SPATIO-TEMPORAL OPTIMAL LAW ENFORCEMENT USING STACKELBERG GAMES

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ABSTRACT

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Every year, road accidents claim the lives of around 1.2 million worldwide (USDOT-NHTSA, 2012). *Deploying speed traps helps bounding vehicles speed and reducing collisions. Nevertheless, deterministic speed traps deployment in both spatial and temporal domains, allow drivers to learn and anticipate covered areas. In this paper, we present a novel framework that provides randomized speed traps deployment schedule. It uses game theory in order to model drivers and law enforcers' behavior. In this context, Stackelberg security game is used to derive best strategies to deploy. The game optimal solution maximizes law enforcer utility. This research work aims to optimize the deployment of speed traps on Lebanese highways according to the accidents probability input data. This work complements the near real time accident map provided by the Lebanese National Council for Scientific Research and designs an optimal speed trap map targeting Lebanese highways.*

Keywords: Stackelberg games, law enforcement, game theory.

INTRODUCTION

The growing mobility of people and goods has a very high societal cost in terms of collisions, fatalities and injured people every year (USDOT-NHTSA, 2012; USDOT-NHTSA-DOTHS 811, 2011; Naja, 2013). The US National Highway Traffic Safety Administration (NHTSA) provides regularly statistics on vehicle type proportions in traffic crashes. The NHTSA statistics on police reported motor vehicle show that the majority of accidents occur with passenger cars (56.3%) (USDOT-NHTSA, 2012). On the other hand, Lebanese Red Cross Road Accidents Statistics for 2014 reported 10866 accidents, 14516 injuries and 229 killed people (Lebanese Red Cross Roads Accidents Report for 2014). These alarming statistics emphasize the road safety problem and stress the urgent need to find a solution that reduces road collisions. *Traffic law enforcement* is the conventional solution adopted by the Internal Security Forces: Traffic law enforcement encloses the police actions to make sure that the law is being obeyed, in our case verifies if the traffic speed limit is respected. Usually it is based on speed trap deployment. Speed traps consist of hidden equipment, generally radar (object-detection system that uses radio waves to determine car speed), used by the police to detect drivers speed.

Nevertheless, speed trap deployment faces four major problems:

- First, current trap deployment is spatially deterministic: drivers may learn the speed traps locations and then can have relevant enforcement information.
- Second, speed traps deployment schedule is temporally deterministic. Consequently drivers anticipate coverage areas ahead of time.
- Third, Lebanese Internal Security Force (ISF) is in possession of a limited number of resources. Thus, speed traps coverage is restricted to certain Lebanese roads, leaving many roads unprotected.
- Fourth, current speed trap deployment does not take into account the distribution of accidents probability on-roads.

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This research paper strives to provide optimal speed traps allocation strategies randomized in space and time. In this context, we propose and evaluate an innovative platform designed to help the ISF in optimally allocating speed traps on Lebanese highways. It aims at scheduling the speed traps in a way that ensures the maximal coverage and at the same time considering randomness to avoid driver's predictability.

This optimal allocation will be achieved by modeling the problem as a Stacklberg security game (SSG) where the players are the law enforcer and the driver.

Law enforcement and game theory

A wide number of research papers was concerned in studying law enforcement .The majority of researchers formulate the problem using game theory, more precisely stackelberg game (Shieh *et al*.; Korzhyk, 2013); Bosansk´y *et al*., 2011; Jain *et al*., 2011; Brown *et al*., 2014).

In fact, Game theory is a mathematical tool developed to understand competitive situations in which rational decision makers interact to achieve their objectives (Turocy and Stengel, 2001; Prisner, 2014). In particular, a Stackelberg security game involves two players: a leader and a follower, where the leader (denoted as the defender) must protect a set of targets from the follower (denoted as the adversary). The defender has a finite number of resources $R=\{r_1,r_2,\ldots r_k\}$ with which to protect the set of targets $T=\{t_1,t_2\ldots,t_N\}$ against the adversary, where k is the number of available resources and N is the number of targets that should be protected such that $k<N$.

