

Cube Root Of A Positive Integer Using LDM

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Abstract

In numerical methods, there are several algorithms to find the square root and cube root of a positive integer. Among all algorithms, one very simple algorithm is 'division algorithm' to find square root of positive integer. In this paper I introduce a modified algorithm of division algorithm to compute the cube root of any positive number. By using this algorithm, we can find the cube root of 5, 10 or up to n-digit number. Some experimental results have been discussed in this paper.

Keywords: - Division algorithm for square root, Convergence criteria, Long division method.

1- INTRODUCTION

In these days there are many numerical approximation methods available to find the cube root, but all of them require an initial value and if the initial value is bit far away from the actual root, the algorithm will work very slowly, i.e. we

need much iteration to find the roots. The algorithm explained in this paper is one of the simplest techniques to find the cube root of any positive large number. The terminology of this algorithm is so easy, that we don't need to understand any higher level concept of mathematics.

2- METHODOLOGY

For Perfect cube numbers Cube root of 4, 5 or 6–digit number

	xy
x	abcdef
	-x ³
<u>3x², 3x, y</u>	mnodef
<u>3x², 3x, 1</u>	-3x ² y, 3xy ² , y ³
* y, y ² , y ³	0000
<u>3x²y, 3xy², y³</u>	

Make group of 3-digits starting from one's place of the number. $x^3 \leq abc$

<u>3x², 3x, y</u>	
* y	
<u>pqrstu</u>	pqrstu ≤ mnodef

Cube root of 7, 8 or 9-digit number

	xyz
x	abcdeghi -x ³
3x ² ,3x,y	rstdef
3x ² ,3x,1 * y,y ² ,y ³	-3x ² y,3xy ² ,y ³ mnopqrghi
3x ² y,3xy ² ,y ³	-3x ² y ² z,3xyz ² ,z ³
3x ² y ² ,3xy,z	00000
3x ² y ² ,3xy,1 * z,z ² ,z ³	
3x ² y ² z,3xyz ² ,z ³	

Make group of 3-digits starting from one's place of the number. $x^3 \leq abc$

$$\begin{array}{r} 3x^2,3x,y \\ * \quad y \\ \hline \end{array}$$

$$\underline{\quad jkluvw} \quad jkluvw \leq rstdef$$

$$rstdef \leq 3x^2y,3xy^2,y^3$$

$$\begin{array}{r} 3x^2y^2,3xy,z \\ * \quad z \\ \hline \end{array}$$

$$\underline{\quad j_1k_1l_1x_1y_1z_1x_2y_2z_2} \quad j_1k_1l_1x_1y_1z_1x_2y_2z_2 \leq mnopqrghi$$

Cube root of 10, 11, 12-digit number or up to n-digit number

The same algorithm will be followed up to n- digit number.

The number of steps increases after increase in 3-digit group of number.

For Non-Perfect cube numbers

	xy.z----
x	abcdef
	-x ³
<u>3x²,3x,y</u>	mnodef
<u>3x²,3x,1</u>	-3x ² y,3xy ² ,y ³
* y, y ² ,y ³	pqrstg000
<u>3x²y,3xy²,y³</u>	-3x ² y ² z,3xyz ² ,z ³
<u>3x²y²,3xy,z</u>	p ₁ q ₁ r ₁ s ₁ t ₁ g ₁ u ₁ v ₁ w ₁
<u>3x²y²,3xy,1</u>	
* z,z ² ,z ³	
<u>3x²y²z,3xyz²,z³</u>	

And these steps will go on till our desired decimal value if it's non-terminating. Three zeroes will be added at back of remainder after every step for decimal quotient.

Make group of 3-digits starting from one's place of the number. $x^3 \leq abc$

$$\begin{array}{r} 3x^2, 3x, y \\ * \quad \quad y \\ \hline \end{array}$$

$$jkluvw \leq mnodef$$

$$mnodef \leq 3x^2y, 3xy^2, y^3$$

$$\begin{array}{r} 3x^2y^2, 3xy, z \\ * \quad \quad \quad z \\ \hline \end{array}$$

$$j_1k_1l_1x_1y_1z_1x_2y_2z_2 \leq pqrstg000$$

$$pqrstg000 \leq 3x^2y^2z, 3xyz^2, z^3$$

3- EXAMPLES

	99	
9	970299	
	-729	243
24579	241299	+ 27
243,27,1	- 241299	2457
* 9, 81,729	000000	24579*9=221211
2187,2187,729		221211<241299
		2187
		2187
		+ 729
		241299

	1234	
1	1879080904	3*12 ² =432
	-1	3*12=36
332	879	432
3,3,1	- 728	+ 36
* 2,4,8	151080	<u>4356</u>
6,12,8	- 132867	
43563	18213904	43563*3=130689
432,36,1	- 18213904	130689<151080
* 3,9,27	00000000	3*123 ² =45387
1296,324,27		3*123=369
4542394		45387
45387,369,1		+ 369
* 4,16,64		<u>45756</u>
181548,5904,64		457564*4=18169576
		18169576<18213904
		181548
		5904
		+ 64
		18213904

4- TABLE

Number	Cube root (exact result)	Cube root (by this algorithm)	Error
2	1.2599	1.2599	0
1728	12	12	0
970299	99	99	0
1404928	112	112	0
287676889	660.1383	660.1383	0
1879080904	1234	1234	0
678777767588	8788.3876	8788.3876	0

5- CONCLUSION

Among all the algorithms, this algorithm is very easy, efficient and simple to find the cube root of any positive integer. Although, the complexity of calculation increases with the increase in the number of digits. In future, our aim is more improvement in this method and resolve its complexity.

6- REFERENCES

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