Large-Scale SDPF Shells Model Calculations for Electron Scattering Form Factors and Charge Density Distribution for the ^{19}F Nuclei

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ABSTRACT

The elastic and inelastic electron scattering form factors, reduced transition probabilities and the charge density distribution for the ^{19}F nuclei have been studied with and without effective charge on the sdpf-model space and Tassie model. The harmonic oscillator and Skyrme potentials have been used to calculate the wave functions of radial single-particle matrix elements. The shell model code NuShell @ MSU is used to obtain the results. The present work includes the transitions from ground state to the ($1/2^+$ 1/2), ($3/2^+$ 1/2), ($5/2^+$ 1/2) states. The results of the reduced transition probabilities and the charge density distribution are reasonable descriptions of the experimental data. Very good agreements are obtained for the form factors in this study.

KEY WORDS: shell model, (e, e) inelastic longitudinal form factors, core-polarization effects, NuShell, Nuclear Structure.

I. INTRODUCTION

Electron scattering is successful tool for studying the nuclear structure which includes the charge, current and magnetization densities for many reasons. Theoretical work on electron scattering starts from 1929 (Mott, 1929), when mott derived the cross section for the relativistic scattering of Dirac particles. The electron is considered as a point particle, so can be accelerated easily. Its interaction with nucleus is weak, perturbation of nucleus is small and reaction mechanism is simple. But the Electron-nucleus scattering have some disadvantages, where it need high intensity, thick targets and large solid angles. There are radioactive effects which described by quantum electrodynamics (QED). The electron-scattering experiments, as well as measurements of electromagnetic transition strengths, indicate that the ground state and low-lying excited states of ^{19}F is an excellent example of a light, odd-A, strongly deformed nuclear system. Large-scale shell model calculations are performed by solving an eigenvalue problem of a large-dimension sparse matrix. In the case of nuclear shell-model calculations, the Hamiltonian matrix in M-scheme basis is very sparse since the Hamiltonian consists of one-body and two-body interactions. The first Born approximation, applied to this nucleus, since $\alpha z << 1$, where α is the finite structure constant and z is the atomic number. According to this approximation the interaction of the electron with the nuclear charge distribution is considered as an exchange of a virtual photon with zero angular momentum along the direction of the momentum transfer q for longitudinal scattering, the interaction of the electron with the nuclear current distribution is considered as an exchange of virtual photon with angular momentum ±1 along q for the Transverse scattering (Uberall, 2012). Some theoretical results of nuclear structure for many light nuclei in p and sd shells have been discussed by Jassim (2016) using shell model calculations. A large-scale shell model calculation also performed for Nuclear Structure of ^{104,106,108}Sn Isotopes (Jassim, 2013) with the Sn100pn interaction.

The aim of the present work is to study the electron scattering form factors, charge density distribution and reduced transition probabilities of the ^{19}F nucleus with extended model space with sdpf now effective interaction using Tassie model and the effect of the effective charge, these calculations are compared with the available experimental data.

2. THEORY

The elastic electron scattering form factor from is simply the Fourier transform of the charge density distributions $\rho_{ch}(r)$, given by (Wong, 2008) **:**

$$F^L(q) = \int \rho_{ch}(r) e^{iqr}\, dV \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (1)$$

$F^L(q)$ is known as the longitudinal form factor. The electron scattering form factor with the corrections, in terms of angular momentum J and momentum transfer q, and inclode isospin, can be written as (Donnelly, 1984)

$$\left|F_J^{\eta}(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \sum_{T=0,1} \begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \langle f \| \hat{T}^{\eta}_{JT}(q) \| i \rangle \, F_{cm}(q)\, F_{fs}(q) \right|^2 \;\ldots\ldots\ldots (2)$$

Where η indicate the longitudinal (C), transverse electric (E), and transverse magnetic (M) form factors. $\hat{T}_J^{\eta}(q)$ is the electron scattering operator.

$F_J^{\eta}(q) = \mathrm{e}^{q^2 b^2/4A}$ is the corrections of the Center-of-mass, A and b are the mass number and the harmonic oscillator size parameter, respectively, and $F_{fs}(q) = e^{-0.43q^2/4}$ is the corrections of nucleon finite-size.