With the Stackelberg model, the defender chooses a mixed strategy first, and the attacker chooses a strategy after observing the defender's choice. The standard solution is then a Strong Stackelberg Equilibrium (SSE), (Korzhyk, 2013).

Law enforcement research work

There have been many models developed to solve the problem of randomizing law enforcement (Adler *et al*., 2013; Curtin *et al*., 2005; Sharma, 2007; Paruchuri, 2007). In the following, our efforts are oriented towards presenting the most important papers tackling the law enforcement randomization. In addition, a synthesis of these works is provided and brings the focus to the added value of our proposal.

A software called ARMOR (Assistant for randomized monitoring over routes) is described (Pita, 2008). The ARMOR system focuses on security measures at the airports and optimizes security resource allocation using Bayesian Stackelberg games. However, the DBOSS algorithm was not scalable due to the absence of the reduction of space of strategies. Another approach, called IRIS (Intelligent randomization in scheduling) is presented (Tsai *et al*., 2009). It randomizes schedules of air marshals on international flights based on Stackelberg game. In IRIS, the payoffs do not depend on the coverage vector, but on whether or not the target is attacked. This idea could not be applicable to the traffic domain where the enforcer's utility is higher when the road is covered. A solution called PROTECT (Port resilience operational tactical enforcement to combat terrorism) is proposed (Eric Shieh, 2013). This approach considers the maritime security of ports and waterways that faces increased risks such as terrorism and drugs trafficking. The linear program for this approach solves only the probability distribution over the defender's strategies and does not give an idea about the pure strategy chosen by the attacker. Yin, *et al*. (2012) presented a new work called TRUSTS (Tactical randomization for urban security in transit systems) designed to protect rail systems. In the train domain the patroller is tied to the predetermined transportation schedules, however in traffic patrolling the situation is more complex because the continuous nature of traffic patrolling should be taken into account.

The majority of these models focus on securing national infrastructure such as airports, historical landmarks, or a location of political or economic importance, and none have focused on traffic patrolling.

In this research work, an innovative patrolling platform is developed targeting Lebanese highways. Our contribution is four-fold:

1. We propose a space randomized speed trap deployment; in fact it takes into account accidental roads via probability of accidents.
2. This platform randomizes temporally resources allocation. Thus it overcomes driver's anticipation.
3. This novel tool avoids applying speed traps deployment in congested roads. This is achieved by taking into account congestion probability.

4. Law enforcement is applied on authentic highways dealing with complex roads. This is achieved by adopting a compact representation.

Accordingly, this study focused on the optimal allocation of speed traps using an innovative platform.

MATERIALS AND METHODS

Game model for speed trap allocation

The present section focuses on optimally allocating speed traps on highways. In fact, the ISF organization has a limited number of resources to deploy on Lebanese territory. Therefore, in order to help the ISF reducing car accidents and optimally deploying the limited speed traps, a dynamic strategy tuned to temporal parameters, accidents statistics, traffic, number of resources should be adopted.

In order to achieve this randomization, the decision of setting the speed traps is modeled as a SSG, where in this case the leader is the law enforcer and the follower is the driver. The law enforcer has a certain number of roads segments that should be covered using a finite number of speed traps, and the driver will choose the segment where to violate. The law enforcer will choose a mixed strategy so that the driver will be unsure of the exact place of the radars.

Payoff matrices

Two payoff matrices are considered in this study, one for the law enforcer and another for the driver related to their utility functions. The enforcer payoff is derived as follows:

- If the enforcer covers a certain road and in case the driver violates it, then the enforcer will get $G>0$ where G corresponds to the social welfare.
- In case the enforcer is not covering a road and if the driver violates it, then the enforcer will get a negative utility $-G*P_a(t)$, where P_a is the probability of accident at the considered road t.

The driver payoff is computed as follows:

- When the driver violates on a covered road segment, then he/she will be punished and will have to pay the ticket. Therefore his payoff in this case will be $g-\lambda s \leq 0$, where g is the net gain of the driver (the time saved by the driver), s is the punitive cost (the price of the ticket paid by the driver) and λ is a parameter transferring the punishment into the negative utility.
- If the driver violates on an uncovered road segment, then he will save time and get a positive utility g.

