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A review on natural convection heat transfer problems by Lattice Boltzmann Method

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ABSTRACT

Natural Convection is one of the major modes of heat transfer that can be classified in terms of being natural, forced, gravitational, granular, or thermomagnetic. In the past decade, several studies on convection heat transfer in many geometries, enhancement of heat transfer by adding nanofluids, effects of magnetic field in heat transfer, heat transfer in a porous medium have been reported. The effects of heater length, as well as the Darcy (Da), Prandtl (Pr), Reynolds (Re), Grashof (Gr) and Rayleigh numbers (Ra), the solid volume fraction of nanoparticles, the permeability of the medium, and the inclination angle of geometry or heat source impacts on heat transfer are also investigated. This paper reviews various researchers work on fluid flow and heat transfer behavior which is carried out by means lattice Boltzmann method (LBM). LBM becomes an important tool in computational fluid dynamics area in recent decades. The easy structure of programming, accuracy and less computational time gives LBM a big advantage. And more over the flexibility of LBM allows to incorporate with conventional CFD tools to create a hybrid LBM is greatly inspired by numerous scholars to work in this field. The principle extent of this paper to present a comprehensive review on the exploration progress on natural convection using lattice Boltzmann method in recent years.

KEY WORDS: Natural convection, LBM, Rayleigh number.

1. INTRODUCTION

Amid late decades, the LBM recreation has gained ground in diverse, complex streams. Recent interest in LBM for complex partial differential equations has been persuaded by the requirement for efficient methods at an extensive variety of troublesome nonlinear issues. To facilitate numerical solution of Boltzmann equation, the complicated non-linear integral collision operator is often replaced by simpler expressions aimed at relinquishing most of the mathematical difficulty without spoiling the basic physics. There are numerous advantages of this method because of its simplicity. One of the major advantages of the LBM is its flexibility in combining with other techniques. Many research studies show that LBM can be coupled with conventional CFD techniques like finite volume method and finite difference method.

By using LBM, an extensive variety of engineering problems is elucidated. The one of the essential application in most of the engineering field is heat transfer using natural convection. In natural convection, the heat transfer is done by density difference. Due to the heat transfer, the flow of circulations is generated inside the domain. By studying the flow behaviors, patterns and heat transfer rate numerous important data can be obtained to model any heating system. Several scholar's shows great interests in studying natural convection heat transfer due to the abundant literature data available to validate their algorithm and also, a problem provides important aspects of flow and thermal behavior in the domain. In general, the streamlines are generated to study the flow characteristics, and the isotherm lines are produced to understand the thermal characteristics. By using LBM techniques various problems are solved in dealing natural convection problem such as natural convection in square cavity, inclination cavity, complex geometries, cavity filled with nanoparticles, porous media effects, in the presence of magnetic field and so on.

Numerical Method:

SRT-BGK: Single relaxation Time-Bhatnagar Gross Krook (1954), collision model is the widely used collision model for its simplicity and stability. For the incompressible thermal problem, two distribution functions need to be solved, f for momentum and g for scalar variable. For the natural convection problem, fluid flow and heat transfer characteristics are able to obtain by solving any of the different lattice arrangements model. One of the stabilized lattice arrangement is two-dimension nine-direction (D2Q9) model as shown in Fig.1 and SRT-BGK collision factor. The following equations need to be solved for momentum Eq.(1) and energy Eq.(2) (Mohammed, 2011),

$$f_i(x + c_i\Delta t, t + \Delta t) = f_i(x,t) - \frac{1}{\tau_v}\left[f_i(x,t) - f_i^{eq}(x,t)\right] + \Delta t F \quad (1)$$

$$g_i(x + c_i\Delta t, t + \Delta t) = g(x,t) - \frac{1}{\tau_c}\left[g_i(x,t) - g_i^{eq}(x,t)\right] \quad (2)$$

Where, c_i denotes the discrete velocity vectors in the ith direction, the lattice time step is defined as Δt which is set to unity, τ_v and τ_c are the relaxation time for the flow and temperature fields respectively. The term ΔF in Eq.(1) is the buoyancy force term. In this simulation the Boussinessq approximation is considered, hence the force term becomes as follows,

$$F = 3.0\omega_i g_y \beta \Delta T c_y \quad (3)$$

The relaxation time factors are related to kinematic viscosity and thermal diffusivity respectively as given in the relation below.

$$v = \left[\tau_v - \frac{1}{2}\right]\frac{\Delta t}{3} \quad ; \quad \alpha = \left[\tau_C - \frac{1}{2}\right]\frac{\Delta t}{3} \quad (4)$$

f_i^{eq} and g_i^{eq} denotes the local distribution functions dependence on the local hydrodynamic properties which are solved by using the local Maxwell-Boltzmann model, the equilibrium distribution function for flow Eq.(1) and temperature Eq.(2) can be obtained using the following equations.