We can rewrite the nuclear many-body matrix elements in terms of the one-body matrix element and the reduced one-body matrix element (Jassim, 2014; Brussaard, 1977).

$$\langle f \| |\hat{T}^{\eta}_{JT}| \| i \rangle = \sum_{ab} OBDM(i,f) \langle j_f \| |\hat{T}^{\eta}_{JT}| \| j_i \rangle \qquad (3)$$

Where quantum numbers (n, l, j) abbreviated by j. The reduced single-particle matrix element in both spin and isospin, can be rewritten in terms of the single particle matrix element reduced in spin only (Donnelly, 1984; Brussaard, 1977).

$$\langle j_f \| |\hat{T}^{\eta}_{JT}| \| j_i \rangle = \sqrt{\frac{2\pi+1}{2}} \sum_{\tau_z} I_T(\tau_z) \left\langle j_f \| \hat{T}^{\eta}_{J\tau_z}(q) \| j_i \right\rangle \qquad (4)$$

where

$$I_T(\tau_z) = \begin{bmatrix} (-1)^{-t_z+\frac{1}{2}}, for\ T = 1 \\ 1 \quad , for\ T = 0 \end{bmatrix} \qquad (5)$$

and t_z = 1/2 and −1/2 for the proton and neutron, respectively. In the present work, the shape of the Tassie Model is employed for core polarization. The effect of core polarization is found to be essential for both the transition strengths and the momentum-transfer dependence and gives a good description of the data [K.S. Jassim, 2014]. The longitudinal form factors for this model are [L.J. Tassie,1956]

$$F^L_J(q) = \sqrt{\frac{4\pi}{2J_i+1}} \frac{1}{Z} \left[\int_0^{\infty} r^2 j_J(qr) \overset{ms}{\rho}_{Jt_z} dr - Nq \int_0^{\infty} dr\, r^{J+1} \rho_o\, j_{J-1}(qr) \right]$$
$$\times F_{cm}(q)\, F_{fs}(q) \qquad (6)$$

Where N is a proportionality constant and $\rho_o$is the ground state two – body charge density distribution, and j is the spherical Bessel function. The reduced electric transition strength is given by (Brown, 1985)

$$B(EJ) = \frac{Z^2}{4\pi} \left(\frac{(2J+1)!!}{K^J} \right)^2 \left| F^L_J(k) \right|^2, \qquad (7)$$

$$\text{where } k = \frac{E_x}{\hbar c}$$

3. RESULTS AND DISCUSSION

In this work, we are calculated various components of electron scattering form factors for the ^{19}F nucleus, which have ground state spin-parity J^{π}= 1/2. sdpf-extended model space with configuration ($1d_{5/2}$ $2s_{1/2}$ $1d_{3/2}$ $1f_{7/2}$ $2p_{3/2}$ $1f_{5/2}$ $2p_{1/2}$) has been adopted in order to distribute the valence particles outside an inert core ^{16}O, by using Nushell code for windows without any restriction imposed on the model space, using sdpf now effective interactions. The radial wave functions for the single-particle matrix elements were calculated with the Harmonic Oscillator (HO) and Skyrme potentials (SK). The oscillator length parameter b = b_{rms}= 1.833 fm. The effective charges which are used 0.35 for each of proton and neutron.

The longitudinal form factor for the ground-state C0 ($1/2^+_1$ 1/2) elastic scattering, shown in Fig. 1. The doted curve indicates the calculations of harmonic-oscillator potential. The dashed curves indicate the calculations of SK potential. The results of the HO model space calculations give a very good agreement with the experimental data (Brown,1985) along the momentum transfer q, while the Our calculations with SK potential have a very good agreement with the experimental data up to the momentum transfer q ≈2.5 fm^{-1}. Any effective charge causes disagreement with the experimental data. Fig. 2 shows the transverse M1 ($1/2^+_1$ 1/2) elastic electron scattering form factor as a function of momentum transfer q. The doted and solid curves indicate the calculations of harmonic-oscillator potential without and with effective charge, respectively. The dashed curves indicate the calculations of Skyrme potential with effective charge. The "+" symbols indicate the calculations of Tassie model. The calculations of the model space are performed without and with effective charge so using Tassie model with HO potential. All the results using model space, model space with effective charge model and Tassie model with HO potential give agreement in the momentum transfer region between (0.5-2.6) fm^{-1} comparing with the experimental data (Brown,1985]. The M1 results with effective charge model using SK give excepting with experimental data at the first peak (0.5-1.3) fm^{-1}. We conclude that the calculations of the effective charge model space with SK potential are identical to the calculating with the HO potential at q ≈ 0-1.3 fm^{-1} range.