For more clarity, the following example gives an idea about setting 2 radars on 3 road segments *A*, *B* and *C*. In this case, we have C_3^2 possible strategies for the ISF which are: covering *A*, *B*; covering *A*, *C* or covering *B*, *C*. Thus, the driver will have to choose between violating segment *A*, violating segment *B* or violating segment *C*. In this case, the payoff matrix is as indicated in Table 1.

Table 1. Enforcer and driver's payoff matrix.

Enforcer/driver	A	B	C
A,B	G; g-λs	G; g-λs	-G*Pa(C);g
A,C	G; g-λs	-G*Pa(B);g	G; g-λs
B,C	-G*Pa(A);g	G; g-λs	G; g-λs

Compact representation

Our main concern is to reduce the space of strategies. In fact, enumerating the complete set of strategies presents a scalability challenge. Indeed, the more the number of roads is, the larger the strategies space is; leading to an increase of the program runtime in a polynomial way. In order to overcome this problem, we did proceed as follows:

- First, the study focused on highways and motorways where the traffic speed could reach more than 30mph≃ 50 km/h (Archer *et al*., 2008).

- Second, the aim was to reduce the space of strategies by developing a distributed algorithm to be executed by a local ISF agent. In fact, the Lebanese ISF organization deploys 25 centers in different Lebanese governorates (ISF-Internal Security Forces Website). Each center runs the tool restricted to its covered region. Thus, the software will manipulate a reduced number of roadways and will decrease the set of strategies.
- Third, our tool did not take into consideration congested road, ie. roads that witness high vehicular density. In fact, a high density impacts the velocity and forces the drivers to decelerate. Thus, speed enforcement does not pose a problem in this scenario.

Stackelberg equilibrium

In order to solve the Stackelberg game, a mixed integer quadratic program (MIQP), was defined and a linearized equivalent mixed integer linear program (MILP) was then presented. We will take the mixed strategy for the enforcer that will give the highest payoff when the driver plays a strategy that maximizes its utility was adopted, whereas:

- X and Q represent the index sets of enforcer and driver's pure strategies, respectively.
- $x=<x_i>$ the enforcer's mixed strategy vector, where x_i is the probability of using strategy i.
- $q=<q_j>$ the driver's strategies vector where $q_j \epsilon \{0,1\}$, qi is equal to 1 when the strategy j is used by the driver.
- X and Q represent the index sets of the leader and follower's pure strategies respectively.

The payoff matrices R and C are defined such that R_{ij} is the reward of the leader and C_{ij} is the reward of the follower when the leader adopts pure strategy i and the follower adopts pure strategy j.

The enforcer's MIQP problem is defined in Equations 1 to 7 as follows:

$$\max_{x,q,a} \quad \sum_{i\in X}\sum_{j\in Q} R_{ij}x_i q_j \qquad (1)$$

$$\text{s.t.} \quad \sum_{i\in X} x_i = 1 \qquad (2)$$

$$\sum_{j\in Q} q_j = 1 \qquad (3)$$

$$0 \le (a - \sum_{i\in X} C_{ij}x_i) \le (1-q_j)M \qquad (4)$$

$$x_i \in [0,1] \qquad (5)$$

$$q_j \in \{0,1\} \qquad (6)$$

$$a \in \Re \qquad (7)$$

The first and fourth constraints define a x_i as a probability distribution over the strategies set. The second and fifth constraints limit the vector q to a pure distribution over the driver's strategies that is q has one coordinate equals to one which corresponds to the pure strategy chosen by the driver, and the remaining coordinates are equal to zero. The third constraint ensures that $q_j=1$ only for strategy j that is optimal for the driver: The leftmost inequality ensures that for all $j\epsilon Q$, $a \ge \sum_{i\in X} C_{ij}x_i$. This means that for a given vector x, a is an upper bound for the driver for any action. The rightmost inequality is inactive for every action where $q_j=0$ since M is a large positive quantity. For the action that has $q_j= 1$ this inequality states that the adversary's payoff for this action must be ≥a, which combined with the previous inequality shows that this action must be optimal for the driver. We linearized the previous MIQP through the change of $z_{ij}=x_i q_j$, thus we will obtain the following MILP in Equations 8 to 16 as follows:

$$\max_{q,z,a} \quad \sum_{i\in X}\sum_{j\in Q} R_{ij}\, z_{ij} \qquad (8)$$

$$\text{s.t.} \quad \sum_{i\in X}\sum_{j\in Q} z_{ij} = 1 \qquad (9)$$

$$\sum_{j\in Q} z_{ij} \le 1 \qquad (10)$$

$$q_{j \le \sum_{i\in X} z_{ij}} \le 1 \qquad (11)$$

$$\sum_{j\in Q} q_j = 1 \qquad (12)$$

$$0 \le (a - \sum_{i\in X} C_{ij}(\sum_{h\in Q} z_{ih})) \le (1-q_j)M \qquad (13)$$

$$z_{ij} \in [0 \ldots 1] \qquad (14)$$

$$q_j \in \{0,1\} \qquad (15)$$

$$a \in \Re \qquad (16)$$

Spatiotemporal optimal patrolling platform

The proposed innovative spatiotemporal speed trap allocation platform derives best strategy of speed trap deployment. It consists of five modules:

- Module 1 manipulates the following inputs: date and time, number of available resources, traffic intensity, and probability of accidents and the set of roads.
- Module 2 identify uncongested roads that are prone to speed traps deployment and proposes law enforcer speed traps deployment strategies on these selected uncongested roads.
- Module 3 computes law enforcers and drivers utilities. It provides payoff matrices needed for equilibrium resolution.
- Module 4 computes probability distribution of resources based on SSE.
- Module 5 of spatiotemporal speed trap allocation tool will produce a schedule to be implemented by the law enforcer.

RESULTS

This section evaluates the results obtained when the linear program with the SSG is applied to the Lebanese Highways domain. The results obtained are based on real data provided by CNRS.

Authentic data inputs are injected into this platform. Several program batches are conducted on 4 different scenarios. Scenario 1 evaluates the impact of shifts and resources on enforcer's utility. Scenario 2 studies the probability distribution with different probability of accidents. Scenario 3 tackles extreme accidents probability and evaluates the impact of resources on enforcer's utility. Scenario 4 presents a comparative study of the enforcer's utility in both cases: deterministic and random deployment.

Scenario 1: Study of enforcer's utility variation

Scenario 1 aims at evaluating the impact of deploying number of resources and the different shifts (Shift1 (from 8AM to 1 PM), Shift2 (from 2PM to 7PM) and Shift3 (From 8PM to 1AM) on the enforcer's utility. Scenario 1 parameters are identified in Table 2.

Table 2. Scenario 1 parameters.

Roads		Ant-elias	Dbaye	Nakash	Naher Elkaleb	Kaslik	Jounieh	Tabarja	Bouar	Nahr Brahim	halat	jbeil	Barb-ara
Traffic Density		0.3	0.35	0.34	0.3	0.45	0.3	0.3	0.2	0.7	0.7	0.65	0,7
Accident proba-bility	**Shift 1**	0.38	0.35	0.34	0.37	0.38	0.38	0.35	0.38	0.37	0.36	0.37	0.37
	Shift 2	0.31	0.32	0.31	0.32	0.29	0.30	0.31	0.30	0.28	0.30	0.31	0.31
	Shift 3	0.11	0.12	0.11	0.12	0.09	0.10	0.11	0.08	0.10	0.11	0.11	0.12
Number of resources		2-3-4-5-6											

Table 2 indicates the different roads, with their traffic density, the probability of accidents on each road, at each shift duration and the different number of resources available. We computed the enforcer's utility for each shift and number of resources as exhibited in Figure 1.

The utility of the enforcer had the lowest values in the first patrolling shift (from 8 AM to 1 PM) because of the high probability of accidents at this interval of time of the day. In the second and third patrolling shift, the probability of accidents starts to decrease and therefore the enforcer's utility is higher than the first shift. It is to be noted that the more we deploy resources on roadways, the more the enforcer's utility increases.

