



Research Paper

Bayesian multiple deferred sampling plan BMDS (0,1) with weighted poisson model using Golub's minimum risks method

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ABSTRACT : Acceptance sampling plans by attributes involve sampling from the weighted poisson distribution and the non-conforming process of average fraction, following a gamma distribution are considered in this article. Our work presents a new procedure for the selection of bayesian multiple deferred state sampling plan (BMDSP) through average probability of acceptance (APA) with weighted poisson distribution (WPD) as a base line distribution and reduced risk. In constructing sample plan, we propose a procedure for constructing a bayesian MDSP using WPD and developed a technique to determine the parameters of the plan by ensuring a specific required protection to both producers and consumers. The performance power of the weighted poisson BMDSP is also discussed by determining the operating characteristic (OC) curve. which are developing under the producer's and consumer's risk for specified acceptable and limiting quality levels, a gamma prior distribution is baseline distribution. The procedure is given for BMDSP with the weighted poisson distribution for given $(\mu_1, 1-\alpha)$ and (μ_2, β) .

KEY WORDS: Bayesian MDS-1 (0, 1), Weighted poisson distribution, Minimum risks plan, Acceptable quality level (AQL), Limiting quality level (LQL)

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INTRODUCTION :

Sampling involves a risk that the sample may not adequately reflect the condition of the lot. i.e., it may not represent the lot correctly. Sampling risks are of two kinds (i) Producer's risk (ii) Consumer's risk. The concept of the Multiple Dependent (or deferred) State (MDS) sampling plan was introduced by Wortham and Baker (1976). MDS sampling plan belongs to the group of conditional sampling procedures. In these producers,

acceptance or rejection of a lot is based not only on the sample from that lot, but also on sample result from past or future lots. Wortham and Baker's MDS sampling plan a specified into four parameters such as n , m , c_1 and c_2 . Single sampling plans involving a minimum sum of risks for the binomial model for the OC curve can be found from Golub's (1953) tables. Soundararajan (1981) has extended Golub's approach to a single sampling under the conditions of the poisson model for the OC curve. The drawback of Golub's approach is that the sampling

plan involving a minimum risks may still result in larger producer's and consumer's risks. Since minimizing the sum of risks is a desirable feature (as it leads to a better shouldered OC curve), one may attempt to design sampling plans involving smaller producer's and consumer's risks. Soundararajan and Govindaraju (1983) have, therefore, modified the Golub's approach of minimizing the sum of risks such that the producer's and consumer's risks are below the specified levels (e.g. 0.01, 0.05, etc.,) in the case of single sampling plans. For the selection of MDS-1 (c_1, c_2) sampling plans Soundararajan and Vijayaraghavan (1990) presented tables for the given AQL and LQL for the purpose of fixed values of α and β . As an extension of their work this presents a table and procedure in order to find the MDS- 1 (c_1, c_2) sampling plans of Wortham and Baker which is involving minimum sum of risks for given AQL and LQL devoiding the fixation of producer's and consumer's risk. Latha and Subbiah (2015) have studied the selection of Bayesian Multiple deferred state (BMDS-1) sampling plan based on quality regions. Latha and Subbiah (2014) have given the procedure for the selection of multiple deferred state sampling (MDS – 1) plan through the weighted Poisson model with gamma prior.

In this paper a selection procedure of Bayesian Multiple Deferred Sampling Plan – (1, 2) is studied using Weighted Poisson Model with minimum risks.

MDS – 1 plan :

The MDS –1 plan is applicable to the case of Type B situations where lots expected to be of the same quality are submitted for inspection seriously in the lot production. MDS –1 plans are extensions of chain sampling plans of Dodge's (1955) type ChSP – 1. Both the MDS – 1 and chain sampling plans achieve a similar reduction in sample size when compared to the unconditional plans, such as single and double sampling plans. The operating procedure of the MDS – 1 plan as given by.

– From each submitted lot, select a sample of n units and test each unit for conformance to the specified requirements.

– Accept the lot if x , the observed number of non-conformities, is less than or equal to c_1 ; reject the lot if x is greater than c_2 .

– If $c_1 < x < c_2$ accept the lot, provided in each of the sample taken from the preceding or succeeding m lots, the number of non-conformities found is less than or equal

to c_1 . The lot otherwise rejected.