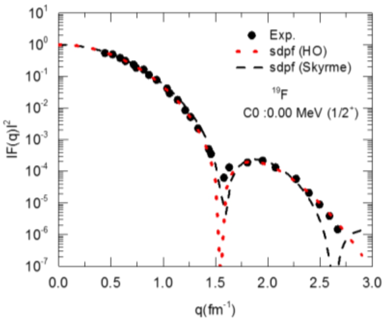

Figure.1. The coulomb C0 form factors for the ground state $(1/2_1^+\ 1/2)$ in the ^{19}F nucleus
Experimental values are indicated by the filled circles (Brown, 1985)

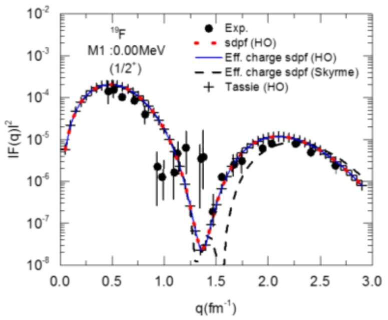

Figure.2. The transverse M1 form factors for the ground state $(1/2_1^+\ 1/2)$ in the ^{19}F nucleus
Experimental values are indicated by the filled circles (Brown, 1985)

In Fig.3, we present calculations of the longitudinal C2 $(3/2_1^+\ 1/2)$ inelastic electron scattering form factor as a function of momentum transfer q. The dotted and solid curves indicate the calculations of harmonic-oscillator potential without and with effective charge, respectively. The dashed curves indicate the calculations of SKX potential with effective charge. The "+" symbols indicate the calculations of TM. The results of the HO model space calculations (without effective charge) are small at the two peak then give a good agreement with the experimental data (Brown, 1985) at the first peak, when we use effective charge. The calculations of the TM with HO give a very good agreement with the experimental data along the momentum transfer q. The form factors of the effective charge obtained by calculating the matrix elements of a q-dependent effective charge operator between the single particle states, rather than those of the true charge operator between the exact states (Uberall, 2012).

Fig.4, shows the calculated transverse M1 $(3/2_1^+\ 1/2)$ inelastic electron scattering form factor as a function of momentum transfer q in comparison with the experimental data (Uberall, 2012). The dotted and solid curves indicate the calculations of harmonic-oscillator potential without and with effective charge, respectively. The dashed curves indicate the calculations of SKX potential with effective charge. The "+" symbols indicate the calculations of TM. The "triangle" symbols indicate the calculations of model space with gsp= 8.2, gsn= -2.1

In this case, all calculations are poor agreement with the data. The agreement of the calculated form factors with experimental data can be made by adjusting the g-factor to be 8.2, -2.1 for proton and neutron, respectively.

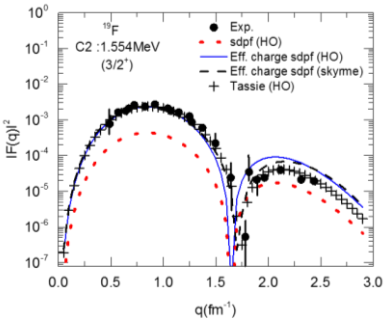

Figure.3. The coulomb C2 form factors for the $(3/2_1^+\ 1/2)$ (1.554 MeV) transition in the ^{19}F nucleus.
Experimental values are indicated by the filled circles (Uberall, 2012)

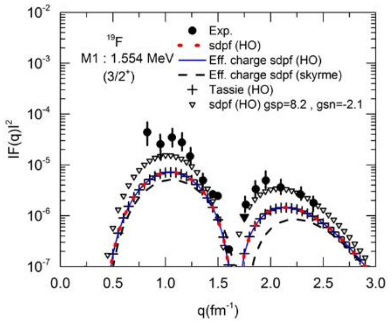

Figure.4. The transverse M1 form factors for the $(3/2_1^+\ 1/2)$ (1.554 MeV) transition in the ^{19}F nucleus. Experimental values are indicated by the filled circles (Uberall, 2012)

The inelastic longitudinal form factors for the C2 to the $5/2_1^+$ 1/2 state in the ^{19}F nucleus are presented in Fig.5. Where, the HO model space calculations (without effective charge) are small at the two peak then give a good agreement with the experimental data (Uberall, 2012) at the first peak when we use effective charge. The calculations of the effective charge model space with SKX potential give agreement with the experimental data (Uberall, 2012) at q < 1.7 fm^{-1}. Calculations of the TM give a very good agreement with the experimental data. In the case of the transverse M3 form factors for these state, theoretical results of the model space and TM with HO potential give a good agreement with the experimental data (Uberall, 2012) as shown in Fig.6, while the calculations of the model space with SKX are in satisfactory agreement with the experimental data.