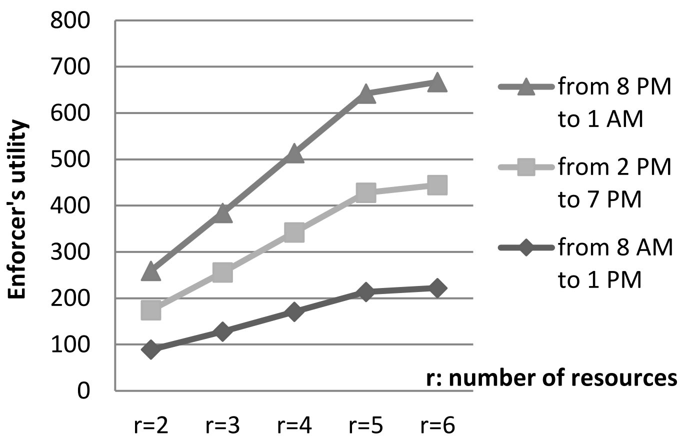

Figure 1. Enforcer's utility in three different epochs of the day.

Scenario 2: Study of the probability distribution over strategies

In this scenario, the probability distribution over the strategies according to various accidents probabilities were evaluated. Table 3 shows the list of parameters considered.

Table 3. Scenario 2 parameters.

Number of resources	2			
List of roads (L) / **Probability of accidents (P)**	Jounieh	Jbeil	Antelias	Dbaye
P1:	0.75	0.85	0.80	0.40
P2:	0.75	0.25	0.25	0.40
Congestion level (%)	30	30	25	30

Figure 2 shows the variation of the strategies probability along the variation of the probability of accidents on each road. When placing 2 resources on Jbeil and Antelias highways, with the probability of accidents equal to 0.8 and 0.85, respectively, our tool converges towards 0.499999989 as a probability of strategy Antelias-Jbeil. This result confirms the relevance of deploying speed traps on accidental roads. When the probability of accidents in Antelias and jbeil is 0.25 for both, the probability of using the strategy Antelias –Jbeil is 7.29E-09. We can conclude that the probability distribution over the strategies varies with the probability of accidents on each road.

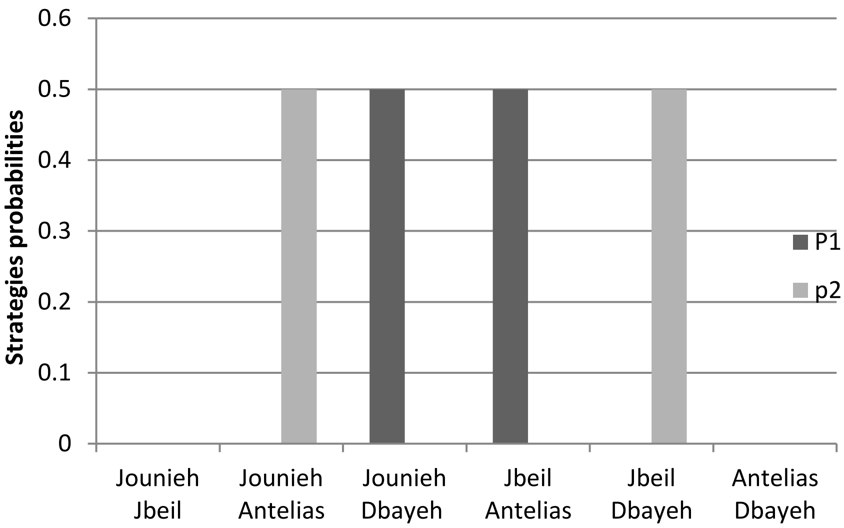

Figure 2. The probability distribution over strategies with various probability of accidents.

Scenario 3: Study of extreme conditions of accident probabilities

This study tackles two extreme cases of accidents probability. For each case, the enforcer's utility is calculated according to various numbers of resources. Table 4 shows the list of parameters considered.

Table 4. Scenario 3 parameters.