Weighted poisson distribution:

In the construction of acceptance sampling plan, size- biased version of random variable about defectives play an important role. The weighted distributions are more suitable distributions than the classical distributions like binomial, poisson and negative binomial. The weighted poisson distribution plays an important role in acceptance sampling, mainly in the construction of sampling plans. Each outcome (number of defectives) is specific but can be assigned different weights based on its importance or usage. The probability mass function of weighted Poisson distribution is given by:

$$p(x, n, p, k) = \frac{x^k p(x)}{\sum x^k p(x)} \quad x = 1, 2, 3 \dots \dots \dots (1)$$

where,

$$p(x) = \frac{e^{-np} (np)^x}{x!} \quad x = 1, 2, 3 \dots \dots \dots (2)$$

Here X^k is the corresponding weight for each outcome and 'k' is a constant. The poisson distribution can be seen as the particular case of the weighted poisson distribution when $k = 0$. The probability mass function of the weighted Poisson distribution for $k = 1$ is :

$$p(x) = \frac{e^{-np} (np)^{x-1}}{(x-1)!} \quad x = 1, 2, 3 \dots \dots \dots (3)$$

When p follows gamma prior distribution with density function:

$$(p) = \frac{e^{-pt} t^p p^{s-1}}{\Gamma s}, \quad p > 0, s, t > 0 \dots \dots \dots (4)$$

where, s and t are the parameters and the mean value of distribution $= \frac{s}{t}$.

Mathematical model:

Bayesian MDS-1 plan:

Based on Hald (1981), APA functions for MDS-1 plan with gamma poisson distribution is obtained as:

$$\bar{P} = \bar{P}c_1 + [\bar{P}c_2 - \bar{P}c_1] [\bar{P}c_1]^m \dots \dots \dots (5)$$

where,

$$\bar{P}c_1 = \sum_{x=1}^{c_1} \frac{1}{\beta(x, s-1)} \frac{s^s}{(s-1)} \frac{\mu^{x-1}}{(\mu+s)^{x+s-1}}; \quad x = 1, 2, 3, \dots$$

$$\bar{P}c_2 = \sum_{x=1}^{c_2} \frac{1}{\beta(x, s-1)} \frac{s^s}{(s-1)} \frac{\mu^{x-1}}{(\mu+s)^{x+s-1}}; \quad x = 1, 2, 3, \dots$$

The average probability of acceptance is given by:

$$\bar{P} = \frac{s^s}{(s + n\mu)^s} + \frac{s^{ms+s+1} n\mu}{(s + n\mu)^{ms+s+1}} \dots\dots\dots(6)$$

Selection of BMDS-1 (0,1) plan given operating ratio (r, s) and s :

- Calculate the operating ratio $\frac{\mu_2}{\mu_1}$
- The value $\frac{\mu_2}{\mu_1}$, is entered it into Table 2 in the row

headed by $\frac{\mu_2}{\mu_1}$, which is equal to or just greater than the computed ratio.

- Based on row identification in step 2 the values of parameter c_1, c_2 and m of the MDS-1 plan are obtained.
- By identifying the column heading corresponding to the parameter c_1, c_2 and m in step 3, the sample size n is obtained.

For example:

- Given $\alpha = 0.05, \beta = .10, \mu_1 = 0.01, \mu_2 = 0.30$ and $s=2$ determine the minimum risks BMDS-1 (0,1) plan solution for the specified $\mu_1, \mu_2, \frac{\mu_2}{\mu_1} = 30$, from Table 2.

- It is observed that the tabulated $\frac{\mu_2}{\mu_1} = 30.99347$ is nearer to the calculated value 30.

- Corresponding to $\frac{\mu_2}{\mu_1} = 30.99347, s=2$ from Table 2, $m=3$

- From Table 2, corresponding to $\frac{\mu_2}{\mu_1} = 30.99347, n\mu_2 = 0.1391$, which gives $n = \frac{n\mu_1}{\mu_1} = \frac{0.1391}{0.01} = 13.9 \sim 14$

Hence, BMDS-1 (0,1) plan selected will have the sample size 14 and $m=3$.