$$f_i^{eq}(x) = \omega_i \rho(x)\left[1 + 3\, c_i.u + \frac{9}{2}(c_i.u)^2 - \frac{3}{2}u^2\right] \quad (5)$$

$$g_i^{eq}(x) = \omega_i \rho(x) T[1 + 3\, c_i.u] \quad (6)$$

Where,ω_i is the weight factor at respective lattice,ρ and u denotes the macroscopic density and velocity, respectively. The weight factors (ω_i) and lattice velocity (c_i)used for D2Q9 model is as follows.

$$\omega_i = \begin{cases} \frac{4}{9} i = 0 \\ \frac{1}{9} i = (1,2,3,4) \\ \frac{1}{36} i = (5,6,7,8) \end{cases} \quad (7)$$

$$c_i = \begin{cases} (0,0) i = 0 \\ c([\sin(i-1)\pi/2], [\cos(i-1)\pi/2]) \quad i = 1,2,3,4 \\ c(\sqrt{2}[\cos(2i-11)\pi/4], \sqrt{2}[\sin(2i-11)\pi/4]) \; i = 5,6,7,8 \end{cases} \quad (8)$$

MRT: Multiple Relaxation Time (MRT) is considered as a scheme which offers higher stability and accuracy than the SRT. The equations and matrixes which are required to imply the MRT scheme is given below (Mohammed, 2011).

The general lattice Boltzmann equation is

$$f_i(x + c\Delta t, t + \Delta t) - f_i(x,t) = -\Omega[f_i(x,t) - f_i^{eq}(x,t)] \quad (9)$$

Where Ω is the collision matrix. The collision step in the velocity space is difficult to continue but it is more convenient to perform the collision process in the momentum space. Hence, Eq.10 can change to the following form,

$$f_i(x + c\Delta t, t + \Delta t) - f_i(x,t) = -M^{-1}S[m(x,t) - m^{eq}(x,t) \quad (10)$$

Where $m(x,t)$ and m^{eq} are vectors of moments, $m = (m_0, m_{01}, m_2, \ldots\ldots m_n)^T$). The relaxation matrix S is a diagonal matrix. The mapping between velocity and moment spaces can be performed by linear transformation (Mohammed, 2011),

$$m = Mf \text{ and } f = M^{-1}m \quad (11)$$

The matrix for D2Q9 is given below

$$m = (\rho, e, \in, j_x, q_x, j_y, q_y, p_{xx}, p_{yy})^T \quad (12)$$

The equilibrium of the moment $\mathbf{m^{eq}}$ is

$$m_0^{eq} = \rho;\; m_1^{eq} = -2\rho + 3(j_x^2 + j_y^2);\;\; m_2^{eq} = \rho - 3(j_x^2 + j_y^2);\, m_3^{eq} = j_x;\, m_4^{eq} = -j_x;$$
$$m_5^{eq} = j_y;\; m_6^{eq} = -j_y;\, m_7^{eq} = (j_x^2 - j_y^2);\; m_8^{eq} = j_x j_y \quad (13)$$

In the above series of terms the j_x and j_y are defined as

$$j_x = \rho u_x = \sum_i f_i^{eq} c_{ix};\;\; j_y = \rho u_y = \sum_i f_i^{eq} c_{iy} \quad (14)$$

If the external force F is included in the problem then the above Eq. 17 can be modified as,

$$j_x = \rho u_x = \sum_i f_i^{eq} c_{ix} - F/2;\, j_y = \rho u_y = \sum_i f_i^{eq} c_{iy} - F/2 \quad (15)$$

In short notation S can be written as,

$$S = diag\ (1.0,1.4,1.4, s3.1.2, s5, 1.2, s7, s8). \tag{16}$$

Where, s7= s8=2/ (1+6v), s3, and s5 are arbitrary, can be set to 1.0.

Magnetic field: To introduce the effect of magnetic field in the domain, the magnetic force is added to buoyancy term. So the force term in the eqn. (3) becomes

$$F = F_x + F_y \tag{17}$$

$$F_x = 3\omega_i\rho\left[A(v\sin(\theta_M)\cos(\theta_M)) - \left(u\sin^2(\theta_M)\right)\right] \tag{18}$$

$$F_y = 3\omega_i\rho\left[g_y\beta(T-T_m) + A(u\sin(\theta_M)\cos(\theta_M)) - \left(v\cos^2(\theta_M)\right)\right]. \tag{19}$$

Where magnetic field is introduced in terms of non dimensionalised Hartmann number (Ha)

$$A = \frac{Ha^2\mu}{L^2} \quad ; \quad Ha = LB_0\sqrt{\frac{\sigma}{\mu}} \tag{20}$$

In the above equation, B_0 refers to magnetic flux density, μ is dynamic viscosity, and σ is electric conductivity.