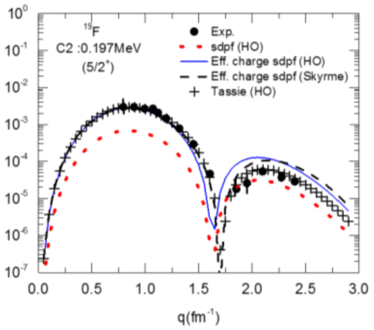

Figure.5. The coulomb C2 form factors for the $(5/2_1^+\ 1/2)$ (0.197 MeV) transition in the ^{19}F nucleus. Experimental values are indicated by the filled circles (Uberall, 2012)

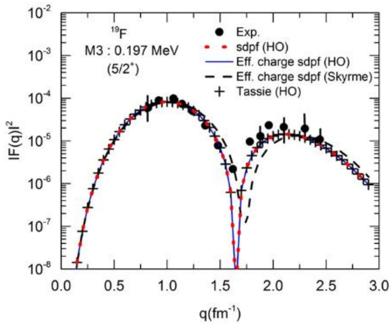

Figure.6. The transverse M3 form factors for the $(5/2_1^+\ 1/2)$ (0.197 MeV) transition in the ^{19}F nucleus. Experimental values are indicated by the filled circles (Uberall, 2012)

Fig.7, shows the dependence of the ground state two body charge density distributions on radius for the ^{19}F, calculated with HO, SKX and WS potential in sdpf model space and with Tassie shape. Table.1, show the energy levels and reduced transition probabilities B (WL), the results of energy levels gives agreement comparing with experimental data (Tilley, 1988). The B (WL) values give acceptable agreement for available experimental data (Uberall, 2012).

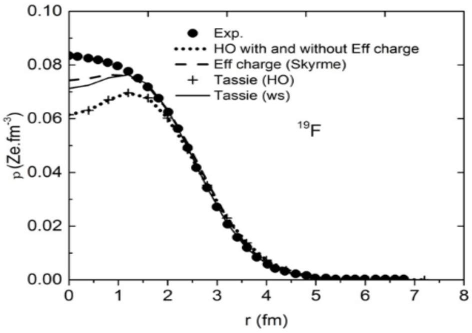

Figure.7. The dependence of the ground state charge density distribution (in fm^{-3}) on radius (in fm) for the ^{19}F nucleus. Experimental values are indicated by the filled circles (De Vries, 1987)

Table.1. Excitation energies and the reduced transition probabilities B (WL) (Ji= 1/2 → J$_f$) for sdpf-model spaces with their corresponding experimental values

Experimental values for Excitation energies and B (WL) were taken (Tilley, 1998; Brown, 1985), respectively. The units of B (EL) and B (ML) are e^{2}fm^{2L} and μ_N^2, respectively.

J_f^π	WL	E$_x$ (MeV)		B (WL) sdpf			B (WL) Tassie	B (WL) Exp.
		Cal.	Exp.	HO	Eff. Charge (HO)	Eff. Charge (Skyrme)		
1/2$^+$	M1	0	0	0.0601	0.0602	0.0595	0.0602	0.0547
3/2$^+$	M1	1.595	1.554± 0.009	0.010	0.010	0.0127	0.010	0.15(9)
3/2$^+$	E2	1.595	1.554± 0.009	8.321	35.32	43.07	35.32	
5/2$^+$	E2	0.229	0.197± 0.004	13.24	54.68	60.53	54.68	62.8(7)
7/2$^+$	E4	5.328	4.377± 0.042	53.58	138.6	72.43	138.6	
9/2$^+$	E4	3.278	2.779±0.034	809.4	3615	4669	3615	

4. CONCLUSIONS

The complete sdpf-shell model space wave functions using the sdpf now effective interaction succeeded in describing all the states of ^{19}F considered in this study, except the transverse M1 (3/2$^+$ 1/2) state. In the present study demonstrated that the best potential that might be used to describe electron scattering form factors is HO. The use of effective charges improved the result for the longitudinal form factors and reduced transition probabilities B (WL), while they didn't affect the transverse form factors and the charge density distribution CDD. The calculations of the model space and TM with HO potential are identical and give a very good agreement with the experimental data for the transverse form factors. These calculations have a little difference for longitudinal form factors, where TM calculations are better. The results of CDD and B (WL) indicate that improvement of the calculations depends on the potential, so we note that the calculations of Skyrme potential are the closer to the experimental values.

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