Selection of BMDS-1 (0,1) plan for minimum sum of risks :

Suppose that v_1 and v_2 are the weights considered such that $v_1 + v_2 = 1$, then $v_1 \alpha + v_2 \beta$ can be minimized for obtaining the parameters of the required plan. Instead of minimizing $v_1 \alpha + v_2 \beta$ the expression $\alpha + v\beta$ can be

minimized, where $v = \frac{v_2}{v_1}$ is the index of relative importance given to the consumer's risk in comparison with the producer's risk. When $v > 1$, the plan obtained

will be more favourable to the consumer compared to the equal weights plan. When $v < 1$, it will be more favourable to the producer than the equal weights plan.

To obtain optimum sample size:

Minimizing $\alpha + v\beta = \bar{P}_{\mu_1}(R) + \bar{P}_{\mu_2}(A)$

is equivalent to minimizing $v\bar{P}_{\mu_2}(R) - \bar{P}_{\mu_1}(A) \dots\dots(7)$

The acceptance quality level (AQL) and limiting quality level (LQL) corresponding to APA curve are referred as μ_1 and μ_2 , respectively. The AQL and LQL are usual quality levels in OC curve corresponding to the probability acceptance $1 - \alpha = 0.95$ and $\beta = 0.10$, respectively. When sample size n is fixed the minimum value of expression is obtained with

$\bar{P}(\mu) < \bar{P}(\mu - 1)$ and $\bar{P}(\mu) > \bar{P}(\mu + 1) \dots\dots\dots(8)$

Substituting and μ_1 and μ_2 in eq. 6 we get,

$$\bar{P}_{\mu_1} = \frac{s^s}{(n\mu_1 + s)^s} + \frac{n\mu_1 s^{ms+s+1}}{(n\mu_1 + s)^{ms+s+1}}$$

$$\bar{P}_{\mu_2} = \frac{s^s}{(n\mu_2 + s)^s} + \frac{n\mu_2 s^{ms+s+1}}{(n\mu_2 + s)^{ms+s+1}} \dots\dots\dots(9)$$

Substituting (10) in (8), we get

Let $f = v\bar{P}_{\mu_2} - \bar{P}_{\mu_1} \dots\dots\dots(10)$

$$\Rightarrow f = s^s [v(n\mu_2 + s)^{-s} - (n\mu_1 + s)^{-s}] + s^{ms+s+1} [v n\mu_2 (n\mu_2 + s)^{-ms+s+1} - n\mu_1 (n\mu_1 + s)^{-ms+s+1}]$$

The minimum value of n can be determined from the following expression

$$v \frac{\mu_2}{\mu_1} \left[\frac{(s + n\mu_2)^{ms+1} + [(n + 1)n\mu_2 - 1]s^{ms+1}}{(s + n\mu_1)^{ms+1} + [(n + 1)n\mu_1 - 1]s^{ms+1}} \right] - \left[\frac{s + n\mu_2}{s + n\mu_1} \right]^{ms+s+2} = 0 \dots\dots(11)$$

Table 3 is constructed using eq. (12) minimizing $\alpha + v\beta$, for given s, m, μ_1, μ_2 .

Example:

Obtain the sample size for the BMDS-1(1,2) plan minimizing $\alpha + \beta, s=1, m=1, \mu_1 = .05, \mu_2 = .1$.

Solution:

From Table 3 it is observed that from $\alpha + v\beta$, for given $\mu_1 = .05, \mu_2 = .1, s=1$, and $m=1$ the optimum sample size is 20.

To obtain acceptance criteria:

Under the conditions for application of weighted poisson model, for given n, AQL and LQL, the expression

for m minimizing the sum of producer's and consumer's risks whose value (integer nearest to) is:

$$m = -\frac{1}{2} - \frac{1}{s} + \frac{1}{s} \frac{\ln \left[\frac{n\mu_2 (s^s - (n\mu_2 + s)^s)}{n\mu_1 [(s^s - (n\mu_1 + s)^s)]} \right]}{\ln \left[\frac{s + n\mu_2}{s + n\mu_1} \right]} + \frac{1}{2} \frac{\ln \left[\frac{s + n\mu_1}{s + n\mu_2} \right]}{\ln \left[\frac{s + n\mu_2}{s + n\mu_1} \right]} \dots\dots(12)$$

where $n\mu_1 = x_1$ and $n\mu_2 = x_2$. The derivation for this expression for m is given in Appendix.

