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A Review

Hierarchical bayes small area estimation under an area level model with applications to horticultural survey data

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ABSTRACT : In this paper we studied Bayesian aspect of small area estimation using Area level model. We proposed and evaluated new prior distribution for the area level model, for the variance component rather than uniform prior. The proposed model is implemented using the MCMC method for fully Bayesian inference. Laplace approximation is used to obtain accurate approximations to the posterior moments. We apply the proposed model to the analysis of horticultural data and results from the model are compared with frequestist approach and with Bayesian model of uniform prior in terms of average relative bias, average squared relative bias and average absolute bias. The numerical results obtained highlighted the superiority of using the proposed prior over the uniform prior.

KEY WORDS : Small area estimation, Area level model, Hierarchical bayes

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INTRODUCTION :

Model-based small area estimation methods have been widely used in practice due to the increasing demand for precise estimates for local regions and various small areas. It is now generally accepted that the indirect estimates should be based on explicit models that provide links to related areas through the use of supplementary data such as census counts or administrative records; see, for example (Rao, 2003 and Jiang and Lahiri, 2006) for more discussion on model-based small area methods. Also, (Adam *et al.,* 2013) summarise the main methodological approaches to SAE and their linkages. (Jiango *et al.,* 2013) investigate two new approaches: one relying on the work of Chambers, and the second using the concept of conditional bias to measure the influence of units in the population. (Chambers *et al.,* 2014) proposed two different analytical mean-squared error estimators for the ensuing bias-corrected outlier robust estimators. (Rao *et al.,* 2013) they relaxed the assumption of linear regression for the fixed part of the model and replace it by a weaker assumption of a semi-parametric regression. The model-based estimates are obtained to improve the direct design-based estimates in terms of precision and reliability, *i.e.*, smaller co-efficients of variation (CVs). There are two broad classifications for small area models: area level models and unit level models. Area level models are based on area direct survey estimates and unit level models are based on individual observations in small areas. In this paper we focus on area level models that borrow strength across regions to improve the direct survey estimates. Among the area level models, the Fay-Herriot model (Fay and Herriot, 1979) is a basic and widely used area level model in practice to obtain reliable model-based estimates for small areas. The Fay-Herriot model basically has two components, namely, a sampling model for the direct estimates and a linking model for the parameters of interest.

The objective of this paper is to consider new improved prior on hyperparameters of variance component A of Area Level model and illustrate the usefulness of this models through an application to horticultural survey data. The paper is organized as follows. In section 2, we first study area level models including EBLUP estimators of area level model. Then in section 3 we propose Bayesian formulation of area level model with new prior for variance component A and obtain HB inference for small area parameters through the MCMC method using Laplace approximation. In section 4, we apply the proposed model to the analysis of small area data from the Horticultural Survey. We compare the performance the proposed model *i.e.* hierarchical Bayes estimate with the proposed prior (HB_{LL}) with the hierarchical Bayes estimate with uniform prior and EBLUP estimates to investigate the effects of incorporating new prior on the area-specific random effects. Bayesian model comparison and model fit analysis are also provided. Finally in section 5, we offer some concluding remarks.

Area level model:

A basic area level model assumes that the small area parameter of interest θ_i is related to area specific auxiliary data x_i through a linear model:

$$\theta_i = x_i^T\beta + v_i,\ i = 1,2,...,m \qquad ....(1)$$

where m is the number of small areas, $\beta = (\beta_1, \beta_2,...,\beta_p)'$ is p x 1 vector of regression co-efficients Further, the V_i'S are area-specific random effects assumed to be independent and identically distributed (iid) with $E_m(v_i) = 0$, $V_m(v_i) = A\ (\geq 0)$. Where E_m denotes the model expectation and V_m the model variance. This assumption can be denoted as $v_i \sim iid\ (0, A)$. Normality of the random effect v_i may also be included. The parameter A is a measure of homogeneity of the areas after accounting for the covariates x_i. The area level model assumes that there exists a direct survey estimator y_i for the small area parameter θ_i such that :

$$y_i = {}^{-}{}_i + e_i, \qquad i=1,2,...,m \qquad ......(2)$$

where, the e_i is the sampling error associated with the direct estimator y_i. We also assume that the e_i's are independent normal random variables with mean $Ep(e_i \mid \theta_i) = 0$ and sampling variance $V_p(e_i \mid \theta_i) = D_i$, it is also customary to assume that sampling variances D_i are known.