List of roads L	Chekka-Jbeil-Antelias-Dbaye.-kaslik-anfeh					
Number of resources	2 – 3- 4- 5					
Probability of accidents (P)	1-0					
Traffic density	Cheka	Jbeil	Antelias	Dbaye	Kaslik	Anfeh
	0.3	0.3	0.25	0.3	0.3	0.35

Scenario 3 results are shown in figures 3 and 4. This scenario considers two extreme cases, where the probability of accidents P is 1 or 0. We compared the enforcer's utility in each case by varying the number of available resources. When the probability of accidents is 1, Figure 3 shows that the use of 2 resources to cover the 6 considered roads gives the enforcer a negative utility, since the probability of accidents is very high; with reduced number of resources, the enforcer is leaving many roads unprotected. Therefore the law enforcer is exposing the driver to a higher risk of accidents. As a result the utility will be negative. When the number of resources increases by 1, the utility of the defender will start increasing and will get a positive value. This is due to the fact of covering more roads, and therefore exposing drivers to a lower risk of accidents.

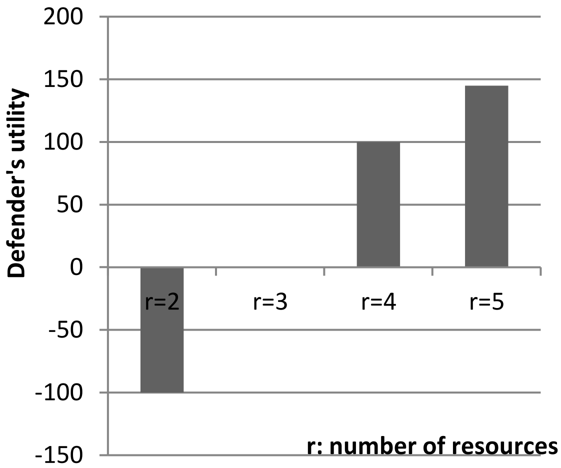

Figure 3. Defender's utility with various number of resources (2,3,4,5) when P=1.

In the extreme case of a null probability of accidents shown in Figure 4, roads are safe and there is no risk of traffic accidents and fatalities. By varying the number of resources from 2 to 5, we notice in figure 4 that the enforcer's utility increases. This utility increase is justified by the fact that when the enforcer is covering a certain road, he will get a positive utility.

The case of a null probability of accidents could be studied after applying the proposed tool for a long period of time on the Lebanese map. We may assume that after being punished for several times, the driver will obey the laws and drive within speed limits. Thus, the probability of accidents caused by traffic speed will decrease and tends to be null.

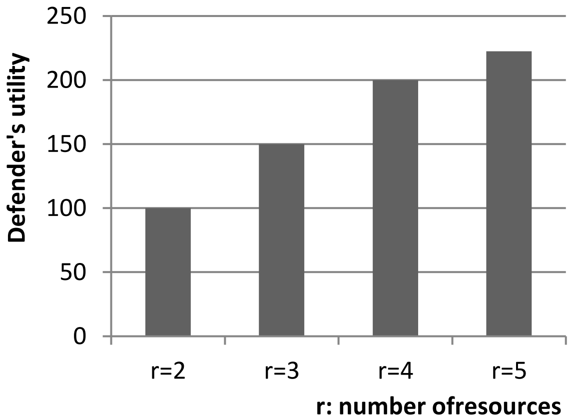

Figure 4. Defender's utility with various number of resources (2, 3, 4, 5) when P=0.

Scenario 4: Study of enforcer's utility with deterministic and random deployment

This scenario presents a comparison between the deterministic law enforcement and the random schedule provided by this novel platform. Table 5 considered a set of roads in 3 different shifts as studied in scenario 1.The table assumed that the deterministic deployment is covering roads: Dbaye-Jounieh-Jbeil all over the 3 shifts. Figure 5 shows the enforcer's utility for the two enforcement types. As we can see, scheduling speed traps deterministically at same location during the 3 shifts of the day gives a lower enforcer's utility from choosing a randomized placement. This result confirms the efficiency of the proposed solution: When placing speed traps at the same location, drivers will learn the places of the covered areas and will anticipate them by reducing their speed in order to avoid the punishments, which consequently gives the enforcer a low utility. However, when changing the placement of speed traps from one shift to another, more roads are covered especially accidental ones which gives the enforcer a higher utility.

Table 5. Scenario 4 parameters.

