \quad \hat{S}_{MS6}^2 = s_y^2 \left[\frac{S_x^2 + (D + Q_{M.A})}{s_x^2 + (D + Q_{M.A})}\right],$$

$$\hat{S}_{MS7}^2 = s_y^2 \left[\frac{S_x^2 + (S_{pw} + Q_{M.A})}{s_x^2 + (S_{pw} + Q_{M.A})}\right], \quad \hat{S}_{MS8}^2 = s_y^2 \left[\frac{S_x^2 + D_{M.A}}{s_x^2 + D_{M.A}}\right],$$

$$\hat{S}_{MS9}^2 = s_y^2 \left[\frac{S_x^2 + (MR + D_{M.A})}{s_x^2 + (MR + D_{M.A})}\right], \quad \hat{S}_{MS10}^2 = s_y^2 \left[\frac{S_x^2 + (HL + D_{M.A})}{s_x^2 + (HL + D_{M.A})}\right],$$

$$\hat{S}_{MS11}^2 = s_y^2 \left[\frac{S_x^2 + (TM + D_{M.A})}{s_x^2 + (TM + D_{M.A})}\right], \quad \hat{S}_{MS12}^2 = s_y^2 \left[\frac{S_x^2 + (G + D_{M.A})}{s_x^2 + (G + D_{M.A})}\right],$$

$$\hat{S}_{MS13}^2 = s_y^2 \left[\frac{S_x^2 + (D + D_{M.A})}{s_x^2 + (D + D_{M.A})}\right], \quad \hat{S}_{MS14}^2 = s_y^2 \left[\frac{S_x^2 + (S_{pw} + D_{M.A})}{s_x^2 + (S_{pw} + D_{M.A})}\right] \qquad \text{..(4)}$$

We have derived here the bias and mean square error of the proposed estimator $\hat{S}_{MSi}^2;\ i = 1,2,....,14$ to first order of approximation as given below:

Let $e_0 = \dfrac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \dfrac{s_x^2 - S_x^2}{S_x^2}$. Further we can write

$s_y^2 = S_y^2(1+e_0)$ and $s_x^2 = S_x^2(1+e_0)$ and from the definition of e_0 and e_1 we obtain:

$$E[e_0] = E[e_1] = 0,\ E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)} - 1),$$

$$E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)} - 1),\ E[e_0 e_1] = \frac{1-f}{n}(\lambda_{22} - 1)$$

The proposed estimator $\hat{S}^2_{MSi};\ i = 1,2,3,...,14$ is given below:

$$\hat{S}^2_{MSi} = s_y^2\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + \alpha a_i}\right]$$

$$\Rightarrow\ \hat{S}^2_{MSi} = s_y^2(1+e_0)\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + e_1 S_x^2 + \alpha a_i}\right] \ \Rightarrow\ \hat{S}^2_{MSi} = \frac{S_y^2(1+e_0)}{(1+A_{MSi}e_1)}$$

where, $A_{MSi} = \dfrac{S_x^2}{S_x^2 + \alpha a_i}$

$a_i;\ i = (Q_{M.A}), (MR + Q_{M.A}), (HL + Q_{M.A}), (TM + Q_{M.A}), (G + Q_{M.A}), (D + Q_{M.A}), (S_{pw} + Q_{M.A}), D_{M.A}, (MR + D_{M.A}), (HL + D_{M.A}), (TM + D_{M.A}), (G + D_{M.A}), (D + D_{M.A}), (S_{pw} + D_{M.A})$.

$$\Rightarrow\ \hat{S}^2_{MSi} = S_y^2(1+e_0)(1+A_{MSi}e_1)^{-1}$$

$$\Rightarrow\ \hat{S}^2_{MSi} = S_y^2(1+e_0)(1 - A_{MSi}e_1 + A^2_{MSi}e_1^2 - A^3_{MSi}e_1^3 + ...)$$

Expanding and neglecting the terms more than 3rd order, we get

$$\hat{S}^2_{MSi} = S_y^2 + S_y^2 e_0 - S_y^2 A_{MSi} e_1 - S_y^2 A_{MSi} e_0 e_1 + S_y^2 A^2_{MSi} e_1^2$$

$$\Rightarrow\ \hat{S}^2_{MSi} - S_y^2 = S_y^2 e_0 - S_y^2 A_{MSi} e_1 - S_y^2 A_{MSi} e_0 e_1 + S_y^2 A^2_{MSi} e_1^2 \quad ....(5)$$