EBLUP estimators of area level model :

For the basic area level model defined above in section (2) the BLUP estimator of θ_i is given as :

$$\tilde{\theta}_i = x_i'\tilde{\beta} + \gamma_i(\theta_i - x_i'\tilde{\beta}) = \gamma_i\theta_i + (1-\gamma_i)x_i'\tilde{\beta} \qquad ....(3)$$

where,

$$\gamma_i = \frac{A}{A + D_i} \text{ and } \tilde{\beta} = \left[\sum x_i x_i' / (D_i + A)\right]^{-1}\left[\sum x_i\theta_i / (D_i + A)\right]$$

The BLUP estimator is dependent on the variance component A. However, replacing A with an asymptotically consistent estimator $\hat{A}$ yields a two stage estimator. This estimator $\hat{\theta}_i$ is called empirical BLUP or EBLUP (Jiang and Lahiri, 2006). It will remain approximately unbiased provided the distribution of v_i and e_i are both symmetrical. This EBLUP estimator of θ_i is given as:

$$\hat{\theta}_i = \hat{\gamma}_i\theta_i + (1-\hat{\gamma}_i)x_i'\hat{\beta} \qquad .....(4)$$

where, $\hat{\gamma}_i$ and $\hat{\beta}_i$ are the values of γ_i and β_i when A is replaced by $\hat{A}$. The MSE of the BLUP estimator $\tilde{\theta}_i$ can be obtained from the general results of MSE of mixed effects model or by direct calculation. It is given by :

$$MSE(\tilde{\theta}_i) = E(\tilde{\theta}_i - \theta_i)^2 = g_{1i}(A) + g_{2i}(A) \qquad ....(5)$$

where,

$$g_{1i}(A) = \frac{AD_i}{D_i + A} = \gamma_i D_i \text{ and } g_{2i}(A) = (1-\gamma_i)^2 x_i'\left[\sum_{i=1}^{m} x_i x_i'/(D_i + A\right]^{-1} x_i \qquad .(6)$$

The second order MSE approximation *i.e.* EBLUP estimate of $\tilde{\theta}_i$ is :

$$MSE(\hat{\theta}_i) = g_{1i}(A) + g_{2i}(A) + g_{3i}(A) \qquad ......(7)$$

where,

$$g_{3i}(A) = D_i^2(D_i + A)^{-3}\bar{V}(\hat{A}) \qquad ......(8)$$

Now regarding the estimation of MSE ($\tilde{\theta}_i$) the estimator of MSE is given by:

$$mse\ (\hat{\theta}_i) = g_{1i}(\hat{A}) + g_{2i}(\hat{A}) + 2g_{3i}(\hat{A}) \qquad .....(9)$$

Bayesian formulation of area level model :

The area level model, extensively used in the small area estimation literature, consists of two levels. In Level 1, a sampling model captures the sampling variability of the regular survey estimates y_i of true small area means θ_i. In Level 2, a linking model relates the true small area means θ_i to a p x 1 vector of known covariates x_i,

$$\theta_i \mid \beta, A \sim N(x_i'\beta, A), \; i = 1,2,...,m \qquad .... (10)$$

In the above model, β is a p x 1 vector of unknown regression co-efficients and A is an unknown variance component. The sampling variances, D_i's are assumed to be known, though in practice they are estimated by some suitable method.