Appendix: Derivation of the expression for 'm':

The operating characteristic function of MDS-1(c_1, c_2), according to Varest (1982) is given by

$$\bar{P} = \bar{P}(\mu, n, c_1) + [\bar{P}(\mu, n, c_2) - \bar{P}(\mu, n, c_1)]\bar{P}^m(\mu, n, c_1) \dots\dots(13)$$

where \bar{P} = probability of acceptance of lot with proportion defective, $\bar{P}(\mu, n, c_i)$ = probability of finding c_i defectives in a sample of n units from product of quality p, where i:1, 2.

Let \bar{P}_1 and \bar{P}_2 be AQL and LQL and let their corresponding risks be α and β . For the condition to apply weighted poisson model, choose $c_1=1$ and $c_2=2$ and the eq. (13) becomes:

$$\bar{P} = \frac{s^s}{(s-1)(s+s)^s} \left[s-1 + \frac{\epsilon}{(s+s)^{ms+1}} s(s-1) \right] \dots\dots\dots(14)$$

$$= \frac{s^s}{(s+s)^s} + s \frac{\epsilon s(s-1)^m}{(s+s)^{ms+1}}$$

$$\bar{P} = \frac{s^s}{(s+s)^s} + \frac{\epsilon}{(s+s)^{ms+1}} s^{s+1} (s-1)^m$$

The best value for m can be chosen for a given n such that the sum of producer's and consumer's risk is a minimum. Alternatively, the best value of m is that value which maximizes the expression:

$$f(m) = \bar{p}(\mu_1) + 1 - \bar{p}(\mu_2) \dots\dots\dots(15)$$

where, $\bar{p}(\mu_1)$ and $\bar{p}(\mu_2)$ reads:

$$\bar{P}(\mu_1) = \frac{s^s}{(n\mu_1 + s)^s} + \frac{n\mu_1}{(n\mu_1 + s)^{ms+s+1}} s^{ms+s+1}$$

$$\bar{P}(\mu_2) = \frac{s^s}{(n\mu_2 + s)^s} + \frac{n\mu_2}{(n\mu_2 + s)^{ms+s+1}} s^{ms+s+1} \dots\dots\dots(16)$$

For calculation simplicity, rescaling to suitable parameters:

$$\text{Let, } \epsilon = n\mu, \omega = s^s \Rightarrow \omega^m = s^{ms}, \lambda s = s^{s+1},$$

$$f_0 = s^s \left[\frac{1}{(\epsilon_1 + s)^s} - \frac{1}{(\epsilon_2 + s)^s} \right] + 1 \dots\dots\dots(17)$$

Eq. (17) can be written as:

$$f(m) = f_0 + \lambda_s \omega^m \left[\frac{\epsilon_1}{(\epsilon_1 + s)^{ms+s+1}} - \frac{\epsilon_2}{(\epsilon_2 + s)^{ms+s+1}} \right] + 1 \dots\dots\dots(18)$$

Shifting m to m-1, the above equation reads:

$$f(m-1) = f_0 + \lambda_s \omega^{m-1} \left[\frac{\epsilon_1}{(\epsilon_1 + s)^{ms+1}} - \frac{\epsilon_2}{(\epsilon_2 + s)^{ms+1}} \right] \dots\dots\dots(19)$$

For maximum at m, it must satisfy the following inequality.

$$f(m) < f(m-1) \dots\dots\dots(20)$$

$$\delta_{1,m} - \delta_{2,m} > \omega(\delta_{1,m+1} - \delta_{2,m+1})$$

$$\delta_{1,m} - \omega\delta_{1,m+1} > (\delta_{2,m} - \omega\delta_{2,m+1}) \dots\dots\dots(21)$$

Simplifying, the above expression which reads:

$$m > -1 - \frac{1}{s} + \frac{1}{s} \ln \frac{\frac{\epsilon_2 \left[\frac{\epsilon_2 + s}{\epsilon_1 + s} \right]^s - s^s}{\epsilon_1 \left[\frac{\epsilon_2 + s}{\epsilon_1 + s} \right]^s - s^s}}{\ln \left[\frac{\epsilon_2 + s}{\epsilon_1 + s} \right]} + \frac{\ln \left[\frac{\epsilon_1 + s}{\epsilon_2 + s} \right]}{\ln \left[\frac{\epsilon_2 + s}{\epsilon_1 + s} \right]} \dots\dots\dots(22)$$