Porous media: To incorporate the effects of porous media certain changes are needed in the LBM equation, which are given below. Using the temporal velocity v to incorporate the presence porous media the fluid velocity is calculated as

$$u = \frac{v}{c_0 + \sqrt{{c_0}^2 + c_1|v|}} \tag{21}$$

Where the temporal velocity and the other two parameters are

$$\rho v = \sum_i c_i f_i + \frac{1}{2}\rho\epsilon G \tag{22}$$

Here G is the buoyancy term

$$c_0 = \frac{1}{2}\left(1+\epsilon\,\frac{v}{2K}\right) \tag{23}$$

ε is porosity of medium and K is permeability of porous media.

$$c_1 = \frac{1}{2}\frac{1.75}{\sqrt{150\ \epsilon^2\ K}} \tag{24}$$

Thus the equilibrium and force term changes to

$$f_i^{eq}(x) = \omega_i\rho(x)\left[1 + 3\ c_i.u + \frac{9}{2\epsilon}(c_i.u)^2 - \frac{3}{2\epsilon}u^2\right] \tag{25}$$

$$F = \omega_i\rho(x)\left(1 - \frac{1}{2\tau_v}\right)\left[1 + 3\ c_i.u + \frac{9}{2\epsilon}(c_i.u)^2 - \frac{3}{2\epsilon}u^2\right] \tag{26}$$

The macroscopic quantities (velocity U, pressure P, mass densityρ, momentumρu) are obtained by evaluating the distribution function$f$.The macroscopic quantities are calculated as

Fluid density : $\rho(x,t) = \sum_i f_i(x,t)$ (27)

Momentum : $\rho u(x,t) = \sum_i f_i(x,t)\, c_i$ (28)

Temperature : $T = \sum_i g_i(x,t)$ (29)

Local Nusselt number (Nu) calculation is essential in this work for predicting the heat transfer quantitatively, which is nothing but the ratio of convective heat transfer to conductive heat transfer. The following equations are used to calculate the local Nusselt number (Eq.25) and average Nusselt number (Eq.26).

$$Nu_{local} = \pm\frac{\partial T}{\partial y}\bigg|_{walls} \tag{30}$$

$$\overline{Nu} = \int_0^1 Nu_{local}\,dy \tag{31}$$

Recent Studies: Natural convection using LBM in recent years has been review briefly in the following section. Table.1, presents the various studies reported in the latest research work related to natural convection and there problem descriptions.

Cavity: Dixit and Babu (2006), studied the characteristics of high Rayleigh number natural convection in a square cavity. The simulation is carried out for insulated horizontal and isothermal vertical walls. Since in LBM uniform grid requires high grid requirements for higher *Ra* they used the interpolation supplemented LBM and the results obtained shows this model is valid for higher *Ra* also. Without using any particular turbulence model, the calculations for turbulence flow is done and validated with existing literature. Since this model incorporates non-uniform grid with LBM, the authors used very fine grid size at the walls where y^+ value is less than 0.3. The authors mentioned that this model is very efficient up to Ra = 10^8, beyond this there are small variations in Nusselt number calculations even though the maximum velocity values are almost accurate. And also, the computing time taken for convergence is very high for these higher level Ra's. The critical finding of this work is the capability of the LBM to predict the mean velocity turbulent boundary layer profile accurately precisely without utilizing any turbulence model. Kao (2008), analyzed the macroscopic and mesoscopic natural convection flows in a rectangular cavity. The simulation is done to recognize the convective-dominated stationary, time-independent steady flow. The spectral information of secondary instability with an oscillatory flow is then investigated using a spectrum analysis of the fast Fourier transform technique. From the outcomes, it is comprehended that the primary instability is generated at the critical Rayleigh number between the ranges of 10^3 to 10^4 irrespective of Knudsen number (*Kn*) and aspect ratio of the cavity. However, the bifurcation to secondary instabilities takes place only at certain aspect ratios and Knudsen numbers. Mohamad et al. 2009 researched the natural convection heat transfer in the open-ended cavity. In this study, they investigated the flow in the open-ended cavity for different aspect ratios and Rayleigh numbers by utilizing unique boundary conditions for the open ended cavity. The results are then validated with the results from FVM and FDM. The results show that expanding aspect ratio for a given Ra, the average Nusselt number diminishes linearly, this shows the heat transfer by the convective regime is getting suppressed. Furthermore, the center of circulation is found nearly at the center for Ra = 10^4 and when the Ra increases the center of circulation moves towards the top corner. And for the same Ra number the strength of circulation increases with increasing aspect ratio.

Mussa (2011), explored the natural convection heat transfer in an enclosure using Cubic-Interpolated Pseudo-particle (CIP) LBM. They utilized D2Q9 lattice model for density distribution field and D2Q4 lattice model for temperature distribution field. The CIP technique is found to be producing stable results. They found that the symmetrical position of the heat source was the best position for high heat convection. The authors proposed that this result could be taken into account when designing a cooling system for high-powered electronic chips.