By taking expectation on both sides of (5), we get

$$E(\hat{S}^2_{MSi} - S_y^2) = S_y^2 E(e_0) - S_y^2 A_{MSi} E(e_1) - S_y^2 A_{MSi} E(e_0 e_1) + S_y^2 A^2_{MSi} E(e_1^2)$$

$$Bias(\hat{S}^2_{MSi}) = S_y^2 A^2_{MSi} E(e_1^2) - S_y^2 A_{MSi} E(e_0 e_1)$$

$$Bias(\hat{S}^2_{MSi}) = \gamma S_y^2 A_{MSi}[A_{MSi}(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \quad ....(6)$$

Squaring both sides of (5) and (6), neglecting the terms more than 2nd order and taking expectation, we get

$$E(\hat{S}^2_{MSi} - S_y^2)^2 = S_y^4 E(e_0^2) + S_y^4 A^2_{MSi} E(e_1^2) - 2S_y^4 A_{MSi} E(e_0 e_1)$$

$$MSE(\hat{S}^2_{MSi}) = \gamma S_y^4[(\beta_{2(y)} - 1) + A^2_{MSi}(\beta_{2(x)} - 1) - 2A_{MSi}(\lambda_{22} - 1)]$$

We use the data of Murthy (1967) page 228 in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories are denoted by Y(study variable).we apply the proposed and existing estimators to this data set and the data statistics is given below:

N=80, Sx=8.4542, n=20, Cx=0.7507, $\bar{X}$=11.2624, $\beta_{2(x)}$=2.8664, $\bar{Y}$ = 51.8264, $\beta_{2(y)}$= 2.2667, ρ=0.9413, $\beta_{1(x)}$=1.05, λ_{22}=2.2209, S_y=18.3569, Q1=9.318, C_y=0.3542, G = 9.0408, D= 8.0138, S_{pw} = 7.9136, TM =9.318, HL= 10.405, MR =17.955, Q_{MA}=11.2893, D_{MA}= 12.95.

RESULTS AND DATA ANALYSIS :

The Table 1 and 2 show that bias and mean square error are less than the already existing estimators in the literature and also by the percentage relative efficiency criterion. Hence, the proposed estimator may be preferred over existing estimators for use in practical applications.

The paper proposes new ratio type variance estimators using known values of an auxiliary variable. Results show that mean square error are less than the already existing estimators in the literature and also by the per cent relative efficiency criteria, the modified estimators are more efficient than the existing estimators. As can be seen from the Table 1 that the lowest MSE is found in the modified estimator MS10 followed by MS2. From Table 2 we can conclude that the highest per cent

Table 1: Bias and mean square error of the existing and the proposed estimators

Estimators	Bias	Means square error
Isaki (1983)	10.8762	3925.1622
Kadilar and Cingi (2006) 1	10.4399	3850.1552
Sumramani and Kumarapandiyan (2015)	6.1235	3180.7740
Proposed (MS1)	5.6814	3122.1149
Proposed (MS2)	1.2374	2690.0492
Proposed (MS3)	2.7244	2792.8138
Proposed (MS4)	2.9745	2814.9565
Proposed (MS5)	3.0267	2816.0900
Proposed (MS6)	3.2834	2843.3445
Proposed (MS7)	3.3128	2844.4801
Proposed (MS8)	5.0965	3043.1968
Proposed (MS9)	0.9659	2678.1262
Proposed (MS10)	2.3568	2762.7225
Proposed (MS11)	2.5947	2782.0264
Proposed (MS12)	2.6320	2785.0264
Proposed (MS13)	2.8795	2804.7368
Proposed (MS14)	2.9029	2804.5006

Table 2 : Per cent relative efficiency of proposed estimators with existing estimators

Estimators	Isaki (1983)	Kadilar and Cingi (2006) 1	Subramani and Kumarapandiyan (2015)
P1	125.7210	123.3188	101.8794
P2	145.9141	143.1258	121.9603
P3	140.5450	137.8593	113.8920
P4	139.4395	136.7749	117.1581
P5	139.3834	136.7198	112.9507
P6	138.0473	135.4093	111.8680
P7	137.9922	135.3553	111.8233
P8	128.9815	126.5167	104.5214
P9	143.5637	143.7630	118.7693
P10	147.8743	139.3609	115.1325
P11	141.0900	138.3939	114.3337
P12	140.9380	138.2448	114.2105
P13	139.9476	137.2733	113.4097
P14	139.9449	137.2755	113.4098

relative efficiency can also be found in MS10 followed by MS2. Hence, these estimators are preferred in real life situations.

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