Visit us - www.researchjournal.co.in ■ DOI : 10.15740/HAS/IRJAES/8.1/183-185

International Research Journal of Agricultural Economics and Statistics

Volume 8 | Issue 1 | March, 2017 | 183-185 ■ e ISSN-2231-6434

Research Paper

Variance estimation using linear combination of Hodges-Lehmann and Quartiles

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Paper History :

Received : *24.01.2017;*

Revised : *17.02.2017;*

Accepted : *26.02.2017*

ABSTRACT : In this paper, we have proposed a class of modified ratio type variance estimator for estimation of population variance of the study variable, when Hodges-Lehmann and Quartiles of the auxiliary variable are known. The bias and mean square error (MSE) of the proposed estimator are obtained. From the numerical study it is observed that the proposed estimator performs better than the existing estimators in the literature.

KEY WORDS : Simple random sampling, Bias, Mean square error, Hodges-Lehmann, Quartiles, Auxiliary variable.

HOW TO CITE THIS PAPER : Bhat, M.A., Maqbool, S. and Raja, T.A. (2017). Variance estimation using linear combination of Hodges-Lehmann and Quartiles . *Internat. Res. J. Agric. Eco. & Stat.,* **8** (1) : 183-185, **DOI : 10.15740/HAS/IRJAES/8.1/183-185.**

INTRODUCTION :

Here we consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on Ui, i=1,2,3,…,N giving a vector $Y = \{y_1, y_2, ..., y_N\}$. The goal is to estimate the population means $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ or its variance $S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2$ on the basis of random sample selected from a population U. Sometimes in sample surveys along with the study variable Y, information on auxiliary variable X, which is positively correlated with Y, is also available. The information on auxiliary variable X, may be utilized to obtain a more efficient estimator of the population. In this paper, our aim is to estimate the population variance on the basis of a random sample of size n selected from the population U. The problem of constructing efficient estimators for the population variance has been widely discussed by various authors such as Isaki (1983), who proposed ratio and regression estimators. Latter various authors such as Kadilar and Cingi (2006) and Sumramani and Kumarapandiyan (2015) improved the already existing estimators.

Natations : N=Population size, n=Sample size. $\gamma = \frac{1}{n}$, Y= Study variable. X= Auxiliary variable. $\bar{X}$, $\bar{Y}$ =Population means. $\bar{x}, \bar{y}$ = Sample means. S_Y^2 , S_x^2 = Population variances. s_y^2 , s_x^2 =Sample variances. C_x, C_y=Co-efficient of variation. ρ=Correlation co-efficient. $\beta_{1(x)}$= Skewness of the auxiliary variable. $\beta_{2(x)}$=Kurtosis of the auxiliary variable. $\beta_{2(y)}$ =Kurtosis of the study variable. M_d=Median of the auxiliary variable. B (.)=Bias of the estimator.

MSE(.)= Mean square error. $\hat{S}_R^2$ =Ratio type variance estimator. $\hat{S}_{Kc1}^2$, $\hat{S}_{jG}^2$, = Existing modified ratio estimators. Q_a = $(Q_3+Q_1)/2$, Population semi-quartile average of the auxiliary variable x. HL=Median [(Yj+Yk)/2, $1 \le j \le k \le N$]. QD=Quartile deviation, $Q_{M.A.}$=Quartile mean average, Q_1= First quartile, Q_2= Second quartile, Q_3=Third quartile, Q_r=Quartile range.

MATERIALS AND METHODS :

In this paper, we first discuss the already existing estimators in the literature and then proposed the modified estimators using the linear combination of Hodges lehmann and Quartiles and the results are compared with the existing estimators.

Ratio type variance estimator proposed by Isaki (1983):

Isaki suggested a ratio type variance estimator for the population variance S_y^2 when the population variance S_x^2 of the auxiliary variable X is known. Its bias and mean square error are given by

$$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \qquad (1)$$

$$\text{Bias}\,(\hat{S}_R^2) = \gamma S_y^2 \{[\beta_{2(x)} - 1] - (\lambda_{22} - 1)\}$$

$$\text{MSE}\,(\hat{S}_R^2) = \gamma S_y^4 \{[\beta_{2(y)} - 1] + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\}$$

Kadilar and Cingi (2006) estimators:

The authors suggested four ratio type variance estimators using known values of C.V. and co-efficient of kurtosis of an auxiliary variable X.

$$\hat{S}_{kc1}^2 = s_y^2 \left[\frac{\beta_{2(x)} S_x^2 + C_x}{\beta_{2(x)} s_x^2 + C_x}\right] \qquad (2)$$

$$\text{Bias}\,(\hat{S}_{kc1}^2) = \gamma S_y^2 A_1 \{A_1 [\beta_{2(x)} - 1] - (\lambda_{22} - 1)\}$$

$$\text{MSE}\,(\hat{S}_{kc1}^2) = \gamma S_y^4 \{[\beta_{2(y)} - 1] + A_1^2 (\beta_{2(x)} - 1) - 2A_1 (\lambda_{22} - 1)\}$$

Recent developments:

Sumramani and Kumarapandiyan (2015) estimators where the authors used median, quartiles and Deciles of an auxiliary variable.

$$\hat{S}_{jG}^2 = s_y^2 \left[\frac{S_x^2 + \alpha w_i}{s_x^2 + \alpha w_i}\right] \qquad (3)$$

$$\text{Bias}\,(\hat{S}_{jG}^2) = \gamma S_y^2 A_{jG} \{A_{jG} [\beta_{2(x)} - 1] - (\lambda_{22} - 1)\}$$

$$\text{MSE}\,(\hat{S}_{jG}^2) = \gamma S_y^4 \{[\beta_{2(y)} - 1] + A_{jG}^2 (\beta_{2(x)} - 1) - 2A_{jG} (\lambda_{22} - 1)\}$$

When α=0 in equ. (3), the above estimator reduces to Isaki (1983) estimator.