Similarly equation (22) can be rewritten as:

$$\omega\delta_{1,m} - \delta_{1,m} < \omega\delta_{2,m} - \delta_{2,m} - 1 \dots\dots\dots(23)$$

$$\delta_{1,m} = \frac{\epsilon_1}{(\epsilon_1 + s)^{ms+s}} \text{ and } \delta_{1,m} - 1 = \frac{\epsilon_1}{(\epsilon_1 + s)^{ms-s+t}}$$

$$\delta_{2,m} = \frac{\epsilon_2}{(\epsilon_2 + s)^{ms+t}} \text{ and } \delta_{2,m} - 1 = \frac{\epsilon_2}{(\epsilon_2 + s)^{ms-s+t}} \dots\dots\dots(24)$$

On simplification, it reduces to:

$$m < -1 - \frac{1}{s} + \frac{1}{s} \ln \frac{\left[\frac{\epsilon_2 \left[\frac{\epsilon_2 + s}{\epsilon_1 + s} \right]^s - s^s}{\epsilon_1 \left[\frac{\epsilon_2 + s}{\epsilon_1 + s} \right]^s - s^s} \right]}{\ln \left[\frac{\epsilon_2 + s}{\epsilon_1 + s} \right]} + \frac{\ln \left[\frac{\epsilon_1 + s}{\epsilon_2 + s} \right]}{\ln \left[\frac{\epsilon_2 + s}{\epsilon_1 + s} \right]} \dots\dots\dots(25)$$

$$-\frac{1}{2} - \frac{1}{s} + \frac{1}{s} \frac{\ln \left[\frac{n\mu_2 \left[\frac{n\mu_2 + s}{n\mu_1 + s} \right]^s - s^s}{n\mu_1 \left[\frac{n\mu_2 + s}{n\mu_1 + s} \right]^s - s^s} \right]}{\ln \left[\frac{n\mu_2 + s}{n\mu_1 + s} \right]} + \frac{\ln \left[\frac{n\mu_1 + s}{n\mu_2 + s} \right]}{\ln \left[\frac{n\mu_2 + s}{n\mu_1 + s} \right]}$$

From (24), it is clear that m is the integer nearest to:

$$m = -\frac{1}{2} - \frac{1}{s} + \frac{1}{s} \frac{\ln \left[\frac{n\mu_2 (s^s - (n\mu_2 + s)^s)}{n\mu_1 (s^s - (n\mu_1 + s)^s)} \right]}{\ln \left[\frac{s + n\mu_2}{s + n\mu_1} \right]} + \frac{1}{2} \frac{\ln \left[\frac{s + n\mu_1}{s + n\mu_2} \right]}{\ln \left[\frac{s + n\mu_2}{s + n\mu_1} \right]} \dots\dots\dots(26)$$

Choosing $n\mu_1 = x_1$ and $n\mu_2 = x_2$, the above expression reduces to the integer nearest to:

$$m = -\frac{1}{2} - \frac{1}{s} + \frac{1}{s} \frac{\ln \left[\frac{x_2 \left[\frac{x_2 + s}{x_1 + s} \right]^s - s^s}{x_1 \left[\frac{x_2 + s}{x_1 + s} \right]^s - s^s} \right]}{\ln \left[\frac{s + x_2}{s + x_1} \right]} + \frac{1}{2} \frac{\ln \left[\frac{s + x_1}{s + x_2} \right]}{\ln \left[\frac{s + x_2}{s + x_1} \right]} \dots\dots\dots(27)$$

Example:

Obtain the acceptance criterion m for BMDS (0,1) plan for given s=1, $n\mu_1=.2$, $n\mu_2=2$ minimizing $\alpha+\beta$.

Solution:

From Table 4, it is observed that for s=1, $n\mu_1=.2$, $n\mu_2=2$ the value of m=3.