In the Bayesian analysis, θ_i is estimated by its posterior mean E (θ_i | y) and the associated uncertainity is measured by the posterior variance, V (θ_i | y). In small area estimation, usually the main objective is to draw inferences about the high-dimensional parameters, *i.e.* θ_i. However, as an intermediate step, estimation of the low-dimensional parameters β and A, usually referred to as hyperparameters, is also of importance. In the Bayesian implementation of the area level model, a prior distribution, often a vague or noninformative prior, is assumed on the hyperparameters. *i.e.*

p (|A)=~1; (,A) ČR^p x [1,ã] .....(11)

The prior distribution (11) for the hyperparameters is simple to interpret and is often recommended. The uniform prior for A is non-informative and yields a posterior distribution of A for which the mode is identical to the residual maximum likelihood (REML) estimator of A (Harville, 1977 and Berger, 1985).

When the hyperparameters β and A are known, the posterior distribution of θ_i is normal with mean and variance given by:

$$E(\theta_i \mid y, A, \beta) = (1 - B_i) y_i + B_i x_i'\beta \qquad .....(12)$$

$$V(\theta_i \mid y_i, A, B) = D_i(1 - B_i) \qquad .....(13)$$

where, $B_i = \frac{D_i}{(A + D_i)}$ is the shrinkage factor which shrinks the direct estimates to a regression surface. Note that the right hand side of (12) is essentially the best predictor (BP) of θ_i, being the conditional mean of θ_i, given data, assuming known hyperparameters. Under the Bayesian approach, to estimate θ_i along with a reliable measure of precision, we need to obtain E (θ_i | y) and V (θ_i | y) . To this end, we first find the conditional posterior distribution of β, given A, and then the posterior distribution of A.

When A is known, the uniform prior on β in R^p, the p-dimensional real space, yields the following posterior distributions for β and θ_i :

$$\beta \mid y, A \sim N[\hat{\beta}_A, \Sigma_A] \qquad ....(14)$$

$$\theta_i \mid y_i, A \sim N[(1 - B_i) y_i + B_i x_i' \hat{\beta}_A, D_i(1 - B_i) + B_i^2 x_i' \Sigma_A x_i] \qquad ....(15)$$

where, $\hat{\beta}_A = (X'W_A X)^{-1}(X'W_A y), \Sigma_A = (X'W_A X)^{-1}$ with W_A= diag [1/(A+D_i)], X is the m x p matrix of area-level covariates and y is the m x 1 vector of small area direct estimates. The subscript A in both $\hat{\beta}_A$ and W_A indicates the dependence of the terms on A. In practice, the variance component A is unknown. In an empirical best linear unbiased prediction (EBLUP) or empirical Bayes approach, no prior distribution is assumed on the variance component A, which is estimated using some suitable classical method. Three most common methods of variance component estimation are widely used in the small area estimation. These are analysis of variance (ANOVA, Prasad and Rao, 1990), the method of moments (Fay and Herriot, 1979; Pfeffermann and Nathan, 1981 and Datta *et al.*, 2005) and likelihood-based method. All of these methods can produce unreasonable estimate of A *i.e.* they can produce negative or zero estimate of A for a given data set. When $\hat{A} = 0$, we come up with several unreasonable implications on the estimators and its measure of uncertainty (Bell, 1999). That is why we intend to implement our proposed Bayesian model, which involves new prior on A, starting from the choice $p(A) = A|X'W_A X|^{1/2}$ by simple approximations, motivated from the application of Laplace's method to ratio of integrals. To this end, note that for the balanced case *i.e.* when the sampling variances are equal $(D_i = D, \forall i)$, we have:

$$p(A) \propto \frac{A}{(A + D)^{p/2}}, A \varepsilon [0, \infty]$$

Our choice of prior for the unequal variance case is heuristically motivated by the equal sampling variance case. If we replace D_i, i =1,...m by a particular representative value d_0 (say, the median of D_i) W_A in and then p(A) becomes $\frac{|X'X|^{1/2}}{(A + d_0)^{p/2}}, A \in [0, \infty]$. Simplifying it further, we obtain

$$p_{LL}(A) \propto \frac{A}{(A + d_0)^{p/2}}, A \in [0, \infty] \qquad .....(16)$$

Note that the prior proposed in (16) does not depend on the individual sampling variance, Di unlike (Datta *et* *al.*, 2005). We use the notation $P_{LL}(A)$ for this prior, since it is motivated from the adjustment term given by (Li and Lahiri, 2008).