Jourabian et al. 2012 studied the melting process with natural convection in an inclined cavity. To eradicate the problem arising during phase change, which occurs in normal LBM, thus, enthalpy based LBM is used to solve this problem. The simulation is done for fixed Stefan number of 10, *Ra* varies from 10^4 to 10^6, and various inclination angles. From the results, an increase in *Ra* to increase the rate of melting at each inclination angle also if the cavity is inclined anti-clockwise, the heat transfer is dominated by convection regime and if the inclination is done on clockwise the heat transfer was dominated by conduction regime.

Zhuo and Zhong (2013), proposed a novel thermal filter-matrix lattice Boltzmann model (FMLBM) based on large eddy simulation (LES) of turbulent natural convection. They introduced the Vreman subgrid-scale eddyviscosity model into the framework of LES to precisely predict the flow in the near-wall zone. The results are analyzed for higher order Rayleigh numbers to obtain the turbulent flows in the given square cavity domain. They showed the transition of laminar to turbulence when Ra increases from 10^7 to 10^8. Hence proper minimization in higher order terms can increase the numerical stability and accuracy of the model. The LES-FMLBM is used to calculate the turbulent flow of Ra 10^9 to $10^{10,}$ and the results are compared with other numerical literature and found to be in agreement. Thus, for simulating higher order Ra, this model can be applied. Dubois et al. 2015 presented anisotropic MRT-LBM simulation of 2D natural convection in a square cavity using a separate distribution function to solve the temperature field. The analysis was performed for anisotropic thermal conditions and compared with isotropic thermal conditions.

Three-dimensional: Peng (2004), analyzed the natural convection of air in a cubical enclosure that is heated differentially at two vertical side walls and other sides are kept adiabatic. They used two types of lattice arrangements (D3Q15 and D3Q19) in the study. They used nonuniform grids to increases the accuracy of the problem, for this purpose Taylor series expansion and least square based LBM (TLLBM) is used. The results indicate that the D3Q19 model provides a more stable solution when the problem involves higher level Rayleigh number (>10^5) even though it occupies more space for same grid size. Li (2016), studied a three-dimensional natural convection heat transfer and fluid flow in a cubic cavity using double MRT-LBM model. Three types of cubic natural convection problem were considered in the analysis at two different Rayleigh numbers (Ra = 1×10^4 and 1×10^5). For all three conditions, two opposite vertical walls are kept at different temperatures (hot and cold walls) and other four walls are maintained either adiabatic or constant temperature conditions. The velocity and temperature fields were solved using D3Q19 – MRT and D3Q7 – MRT models, respectively. For the selected three conditions with different Ra, the temperature field, hot surface Nusselt number, average Nusselt number, velocity field and maximum velocities in different directions were presented.

FDLBM: Jami (2006), simulated natural convection heat transfer in an inclined enclosure, differentially heated vertical walls, with inclined partitions attached to its hot wall. The top and bottom horizontal walls of the cavity are insulated. The simulation is done by using the hybrid method of incorporating LBM and FDM. The examination is primarily centered on the impacts of partition length, partition inclination angle, partitions number and aspect ratio of the cavity. The outcome demonstrates that the heat transfer rate reduction increases with increasing in the length of partition when the partition is inclined. The average Nusselt number is constant when the length of the partition is beyond$3\sqrt{2/8}$, even though the partition angle is varied. For aspect ratio 6, the average Nusselt number decreases with the number of partitions mainly when the partition inclination is at 45 degrees. Shi (2006), analyzed the natural convection heat transfer in a horizontal concentric annulus bounded by two stationary cylinders with different temperatures. The simulation is done by utilizing finite difference based lattice Boltzmann model using BGK collision factor. The authors used diverse grid sizes for distinctive Ra values to acquire precision and effective computational time. Also to maintain numerical stability Courant-Friedricks-Lewey (CFL) is maintained to be smaller than 1.0. The results show that for lower Ra the isotherms observed are just marginally changed from the isotherms obtained for pure conduction. Whereas for higher Ra, the isotherms shows the critical change that affirms the higher rate of convective heat transfer. The simulation is carried out the up-to-the scope of $2.0x10^5$ only, henceforth the turbulence flow is not observed in the study. It is likewise noted that for the current study, the FDLBGK model becomes unstable when the coefficient of ‘a’ used in the governing equations is equal to or below 0.5.