Table 1: Values of 'np' for given s, m and the average probability of acceptance \bar{P}

S	M \ \bar{P}	0.99	0.95	0.75	0.5	0.1	0.05
1	1	0.0787	0.1976	0.6514	1.4809	9.8128	19.7394
	2	0.0618	0.1614	0.5302	1.2223	9.0517	18.8159
	3	0.0544	0.1401	0.4672	1.1113	8.9617	18.7564
	4	0.0514	0.1257	0.4286	1.0566	8.9436	18.7499
	5	0.0444	0.1162	0.4029	1.0285	8.9342	18.7472
	6	0.0420	0.1083	0.3847	1.0139	8.9290	18.7458
	7	0.0393	0.1022	0.3714	1.0063	8.9261	18.7450
	8	0.0374	0.0972	0.3631	1.0025	8.9246	18.7446
	9	0.0333	0.0936	0.3559	1.0005	8.9238	18.7443
2	1	0.0837	0.2008	0.6053	1.2136	4.7017	7.1975
	2	0.0681	0.1606	0.4913	1.0056	4.3543	6.9552
	3	0.0565	0.1391	0.4328	0.9190	4.3108	6.9383
	4	0.0502	0.1261	0.3974	0.8751	4.3027	6.9374
	5	0.0462	0.1160	0.3738	0.8524	4.2996	6.9372
	6	0.0423	0.1063	0.3572	0.8406	4.2978	6.9372
	7	0.0393	0.1010	0.3451	0.8345	4.2969	6.9372
	8	0.0376	0.0960	0.3361	0.8313	4.2964	6.9372
	9	0.0355	0.0919	0.3293	0.8297	4.2961	6.9371

Table 2: Operating ratios for $\frac{\mu_2}{\mu_1}$ constructing with weighted poisson distribution for BMDS-1(0, 1) Plan

S	m	$\frac{\mu_2}{\mu_1} = 0.05$	$\frac{\mu_2}{\mu_1} = 0.05$	$\frac{\mu_2}{\mu_1} = 0.25$	n_{-1}	$\frac{\mu_2}{\mu_1} = 0.01$	$\frac{\mu_2}{\mu_1} = 0.01$	$\frac{\mu_2}{\mu_1} = 0.25$	n_{-1}
		$= 0.10$	$= 0.05$	$= 0.10$		$= 0.10$	$= 0.05$	$= 0.10$	
1	1	49.67174	99.91927	15.06379	0.1976	124.6761	250.7974	124.6761	0.0787
	2	56.07914	116.5723	17.07119	0.1614	146.4007	304.3246	146.4007	0.0618
	3	63.95321	133.8514	19.1803	0.1401	164.8415	345.0063	164.8415	0.0544
	4	71.17676	149.2197	20.86498	0.1257	173.9866	364.757	173.9866	0.0514
	5	76.91141	161.389	22.177	0.1162	201.2198	422.2349	201.2198	0.0444
	6	82.44375	173.0855	23.21307	0.1083	212.5324	446.1986	212.5324	0.0420
	7	87.29853	183.329	24.03589	0.1022	227.1267	476.9715	227.1267	0.0393
	8	91.78163	192.772	24.5785	0.0972	238.7049	501.3597	238.7049	0.0374
	9	95.28936	200.1549	25.07445	0.0936	267.9809	562.8928	267.9809	0.0333
2	1	23.41763	35.84854	7.767287	0.2008	56.15041	85.95703	56.15041	0.0837
	2	27.10523	43.29508	8.862002	0.1606	63.92109	102.1009	63.92109	0.0681
	3	30.99347	49.88491	9.959521	0.1391	76.32231	122.843	76.32231	0.0565
	4	34.11426	55.0034	10.82818	0.1261	85.6268	138.0585	85.6268	0.0502
	5	37.06818	59.80865	11.50324	0.1160	93.04112	150.1197	93.04112	0.0462
	6	40.43991	65.27441	12.03367	0.1063	101.5042	163.8388	101.5042	0.0423
	7	42.53245	68.6671	12.45146	0.1010	109.3355	176.5183	109.3355	0.0393
	8	44.76194	72.27522	12.78348	0.0960	114.3776	184.6807	114.3776	0.0376
	9	46.74549	75.48278	13.04713	0.0919	120.8873	195.204	120.8873	0.0355