The posterior moments of β_i and θ_i under the priors described above, are not in a closed-form, but can be obtained either by numerical integration or by Monte Carlo Markov Chain (MCMC) method (Spiegelhalter *et al.*,1997). We have written a programme, using the R2JAGS package in R (R Development Core Team, 2008), that allows the hierarchical Bayes analysis for our new prior using the MCMC method the codes for the model are given in the introduction chapter. But its slow computation speed does not permit its evaluation by repeated use in simulation. Thus, for convenient implementation and evaluation of our hierarchical Bayes method, we approximate the posterior moments of β_i and θ_i using Laplace's method.

Laplace approximation :

Laplace's method is a technique of classical applied mathematics and very useful for asymptotic evaluation of integrals. This remarkable method provides accurate approximations to the posterior means and variances of any real function of parameter vector θ in Bayesian analysis. Posterior moments can be expressed as ratio of integrals and the application of Laplace's method to ratio of integrals leads to accurate approximation for the posterior moments. This method has been applied by many authors in the context of Bayesian analysis (Tierney and Kadane, 1986; Tierney *et al.*, 1989; Kass and Staffey, 1989; Butar and Lahiri, 2002 and Datta *et al.*, 2005). Following (Kass and Staffey, 1989) the first order approximation to the posterior mean and variance of under the area level model described above in section-2 is given by:

$$g_i(A) = E(\theta_i \mid y, A) = (1 - B_i)y_i + B_i x_i' \hat{\beta}_A \quad ....(17)$$

and

$$h_i(A) = V(\theta_i \mid y, A) = D_i(1 - B_i) + B_i^2 x_i' \textstyle\sum_A x_i \quad ......(18)$$

Using iterative expectation technique on $E(\theta_i | y_i,)$, we write the first order approximation to the posterior mean θ_i of as:

$$E(\theta_i \mid y) = E\{g_i(A) \mid y\} = E(\theta_i \mid y, \hat{A}) + O(m^{-1}) \quad ......(19)$$

Using similar iterative expectation and variance technique on $V(\theta_i | y_i, A)$, the first order approximation to the posterior variance can be written as:

$$V(\theta_i \mid y) = E\{h_i(A) \mid y\} + V\{g_i(A) \mid y\}$$

$$= \left[V(\theta_i \mid y, \hat{A}) + O(m^{-1})\right] + \left[\{E'(\theta_i \mid y, A)\}^2_{A=\hat{A}} \frac{1}{i_0} + O(m^{-2})\right] \quad ....(20)$$

$$= D_i(1 - B_i) + \hat{B}_i^2 x_i' \textstyle\sum_{\hat{A}} x_i + \left\{y_i - x_i' \hat{\beta}_A + (A + D_i x_i' u\right\}^2_{A=\hat{A}} V(B_i \mid y) + O(m^{-1}) \quad ....(21)$$

where, $\hat{\beta}_i$ and $\Sigma_{\hat{A}}$ are $_i$ and Σ_A evaluated at $\hat{A}; u = \frac{\partial \hat{\beta}_A}{\partial A}$. The first order approximation of the posterior variance given by (21) has three clearly defined terms. The first term, $T_1 = D_i (1-B_i)$, measures the uncertainty in the model for estimating θ_i , the second term, $T_2 = \hat{B}_i^2 x_i' \sum_{\hat{A}} x_i$, measures the uncertainty in the estimation of β and the third term,

$$T_3 = \left\{y_i - x_i' \hat{\beta}_A + (A + D_i x_i' u\right\}^2_{A=\hat{A}} V(B_i \mid y) + O(m^{-1}),$$

accounts for the uncertainty in estimating A. In many applications, the third term is quite small and often ignored (Bell, 1999). Kass and Staffey (1989) emphasized the importance of the third term in their data analysis. They considered the standard form for the first order Laplace approximation under a similar kind of model as area level model, but with uniform prior for the variance component, under the label CIHM (conditionally independent hierarchical models).