Moufekkir (2012), analyzed the natural convection and volumetric radiation in a tilted square enclosure using hybrid LBM. In this work multiple relaxation times LBM model is used to calculate the velocity fields, and FDM is used to calculate the temperature fields. Also, to that to analyze the radiation, the radioactive transfer equation is treated by the discrete ordinates method using the S8 quadrature to evaluate the source term of the energy equation. The simulation is done for various ranges of Ra and inclination angle of the cavity with the presence of radiation or not. Kefayati (2014), investigated the effect of magnetic field on natural convection of non-Newtonian power-law fluids in a cavity by linear heat treatment. The simulation is carried out for a wide scope of Rayleigh numbers, Hartmann number, and power law index. The author noted that the heat transfer rate increases with increasing Rayleigh number regardless of power indexes and at the same time decreases with increases in Hartmann number. The impacts of dilatant and pseudo-plastic fluids on heat transfer increase when Ra increases. In later days (Kefayati, 2014), researched the above conditions with sinusoidal heat treatment with inclined magnetic field and in another research work he used molten polymer non-Newtonian fluid. Kefayati (2016), numerically investigated the heat transfer and entropy generation on MHD laminar natural convection of non-Newtonian nanofluids in a square cavity using Finite Difference Lattice Boltzmann Method (FDLBM). The volume fraction of Cu nanoparticles was varied from 0 to 0.06 with water (base fluid) to analyze the augmentation of heat transfer rate. Similarly, Rayleigh number (Ra = 10^4 to 10^5), power law index (n = 0.6 to 1), Hartmann number (Ha = 0 to 90) was also varied to investigate the effects on fluid flow and heat transfer rate in a non-Newtonian fluid. They observed the following few important conclusions that the increase in Ha resulted in a decrease in entropy generation due to the magnetic field. The increase in nanoparticle enhances the heat transfer rate however, it is found to decrease with increase in power law index.

FVLBM: Mondal and Li (2010), studied the effect of volumetric radiation on the natural convection in a square cavity containing on absorbing, emitting and scattering medium. In this simulation, they used non-uniform lattices. In this model, FVM is used to compute the radiation phenomenon in the cavity. The results obtained is compared with the uniform lattice and found that non-uniform lattice LBM gives accurate results and also its computational effectiveness is high. The results show that the impact of radiation on streamlines are more significant for higher values Ra. Although, this effect is not very sensitive to the values of the scattering albedo and the extinction coefficient. They likewise found that the extinction coefficient high impact on isotherms. Li (2014), researched the natural convection in a differentially heated square cavity by combining lattice Boltzmann and finite volume method. In this study, they used D2Q9 and D2Q5 lattice model for velocity and thermal fields respectively. For the velocity field lattice Boltzmann model was used and for the temperature field,SIMPLE algorithm is applied to the finite volume method. To the couple, the two diverse systems, non-equilibrium extrapolation scheme was used. The recreations were carried out for diverse Rayleigh numbers, and results were contrasted with pure FVM and pure LBM method.

FEDBE: Seino (2011), explored the natural convection heat transfer using thermal finite element discrete Boltzmann equation. Due to the disadvantage of not easily able to apply to body fitted coordinates for its formulation, the LBM is combined with finite element discrete method. By using this model the authors studied the natural convection in a square cavity and the Rayleigh-Benard convection. By validating the results with other literature, this method is authenticated. The results show that by using this method the simulation using complex lattices can be achieved with good accuracy. They conclude that this finite element discrete Boltzmann equation can overcome the weakness of LBM and can be applied to complex boundaries and problem with relative ease.

Porous media: Seta (2006), concentrated on the effects of natural convection in porous media. The influence of porous media is considered by introducing the porosity to the equilibrium distribution function and by adding force term to the evolution equation. The simulation is carried out for an extensive variety of Ra and Darcy numbers. For a given Da and porosity, the average Nusselt number increases with Ra and for a given Ra and porosity. The results are compared with the finite difference method and found out that LBM method is more effective and less time consumption than FDM. Haghshenas (2010), explored the natural convection heat transfer in an open-ended cavity filled with porous media. In this study, additional to the general double population thermal model, Taylor series expansion and least square based thermal model is implemented. The porous media is included in the physical domain by adding the porosity into the equilibrium distribution function and involving a force term to the evolution equation. The simulation is done for a various range of Ra and porosity. From the results, it is understood that for low Rayleigh number, the average Nusselt number is practically free from all the values of porosity. Zhao (2010), investigated the natural convection in porous media. They studied in detail the flow and heat transfer at the pore level. The effects of pore density and porosity on the natural convection are analyzed. The aftermath of the study shows that the overall heat transfer will be increased by lowering the porosity and cell size. They also found that the square porous medium can have a higher heat transfer performance than sphere porous medium, which is due to the strong flow mixing and more surface area.