Table 3: Sample size minimizing (+) with s=1, m=1

$\frac{\bar{X}_1 \rightarrow}{\downarrow \bar{X}_2}$	0.01	0.02	0.03	.04	.05	.06	.07	.08	.09	.1	.11	.12	.13	.14	.15	.16	.17	.18	.19	.2
.02	99																			
.03	82	57																		
.04	72	50	40																	
.05	65	45	36	31																
.06	60	41	33	29	26															
.07	57	38	31	27	24	22														
.08	54	36	29	25	22	20	19													
.09	51	34	28	24	21	19	18	17												
.1	49	33	26	23	20	18	17	16	15											
.11	47	32	25	22	19	18	16	15	1	14										
.12	46	30	24	21	19	17	16	15	14	13	12									
.13	44	29	23	20	18	1	15	14	13	13	12	11								
.14	43	29	23	19	17	1	15	14	1	12	12	11	11							
.15	42	28	22	19	17	1	14	13	1	12	11	11	10	10						
.16	41	27	21	18	16	1	14	13	1	11	11	10	10	10	9					
.17	40	26	21	18	16	1	1	12	1	11	11	10	10	9	9					
.18	39	26	20	17	15	1	13	12	11	11	10	10	10	9	9					
.19	39	25	20	17	15	1		12	1	11	10	10	9	9	9					
.20	38	25	20	17	15	1		12	11	10	10	9	9	9	8					
.25	35	23	18	15	13	1		10	10	9	9	9	8	8						
.30	33	21	17	14	12	1		10	9	1		8	8							
.35	31	20	16	13	12	1		9	9	1		7								
.40	30	19	15	13	11	1		9	8	1										
.45	29	18	14	12	11	1		8	8	7										

Table 4 : Acceptance criterion ‘m’ of BMDS-1 (0,1) plan when the sample size s=1 is fixed

$\frac{X_1 \rightarrow}{\downarrow X_2}$	0.01	0.015	0.025	0.05	0.075	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.6	0.75	1
0.25	29	26	22	17	14	13	11	9	-	-	-	-	-	-	-
0.30	25	23	19	15	13	12	10	9	8	-	-	-	-	-	-
0.40	21	19	16	13	11	10	8	7	7	6	-	-	-	-	-
0.50	18	16	14	11	10	9	8	7	6	6	5	-	-	-	-
0.60	16	15	13	10	9	8	7	6	6	5	5	4	-	-	-
0.75	14	13	11	9	8	7	6	6	5	5	4	4	3	-	-
1.00	12	11	10	8	7	6	5	5	4	4	4	3	3	3	-
1.25	11	10	8	7	6	6	5	4	4	4	3	3	3	3	2
1.50	10	9	8	6	6	5	4	4	4	3	3	3	3	2	2
1.75	9	8	7	6	5	5	4	4	3	3	3	3	2	2	2
2.00	8	8	7	6	5	4	4	4	3	3	3	2	2	2	2
2.50	7	7	6	5	4	4	4	3	3	3	2	2	2	2	2
3.00	7	6	6	5	4	4	3	3	3	3	2	2	2	2	2
3.50	6	6	5	4	4	4	3	3	3	2	2	2	2	2	2
4.00	6	6	5	4	4	3	3	3	7	2	2	2	2	2	2
5.00	5	5	4	4	3	3	3	2	2	2	2	2	2	2	1
6.00	5	5	4	4	3	3	3	2	2	2	2	2	2	1	1
7.00	5	4	4	3	3	3	2	2	2	2	2	2	2	1	1
8.00	5	4	4	3	3	3	2	2	2	2	2	2	1	1	1
9.00	4	4	4	3	3	3	2	2	2	2	2	2	1	1	1
10.00	4	4	4	3	3	2	2	2	2	2	2	2	1	1	1

Construction of the table:

From the Table 1, the np values (where p is the product quality) for which the proportion of lots that are expected to be accepted is a stated value $\bar{p} = 0.05, 0.1, 0.5, 0.75, 0.95$ and 0.99 . For chosen set of values for α and β , the operating ratios in Table 2 are used to compute np values for the given values of $(=1,2,\dots,10)$.

Conclusion:

It is observed that for $nAQL=0.01$ and $nLQL=3$. The value of m which minimizes the sum of the risks when they both have equal importance is 7 for $s=1, 5$ for $s=5$ and 4 for $s=9$. BMDS Plan is designed for a benefit of consumer. From the table values, it is observed that as the value of s, the parameter of the prior distribution decreases, consumer has more benefit than using conventional plans.

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