Numerical illustration:

In this section, we compare frequentist and Bayesian approaches with our proposed Bayesian method. The conceptual difference between a frequentist and a Bayesian analysis are avoided by studying the frequentist properties of the estimators under two approaches. The frequentist properties of estimators are derived from the distribution of the estimators under repeated simulations of observations following the statistical model with known hyperparameters. We investigate the small sample ($m = 12$) frequentist properties of our proposed prior $p_{LL}(A)$ in estimating A, B_i and θ_i, $i=1,...,m$, using a Monte Carlo simulation study. In the tables, we denote the hierarchical Bayes methods resulting from $p_{LL}(A)$ by HB_{LL}. This is generic and is used for estimation of any parameter. The hierarchical Bayes methods are approximated using the Laplace approximation already discussed. For inference about the small area means θ_i we compare our Bayesian estimator with empirical best linear unbiased predictors using REML and Heirarchical Bayes (uniform gamma prior) estimators of A. The values of D_i are assumed to be know for all 12 small area. They are actually obtained as variances of direct estimator.

Data collected through pilot survey conducted by the Division of Agri-Statistics on estimation of area and yield of apple in district Baramulla has been used for the purpose of our proposed small area estimation. The district Baramulla comprises of 12 blocks *viz.*, Zanigeer, Boniyar, Tangmarg, Wagoora, Sopore, Baramulla, Uri, Pattan, Rohama, Singphora, Rafiabad and Kunzer. Each block consists of different number of villages. A fixed number of five villages were selected at random from each block by simple random sampling. The data set was named apple-1 for analysis and modeling in R/SAS software's. It has 61 rows and 7 columns. The columns names are blocks, N, n, yield, area, trees, actual yield for names of blocks, total number of villages in each block, number of villages selected from each block, yield of apple from each selected village in metric tons, area under apple orchards and total number of apple trees in each of the selected village, actual yield obtained as per census records. This data set has been used for unit level estimation.

Comparison of different estimators :

The performance of different estimators is examined from the accuracy of the point estimates. This is considered through the relative bias and absolute relative bias of different estimators. The different estimators are compared according to four different criteria recommended by the panel on small area estimates of population and income set up by the United States committee on National Statistics (1978); Datta *et al.* (2002) and Ghosh *et al.* (1996). We compare different estimators on the basis of average relative bias, average squared relative bias, average absolute bias and average squared deviation. Suppose act_i denotes the true value of the variable for the ith small area and est_i is any estimate of act_i i=1,2,...,m. Then average relative bias:

$$\mathbf{ARB} = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{est_i - act_i}{act_i}\right|$$

average squared relative bias:

$$\mathbf{ASRB} = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{est_i - act_i}{act_i}\right)^2$$

average absolute bias:

$$\mathbf{AAB} = \frac{1}{m}\sum_{i=1}^{m}\left|est_i - act_i\right|$$

average squared deviation :

$$\mathbf{ASD} = \frac{1}{m}\sum_{i=1}^{m}\left(est_i - act_i\right)^2$$

Now using the above four criteria on the apple data set already discussed in chapter-1. A Comparison of HB estimates using proposed prior with EBLUP estimates and HB estimates with uniform prior is made using above discussed four different criteria and is reported in Table 1.

It is clear from the Table 1 that hierarchical Bayes estimate with the proposed prior *i.e.* HB_{LL} performed significantly better than the hierarchical Bayes estimate with uniform prior and EBLUP estimates in terms of all the four criterion. This motivates us to argue that our proposed prior is superior even in the cases where the standard hierarchical Bayes estimate with uniform prior seems appropriate. Also the average relative error is 2.51 per cent with HB_{LL} compared to 3.53 per cent for HB and 4.71 per cent for EBLUP.

Now an empirical comparison of EBLUP and HB estimates with uniform and proposed prior for all the small areas separately are made using per cent absolute relative bias (ARB) and absolute bias (AB) and the results are depicted in Table 2.