When α=1 in equ. (3), the above estimator reduces to Kadilar and Cingi (2006) estimator.

Proposed estimator:

We have proposed a new modified ratio type variance estimator of the auxiliary variable by using linear combination of measure based on the median of the Walsh averages which is defined as HL=Median [(Yj+Yk)/2, $1 \le j \le k \le N$]. and population semi inter quartile average of the auxiliary variable. It is highly sensitive to outliers and is robust against outliers.

$$\hat{S}_{MS1}^2 = s_y^2 \left[\frac{S_x^2 + (HL + Q_1)}{s_x^2 + (HL + Q_1)}\right] \qquad \hat{S}_{MS2}^2 = s_y^2 \left[\frac{S_x^2 + (HL + Q_2)}{s_x^2 + (HL + Q_2)}\right]$$

$$\hat{S}_{MS3}^2 = s_y^2 \left[\frac{S_x^2 + (HL + Q_3)}{s_x^2 + (HL + Q_3)}\right] \qquad \hat{S}_{MS4}^2 = s_y^2 \left[\frac{S_x^2 + (HL + Q_d)}{s_x^2 + (HL + Q_d)}\right]$$

$$\hat{S}_{MS5}^2 = s_y^2 \left[\frac{S_x^2 + (HL + Q_a)}{s_x^2 + (HL + Q_a)}\right] \qquad \hat{S}_{MS6}^2 = s_y^2 \left[\frac{S_x^2 + (HL + Q_r)}{s_x^2 + (HL + Q_r)}\right] \qquad (4)$$

The bias and mean square error of the above estimator after simplification is given as

$$\text{Bias}\,(\hat{S}_{MSi}^2) = \gamma S_y^2 A_{MSi} \{A_{MSi} [\beta_{2(x)} - 1] - (\lambda_{22} - 1)\}$$

$$\text{MSE}\,(\hat{S}_{MSi}^2) = \gamma S_y^4 [(\beta_{2(y)} - 1) + A_{MSi}^2 (\beta_{2(x)} - 1) - 2A_{MSi} (\lambda_{22} - 1)]$$

where,

$$A_{MS1} = \frac{S_X^2}{S_X^2 + (HL + Q_1)} \qquad A_{MS2} = \frac{S_X^2}{S_X^2 + (HL + Q_2)}$$

$$A_{MS3} = \frac{S_X^2}{S_X^2 + (HL + Q_3)} \qquad A_{MS4} = \frac{S_X^2}{S_X^2 + (HL + Q_d)}$$

$$A_{MS5} = \frac{S_X^2}{S_X^2 + (HL + Q_a)} \qquad A_{MS6} = \frac{S_X^2}{S_X^2 + (HL + Q_r)}$$

RESULTS AND DATA ANALYSIS :

We use the data of Murthy (1967) page 228 in which

Table 1 : Bias and mean square error of the existing and the proposed estimators

Estimators	Bias	Mean square error
Isaki (1983)	10.8762	3925.1622
Kadilar and Cingi(2006) 1	10.7222	3898.5560
Sumramani and Kumarapandiyan (2015)	6.1235	3180.7740
Proposed Estimator (MS1)	3.1879	2850.1552
Proposed Estimator (MS1)	3.6347	2880.2490
Proposed Estimator (MS1)	1.5635	2707.0820
Proposed Estimator (MS1)	4.0169	2927.9408
Proposed Estimator (MS1)	2.7735	2796.7882
Proposed Estimator (MS1)	2.6022	2781.4586

Table 2 : Per cent relative efficiency of proposed estimators with existing estimators

Estimators	Isaki (1983)	Kadilar and Cingi (2006) 1	Sumramani and Kumarapandiyan (2015)
P1	137.7374	136.7839	111.6007
P2	136.2785	135.3548	110.4346
P3	144.9961	144.0132	117.4989
P4	134.0587	133.1500	108.6358
P5	140.3453	139.3940	113.7302
P6	141.1188	140.1622	114.3570

fixed capital is denoted by X (auxiliary variable) and output of 80 factories are denoted by Y (study variable). We apply the proposed and existing estimators to this data set and the data statistics is given below in Table 1 and the per cent relative efficiency is given in Table 2.

$N=80$, $Sx=8.4542$, $n=20$, $Cx=0.7507$, $\overline{X}=11.2624$, $\beta_{2(x)}=2.8664$, $\overline{Y}=51.8264$, $\beta_{2(y)}=2.2667$, $\rho=0.9413$, $\beta_{1(x)}=1.05$, $\lambda_{22}=2.2209$, $S_y=18.3569$, $Md=7.5750$, $Q_1=9.318$, $C_y=0.3542$, $Q_2=7.5750$, $Q_3=16.975$, $QD=5.9125$, $Q_a=11.0625$, $Q_R=11.82$, $HL=10.405$.

The Table 1 and 2 shows that the bias and mean square error are less than the already existing estimators in the literature and also by the percentage relative efficiency criteria the proposed estimators are more efficient than the existing estimators. Hence, the proposed estimator may be preferred over existing estimators for use in practical applications.

Conclusion:

The paper proposes a new ratio type variance estimator using known values of an auxiliary variable. Results show that the bias and mean square error are less than the already existing estimators in the literature and also by the percentage relative efficiency criteria are more efficient than the existing estimators. Hence, the proposed estimator may be preferred over existing estimators for use in practical applications.

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