Barania (2011), studied the natural convection around horizontal elliptic cylinder inside a square enclosure. They simulated to analyze the thermal behavior of the fluid at various vertical positions of the inner cylinder for different Rayleigh numbers. From the results, it is clear that location of the inner cylinder and Ra have a huge impact on the streamline and isotherms. They also observed that the average Nusselt number increases with the increase of Rayleigh number and its minimum value become more distant from the top of the inner cylinder with increasing Ra. Gao (2014), simulated natural convection in porous media under local thermal non-equilibrium conditions by choosing an appropriate selection of equilibrium distribution functions and discrete source terms. The study is done for both steady state and transient conditions. The outcome of the study illustrates that local thermal equilibrium is not assured when there is a high ratio of solid to fluid thermal conductivities, Rayleigh number, and Darcy number. The authors presume that this model can be directly applied to investigate forced convection and mixed convection in porous media since the model is more summed up in structure.Hu et al. 2016 have performed the numerical simulation of natural convection in a square enclosure with a cylinder covered with a porous layer. In order to avoid the complexity, single domain LBM method was adopted for both fluid and porous media region. They have proposed a new technique to predict the velocity and temperature at the curved boundaries using immersed boundary method. The effects of thermal conductivity ratio ($1 \leq Rc \leq 10$), Darcy number ($10^{-1} \leq Da \leq 10^{-6}$) and Rayleigh number ($10^3 \leq Ra \leq 10^6$) on streamlines, isotherms and Nusselt number were analyzed.

Nanofluid: Kefayati (2011), investigated the natural convection in the tall cavity using water/SiO_2 nanofluid. In this work the properties of the nanofluid are considered to be temperature-dependent. The simulation is carried out for different Rayleigh numbers, volume fractions and aspect ratio of the cavity. They found that a critical value of Ra existed around 10^4 to 10^5 for the effect of nanoparticles, from that we can understand that the Nusselt number increases when the Ra increases from 10^3 to 10^4 and decreases when Ra is further increased to 10^5. Lai and Yang (2011), analyzed the impacts of natural convection heat transfer in a square cavity loaded with Al_2O_3/water nanofluids. They have taken the nanoparticles size of 42 nm for the simulation. From the results, the average Nusselt number increases with an increasing average fluid temperature and the increase of temperature difference imposed between the side walls. Fattahi (2012), researched the natural convection in a square cavity filled with water-based nanofluid containing Al_2O_3 or Cu nanoparticles. They used Chon and Brinkmann model to calculate the effective thermal conductivity and viscosity of the nanofluids, respectively. The outcome demonstrates that the average Nusselt number increases with increasing the solid volume fraction for both types of nanoparticles. They also found that average Nusselt number increases with increasing Ra but the only upto a certain range of solid volume fraction.

Kefayati (2012), studied natural convection in an open-ended enclosure that contains water/copper nanofluid. The simulation is carried out for various ranges of parameters like Rayleigh number, the volume fraction of nanoparticles, and aspect ratio of the cavity. The results demonstratethat the decrease in aspect ratio and increase in Rayleigh number enhance the heat transfer rate. They found that the impact of nanoparticles was observed to be maximum when the aspect ratio is 2. Kefayati (2013), analyzed the impact of magnetic field on natural convection in a nanofluid cavity filled with sinusoidal temperature distribution on one side of the vertical wall. The simulation is carried out for various ranges of parameters like Rayleigh number, the volume fraction of nanoparticles, phase deviation for temperature distribution and various Hartmann number while considering the magnetic field is applied horizontally to the enclosure. Sheikholeslami (2013), concentrated the natural convection heat transfer in a square cavity of curve boundaries which is filled with Cu-water nanofluid. In this study to calculate the effective thermal conductivity and viscosity of nanofluid by Maxell-Garnetts and Brinkman models respectively. Special boundary conditions are used to treat the curved boundaries. They conclude that maximum average Nusselt numbers obtained when the cavity is inclined at 30 degrees. Sheikholeslami (2013), studied the magneto hydrodynamic effects on natural convection heat transfer Al_2O_3/water nanofluid flow and heat transfer in a semi-annulus enclosure. In this study, Koo-Kleininstreur-Li correlation was used to calculate the effective thermal model conductivity and viscosity of nanofluid. Hussein (2014), studied the magneto hydrodynamic natural convection heat transfer in an open enclosure that is filled with Cu-water nanofluid. To simulate the effect of uniform magnetic field, the MDF model as used. The results demonstrate that by increasing Ra for low Hartmann number the rate of heat transfer is pure because of convection and if Ha was increased for lower Ra, the conduction effect dominates the heat transfer rate. And also, the effects of nanoparticles on maximum stream function values decreases with increasing in Hartmann number.

Sheikholeslami (2014), researched the MHD flow in concentric annulus filled with Cu-water nanofluid. The simulation is done for various ranges of Hartmann number, nanoparticle volume fraction, Rayleigh number and aspect ratio of the cavity. They used different shapes of nanoparticles, spherical and cylindrical nanotubes. The authors concluded that the average Nusselt number increases with increasing in Rayleigh number, volume fraction and decreases with the increase of Hartmann number. Yoshida and Nagaoka (2014), proposed a curvilinear grid system to simulate convection and diffusion. They stated that there was no need to utilize coarse-graining or an interpolation procedure for this model, thus keeping the algorithm as simple as original lattice Boltzmann method. The simulation was done by implementing MRT model. They solved some numerical problems to validate this model and mainly an axially symmetric problem in which the diffusion flux at an oblate hemispheroid is simulated using a body-fitted orthogonal curvilinear grid system. Zhang and Che (2016), presented a two-dimensional, magneto hydrodynamic fluid flow and heat transfer by natural convection of copper-water nanofluids in an inclined square cavity with four internal heat sources. Double MRT thermal LBM was used to simulate the problem. The horizontal walls were thermally insulated and the vertical walls were maintained constant at cold temperatures. Hot temperatures were maintained at four inner heat sources. The nanofluids were filled between the outer square cavity and the inner heat sources. The effects of Hartmann and Rayleigh numbers, inclination angle and the volume fraction of nanoparticles on fluid flow and heat transfer were analyzed. The heat transfer rate was suppressed in the presence of magnetic field for all Ra and inclination angles.