From the values in the Table 2 it is evident that HB_{LL} appears to be a good compromise as it exhibits smaller errors and a lower incidence of extreme error than either of the HB or EBLUP estimates. Thus it can be concluded that in terms of ARB and AB the performance of the HB_{LL} is the best.

Fig.1 shows the comparison of the values of per cent relative bias and absolute bias for EBLUP,

Table 1 : Comparison estimates using different criteria

Estimates \ Criteria	ARB	ASRB	ABS	ASD
EBLUP	0.047	0.0051	78.34	16444.12
HB	0.0353	0.0037	62.59	12517.53
HB_{LL}	0.0251	0.0029	47.42	9642.63

hierarchical Bayes and proposed Heirarchical Bayes (HB_{LL}).

Fig.1 plots the values of per cent ARB and AB of EBLUP, hierarchical Bayes and proposed Heirarchical Bayes against small areas. Significant disparity is observed

Table 2: Empirical comparison of estimators

Estimators / Small areas	EBLUP		HB		HB_{LL}	
	ARB	AB	ARB	AB	ARB	AB
1	0.79	23.72	0.51	15.31	0.24	7.25
2	1.20	8.13	0.55	3.85	0.28	1.95
3	2.18	12.57	0.77	4.43	0.14	0.81
4	2.32	20.25	1.41	12.33	0.10	0.9
5	4.23	155.43	3.64	133.99	2.37	87.14
6	3.37	24.04	1.83	13.01	0.06	0.41
7	3.37	18.76	1.10	6.83	0.07	0.433
8	16.81	285.17	13.50	228.96	12.27	208.06
9	16.33	293.04	15.57	279.34	14.18	254.43
10	2.08	36.25	1.57	27.39	0.29	4.99
11	2.00	48.11	0.76	18.3	0.11	2.53
12	2.23	14.58	1.12	7.33	0.02	0.1