Roads	Dbaye	Jounieh	Tabarja	Bouar	Jbeil
Probability of accidents in shift 1	0.32	0.35	0.36	0.37	0.33
Probability of accidents in shift 2	0.31	0.3	0.33	0.34	0.31
Probability of accidents in shift 3	0.11	0.12	0.13	0.11	0.1
Number of resources	3				

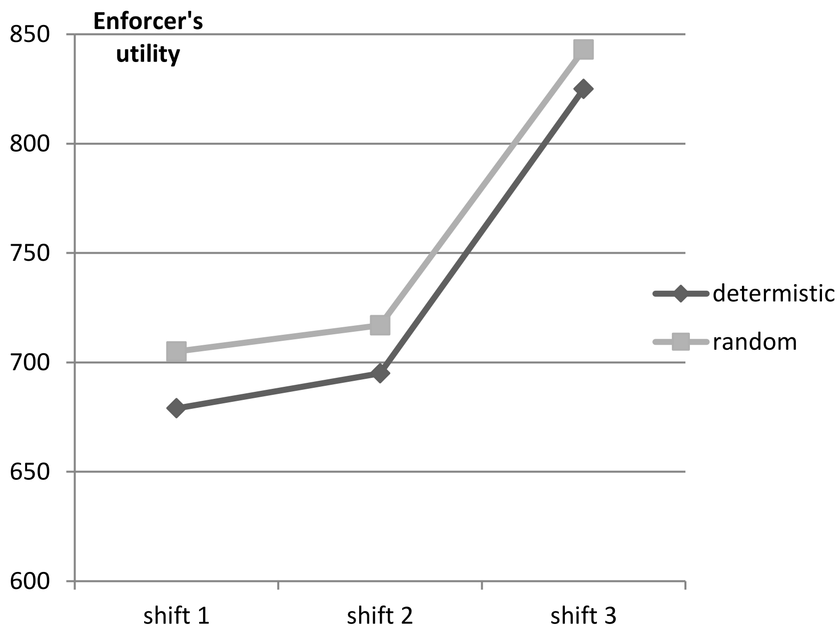

Figure 5. Enforcer’s utility with both deterministic and random schedules.

DISCUSSION

Establishing traffic security is a challenge that is faced by the ISF in Lebanon. The deterministic allocation of speed traps, leads not only to a lack of temporal randomization, but also to a lack of spatial randomization, since it is covering some specific roads leaving many other roads unprotected.

While randomized patrolling is important- as drivers can observe and exploit before violating- randomization must use different weighing functions to reflect the complex costs and benefits of allocating speed traps.

Our designed tool is developed to help the ISF in optimally deploying speed traps on the Lebanese highways. Based on game theory, more specifically on SSG, a game between the law enforcer and the driver is modeled in order to find the optimal strategy that ensures the coverage for the maximum number of roads, and at the same time avoids the deterministic law enforcement.

After dividing the Lebanese map into several parts according to the nearest ISF center, the proposed tool is applied in each of these centers, Lebanese drivers will then be confused about the radars place, and will be punished in case of laws violation. This will discourage drivers from exceeding regulatory speed.

This work tackled 4 scenarios; first the enforcer's utility variation was studied and showed that the enforcer's utility was enhanced by adding more resources and reduced accidents probability. Second, after studying the probability distribution over strategies, it was concluded that this distribution varied with accidents probability on each road, and more specifically the relevance of deploying speed traps on accidental roads was confirmed. Third, the analysis tackled extreme conditions of accident probabilities: it confirmed that using a few number of resources on accidental roads will give a negative utility to the enforcer. Fourth, the study showed that the enforcer's utility is higher in case of using the random schedule instead of the deterministic enforcement.

After analyzing the results obtained, one can see that this approach overcomes the problems presented in previous research papers. For example, the scalability problem (J. Pita, 2008) is solved by the reduction of the strategies space. Furthermore, in contrast to the findings of Eric Shieh (2013), the solution provided by this study was for both players and not only for the driver. Finally, this work dealt with the complexity of roads and took into account the continuous nature of traffic patrolling; it considered congestion level as an input on the road segments. Moreover, the accidents probability on the roads was used as an important input in order to provide optimal solution.

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