Sheikholeslami and Ellahi (2015), have presented a three-dimensional mesoscopic simulation of magneto-hydrodynamics nanofluid natural convection heat transfer in a cubic cavity heated from below. Water-Al_2O_3 nanofluid is considered in the problem. The Koo-Kleinstreuer-Li (KKL) correlation is proposed to find the effects of Brownian motion on the effective viscosity and thermal conductivity of nanofluid. The D3Q19 model was used to find the velocity and temperature field in LBM model. The effects of Hartmann number, nanoparticle volume fraction and Rayleigh number on hydrothermal behaviour were studied. It was observed that the average Nusselt number increases with the increase of Rayleigh number and decrease of Hartmann number. Therefore the magnetic field can be used as flow and heat transfer controlled mechanism inside the cavity. Further (Sheikholeslami, 2015) extended their numerically investigated in lattice Boltzmann simulation of MHD natural convection heat transfer using water-Al_2O_3 nanofluid in a two-dimensional horizontal cylinder enclosed with an inner triangular cylinder.

D2Q4, D2Q5, D2Q9

D3Q15, D3Q19

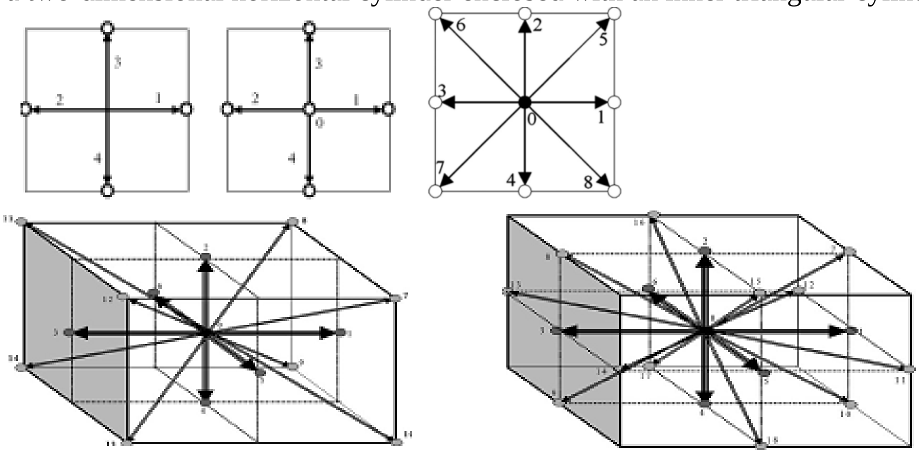

Figure 1. Lattice arrangements

Rayleigh-Benard: Kao and Yang (2007), analyzed Rayleigh-Benard natural convection using s a simple LB model with the Boussinesq approximation. This study investigates the Rayleigh-Benard from the threshold of the primary instability with a theoretical value of critical Rayleigh number 1707.76 to the regime near the flow bifurcation to the secondary instability.The study was carried out for a wide scope of Prandtl number 0.71 to 70 and Ra <10^5. From the results it is observed that bifurcation to secondary instability takes place at certain Prandtl numbers with an appropriate Rayleigh number, like Ra 48,000 for Pr = 6 and Ra 76,000 for Pr = 25. Some instabilities occur for higher Ra due to turbulence, which shows the employed LB model requires separate turbulence model to overcome the instabilities. Fu (2011), studied the characterization of Rayleigh convection in interfacial mass transfer using LBM and compared the results with experiment results. Sun (2011), analysed the dendritic growth of during alloy solidification in the presence of forced and natural convection. The dendritic growth is modeled using a solutal equilibrium approach, in which the evolution of the solid/liquid interface is driven by the difference between the local equilibrium composition and the local actual liquid composition. Lin (2012), proposed a model to simulate natural convection embedded with the complex solid object. They achieved this model by applying the closet nodes next to the boundary of the fluid domain as boundary nodes of the flow domain, and the temperature of the boundary node is obtained by linear interpolation between temperatures of a solid object and second fluid node far away. They simulated the Couette flow and an annulus between the square outer cavity and circular inner cylinder. The cavity wall temperature and cylinder wall temperature are maintained at different conditions. The position of the cylinder also changes.