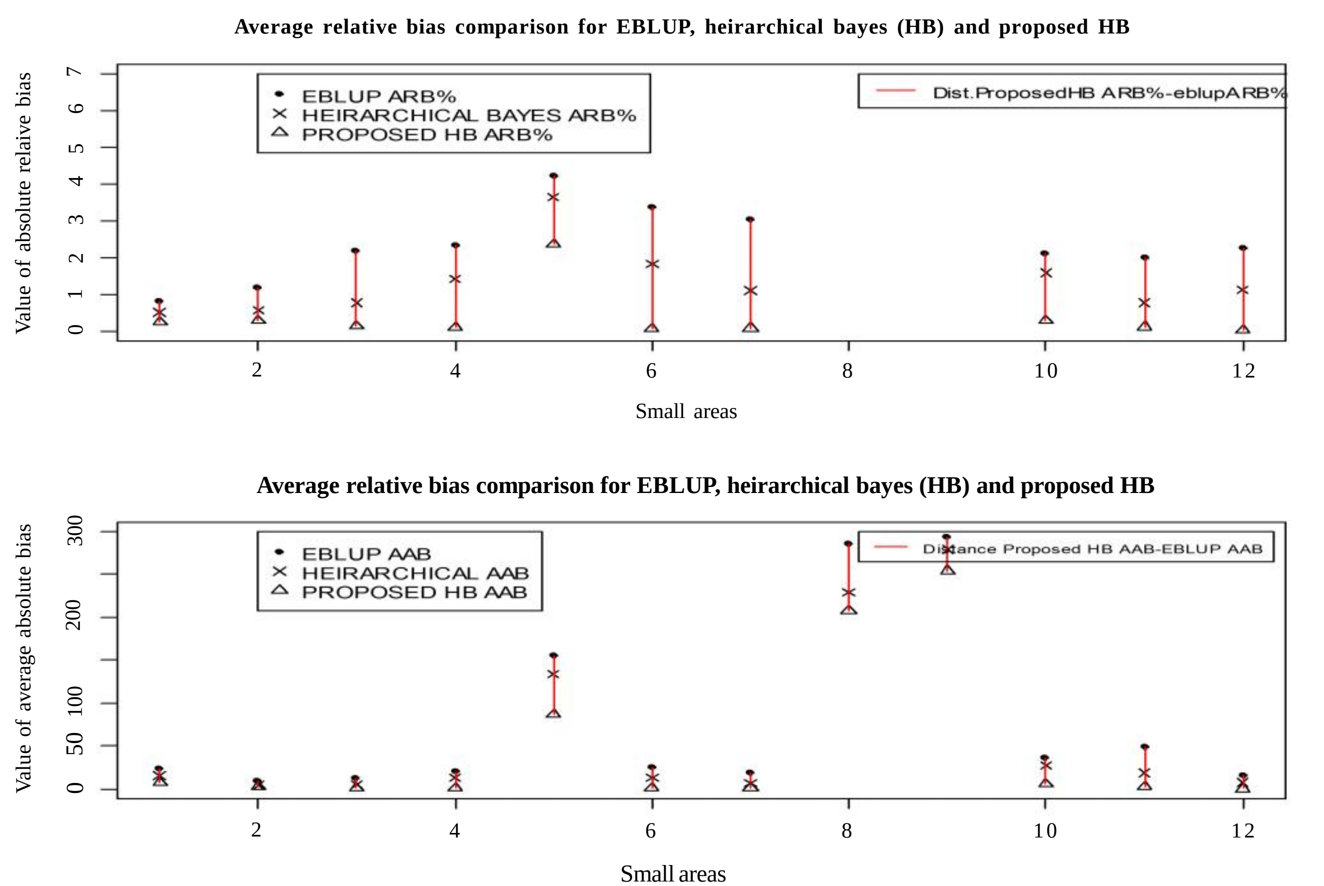

Fig. 1 : Comparison of per cent average relative bias and absolute bias for EBLUP, HB and HB_{LL}

among the three estimators. HB_{LL} performed better by providing the lowest value of both per cent ARB and AB for each of the small areas compared EBLUP and hierarchical Bayes.

Mean square error (MSE) of estimators of variance components for 12 small areas are reported below in Table 3.

From the Table 3 it is clear that on the basis of MSE the performance of HB_{LL} is better than EBLUP and HB for all the small areas. Or in other word we can emphatically say that the HB_{LL} is the best technique than the other two. The value of mse obtained by HB_{LL} is as low as 28.19, compared to 30.05 and 60.03 in HB and EBLUP, respectively.

Table 4 reports the EBLUP, HB and HB_{LL} estimates and their associated standard errors for all the 12 small areas separately. It is clear from the table that HB_{LL} estimates are better than HB and EBLUP estimates. In terms of standard error also HB_{LL} performed better than the HB and EBLUP estimates for all the 12 small areas. Also we can see from the results that the estimates of the yield obtained by HB_{LL} technique are very much closer to the actual value of yield obtained.

Fig.2 plots the point estimates of θ_i against the

Table 3 : MSE of estimators of variance components

Small areas \ Estimators	EBLUP	HB	HB_{LL}
1	6381.78	5919.82	5128.00
2	631.16	552.19	506.88
3	158.44	128.81	109.52
4	118.45	95.21	82.64
5	10394.80	8848.25	7931.26
6	60.03	30.05	20.19
7	145.26	110.61	95.82
8	1578.17	1265.80	1080.61
9	312.97	208.01	165.87
10	783.00	700.00	650.18
11	1099.67	995.87	806.51
12	148.28	134.21	108.31

Table 4 : EBLUP and HB estimates and associated standard errors

Small areas \ Estimators	EBLUP		HB		HB_{LL}		Actual value
	Estimate	S.E	Estimate	S.E	Estimate	S.E	Yield
1	3030.52	79.14	3022.11	76.28	3014.05	70.91	3006.8
2	690.81	25.04	695.09	23.45	696.96	22.40	698.94
3	563.97	12.5	572.11	11.34	575.73	10.46	576.54
4	854.19	10.87	862.11	9.75	873.54	9.09	874.44
5	3521.17	99.22	3542.61	88.57	3589.46	81.43	3676.6
6	736.84	7.71	725.81	5.48	713.21	5.01	712.8
7	639.94	12.02	628.01	10.51	621.613	9.78	621.18
8	1981.31	35.80	1925.10	34.14	1904.20	31.31	1696.14
9	1501.70	9.22	1515.35	8.37	1540.30	8.11	1794.74
10	1777.11	27.77	1768.25	26.45	1745.85	25.49	1740.86
11	2355.29	32.46	2385.10	31.39	2405.93	28.39	2403.4
12	638.62	12.17	645.87	11.58	653.10	10.40	653.2

small areas and also provides a comparison of these values with the actual value of yield obtained in each of the small area.

Fig.2 displays EBLUP, HB and HB_{LL} estimates and their deviation from the actual mean. Here we can see that the values of θ_i obtained by HB_{LL} are more closer to actual values as compared to HB and EBLUP. Thus, for the plot also we conclude that among the three techniques discussed the HB_{LL} is the best technique for obtaining the estimates.

Table 5 repots the relative contribution of the three terms to the posterior variance of θ_i obtained using the prior $p_{LL}(A)$. The three columns T_1, T_2, T_3 exhibits the relative contribution of term1, term 2, term 3, respectively. From the values in the table we conclude that the contribution of the term which accounts for the uncertainty in estimating the variance component is substantially small relative to the first term which accounts for the uncertainty in the model in estimating the small area means.

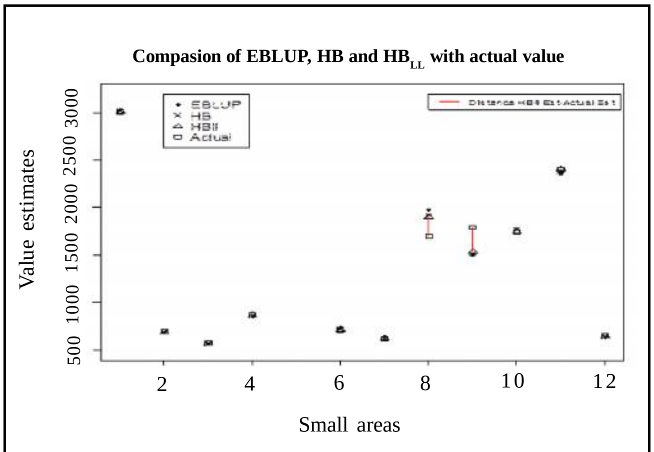

Fig. 2 : **EBLUP, HB and HB_{LL} estimates compared to the true means**

Conclusion:

In this paper Bayesian implementation of area level model is carried out and new prior is proposed on the variance component A. Besides being simple, this prior has two main advantages. It removes the possibility of yielding zero estimates for the variance component; the popular choice of uniform prior on A suffers from this drawback if posterior mode is considered as an estimator. This prior also enjoys good small sample frequentist properties; real agricultural study results justify this conclusion. Also, in order to have closed form expressions of the posterior mean and variance of the true small area mean, Laplace approximation to ratio of integrals, following Kass and Steffey (1989) is being used. To illustrate the method numerically a real data set on apple has been used and the results showed that the Bayes estimators (with

Table 5 : Relative contribution of the three terms to the posterior variance of θ_i using prior $p_{LL}(A)$ on A

Small areas	$T_1^1(g_{i1})$	$T_2^2(g_{i2})$	$T_3^3(g_{i3})$
1	4678.13	160.58	125.85
2	504.15	0.5500	1.10
3	108.85	0.0054	0.1031
4	81.64	0.0075	0.0571
5	6851.22	521.01	281.34
6	24.85	0.0023	0.0147
7	94.82	0.0066	0.0768
8	1030.61	15.18	17.59
9	164.55	0.0322	0.0601
10	643.11	1.95	2.70
11	785.10	5.44	5.01
12	106.31	0.0066	0.0900

[1] Uncertainty in the model in estimating $\bar{}_i$

[2] Uncertainty in estimating co-efficient

[3] Uncertainty in model in estimating the variance component A

new prior) of small area means have good frequentist properties such as MSE and ARB as compared to other traditional methods *viz.*, Direct, Synthetic and Composite estimators.

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