Table.1. Geometry and the conditions used in the literatures

Authors and year	Lattice arrangement	Model and conditions
Peng (2004)	D3Q15, D3Q19.	TLLBM
Dixit and Babu (2006)	D2Q9	interpolation supplemented lattice Boltzmann method
Kao and Yang 2007	D2Q9	SRT-BGK and FFT
Mohamad (2009)	D2Q9, D2Q4	SRT-BGK
Mussa (2011)	D2Q9, D2Q4	CIP-LBM
Jourabian (2012)	D2Q9, D2Q5	SRT-BGK
Zhuo and Zhong (2013)	D2Q9	FMLBM Natural convection for turbulent flows
Jami (2006)	D2Q9	FDLBM Heated partitions
Shi (2006)	D2Q9	FDLBM Horizontal concentric annulus
Moufekkir (2012)	D2Q9	MRT-FDLBM Volumetric radiation, inclination
Kefayati (2014)	D2Q9	FDLBM Magnetic field, non-Newtonian fluid, sinusoidal temperature profile.
Kefayati (2014)	D2Q9	FDLBM Molten polymer non-Newtonian fluid, sinusoidal temperature profile.
Mondal and Li (2010)	D2Q9	FVLBM non-uniform grid, radiation
Li (2014)	D2Q9, D2Q5	FVLBM
Seino (2011)	D2Q9	FEDBE-LBM
Seta (2006)	D2Q9	SRT-BGK Porous media
Haghshenas (2010)	D2Q9	SRT-BGK porous media, open ended cavity
Zhao (2010)	D2Q9	SRT-BGK porous media
Bararnia (2011)	D2Q9	SRT-BGK Heat transfer between square outer cylinder and heated elliptic inner cylinder.
Gao and Chen (2011)	D2Q9	SRT-BGK Porous media, Phase change medium
Gao (2014)	D2Q9	SRT-BGK Porous media, non-equilibrium thermal condition
Kefayati (2011)	D2Q9	SRT-BGK Nanofluid water/$Si0_2$
Lai and Yang (2011)	D2Q9	SRT-BGK Al_2O_3/water nanofluid
Fattahi (2012)	D2Q9	SRT-BGK Cu and Al_2O_3/water nanofluid
Kefayati (2012)	D2Q9	SRT-BGK Cu /water nanofluid, open ended cavity
Kefayati (2013)	D2Q9	SRT-BGK Al_2O_3/water nanofluid, Magnetic field, Sinusoidal heat treatment
Sheikholeslami (2013)	D2Q9	SRT-BGK Al_2O_3/water nanofluid, semi annulus enclosure, MHD
Hussein (2014)	D2Q9	SRT-BGK Cu/water nanofluid, open end cavity
Sheikholeslami (2013)	D2Q9	SRT-BGK Cu/water nanofluid, square cavity with curved boundaries
Lin (2012)	D2Q9	SRT-BGK Complex geometry, special boundary conditions.
Sun (2011)	D2Q9	SRT-BGK Dendritic growth during alloy solidification
Kao and Yang (2007)	D2Q9	SRT-BGK Rayleigh-Benard
Fu (2011)	D2Q9	SRT-BGK Rayleigh-Benard

Ren and Chan (2016), presented a double-diffusive convection with Soret and Dufour effects in a rectangular enclosure was presented using LBM model. NVIDIA’s CUDA platform was used to accelerate the computation time. Parallel tasks were taken care by graphics processing units (GPU) and sequential steps in the computation were handled by CPU. The effects of Rayleigh number, Buoyancy ratios, Prandtl numbers, Lewis numbers, aspect ratios, and, Soret and Dufour coefficients in the fluid flow, temperature and concentration fields are analyzed. The results showed that the average Nusselt and Sherwood numbers were found to be increased with Rayleigh number, Buoyancy ratios, Prandtl numbers, Lewis numbers, aspect ratios, Soret and Dufour Coefficients. Ren and Chan, (2016), the conjugate natural convective heat transfer in a square cavity with solid obstacles was investigated using D2Q9 LBM. Temperature distributions and streamline patterns were analyzed for different Ra, thermal diffusivity ratios and a number of solid blocks. They were solved the above problem using parallel hardware of graphic processor units (GPU) using a CUDA platform and it was found that parallel computing accelerates the computation by a factor up to 20 as compared to non-parallel CPU.

2. CONCLUSION

A comprehensive review of previous studies on natural convection heat transfer and fluid flow by lattice Boltzmann method was presented in this article. The impact of several parameters, such as *Ra* number, aspect ratio, geometrical parameters, *Re*number, and heat flux, were also extensively investigated. In addition, extensive reviews for preparation, parameters, mechanisms, characteristic, and heat transfer enhancement with various applications were reported. The mechanisms involved in the heat transport phenomena remain not fully understood; nevertheless, the increased heat transfer capabilities continue to attract the attention of researchers. The review shows the importance and simplicity of the natural convection problem and the capability of lattice Boltzmann method to adopt in the computational field.

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