

**Expansion Of Space Geometry And Contraction Process**

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**Abstract**

It turnout that  $q$  is related to the curvature of space. for the model of the Universe, with positive curvature (one of the Fried man models) the declaration will have to be sufficient as to bring the expansion to a halt and reverse it. This is then the closed or oscillatory Universe with an oscillation time of

$$T = \frac{2\pi q}{H(2q-1)^{3/2}}$$

This time becomes meaningful only when  $q > 1/2$ , which therefore is the condition for a closed universe.

**Keywords:-** Binary, Model, Luminous Matter, Space Energy.

**1- A BINARY SETTING**

When distances to galaxies and clusters of galaxies are known accurately, it becomes a simple matter to calculate intergalactic distances and compute the number density of galaxies over a given volume of space. The average mass density will follow from a multiplication of the average number density be the average mass of a galaxy. On the basis of the current observational data, the observed mass density of the universe has been computed to be  $\rho_{pbs} = 1.5 \times 10^{-30} \text{ gmcm}^{-3}$ . The observed density of the luminous matter is – 20 per cent of this value. If the present Universe was created with a gigantic explosion of a super massive body as the Big Bang theory postulates, the expansion velocity will be decelerated by the self gravitation of the matter. Whiter the expansion of the Universe will altogether be halted at some future time is actually determined by computing by computing the magnitude of the deceleration parameter defined by

$$Q = - \frac{RR}{R^2} = - \frac{R}{H^2R}$$

Where  $R = HR \text{ cZ}$  which represents the Hubble law. We assume that all the

quantities considered are the present- day values,  $R(t)$  being the radius of the present-day Universe. The present day matter density  $\rho_u$  is related to  $q$  by

$$\rho_u = \frac{2H^2q}{4\pi G}$$

$G$  being the constant of gravitation.

For  $H = 70 \text{ kms}^{-1} \text{ Mpc}^{-1}$

$$R_u \approx 2 \times 10^{-29} q \text{ gmcm}^{-3}$$

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The open (hyperbolic) model of the Universe is characterized by the values of  $q < 1/2$ . The transition between open and closed models of the Universe is characterized by the flat (parabolic) model which corresponds to the critical value  $q = 1/2$ . In terms of the mass density it follows

from that the critical mass density in the present day Universe is given by which marks the flat model of the Universe. The closed and open models are similarly characterized respectively by  $r_u > 1.0 \times 10^{-29} \text{ gm cm}^{-3}$  and  $r_u < 1.0 \times 10^{-29} \text{ gm cm}^{-3}$ . The currently observed density of  $r^{\text{obs}} = 1.5 \times 10^{-30} \text{ gm cm}^{-3}$  then points in favor of the open model: but as we have already mentioned, the present day observational status is not sufficient to draw a definite conclusion in this regard. Extensive search, of course, is going on currently tundra us definite conclusion in this regard. Extensive search, of course, is going on currently in order to establish a more meaningful value of the mass density of the Universe. The is very important as it will determine whether our universe is open, closed or flat.

The actual discrimination between the open, flat and closed models of the Universe requires the determination of a distinct value of  $q$ . this can we achieved only by observations at very great distances where  $Z \geq 0.4$  with  $H = 70 \text{ km s}^{-1}$ . the above redshift corresponds to a distance of about  $D = 2000 \text{ Mpc}$ . At these distances, only the brightest galaxies in clusters can be used assistance discriminators. We chantry to understand the concepts of the open, flat and close cosmological models by a quite common analogy derived from of Newtonian mechanics. We know that if a body on the Earth is thrown with a kinetic energy corresponding to a velocity less than the escape velocity ( $11.2 \text{ km s}^{-1}$ ), it will return back to the Earth. This case is analogous to the closed universe model in which the sufficiently high density of matter in the Universe ( $\rho_u > 1 \times 10^{-29} \text{ gm cm}^{-3}$  and  $q > \frac{1}{2}$ ) produces enough gravitational deceleration to halt its expansion. the flat cosmic model is analogous to the case in which the kinetic energy given it the body is just sufficient to provide it the escape velocity which the kinetic energy given to the body is just sufficient to provide it the escape velocity which subsequently moves in a

parabolic orbit and comes to rest at infinity. The mass density in this model equals the critical value of  $1 \times 10^{-29} \text{ gm cm}^{-3}$  and  $q = \frac{1}{2}$ . If, on the other hand, the kinetic energy given to the body corresponds to a velocity in excess of the escape velocity, it will move to infinity in a hyperbolic orbit with some finite magnetic energy. This base finds analogy with the open Universe model which corresponds to ( $\rho_u > 1 \times 10^{-29} \text{ gm cm}^{-3}$  and  $q < \frac{1}{2}$ ). The following simple mathematical treatment will make the point more easily comprehensible.

Let us suppose that the Universe is an expanding sphere of constant mass  $M$  but whose radius  $R$  ( $t$ ) and density  $\rho$  ( $r$ ) are changing in time as it expands. Then,

$$M = \frac{4}{3} \pi \rho(t) R^3(t) = \text{constant}$$

And its equation of motion is

$$R = - \frac{GM}{R^2}$$

Yields the energy integral

$$\frac{1}{2} R^2 - \frac{GM}{R} = E$$

$E$  being the total energy which is constant.

Combining we get

$$q = \frac{1}{2} - \frac{E}{R^2}$$

Where  $R > 0$  in an expanding Universe.

Thus Eq. (21.8) yields the following three cases:

1.  $E < 0, q > \frac{1}{2}$ ,
2.  $E = 0, q = \frac{1}{2}$ ,
3.  $E > 0, q < \frac{1}{2}$ ,

The first case corresponds to the closed universe; the second represents a flat Universe while the third represents an open Universe. These are shown schematically.

## 2- THE PRIMARY GOAL OF COSMOLOGY

The primary goal of Cosmology is to construct models of the Universe that will survive the tests of current observations and of those that may be made in future. many model shave so far been constructed which represent various outlooks regarding the overall view of the Universe. All these models are however, based on the validity

of a fundamental postulate which has been called the Cosmological Principle. The main contention of this principle is that the Universe presents the same picture at any particular epoch in whichever direction we may look from whatever position, except for local irregularities, which are of statistical nature. The above statement physically embodies the isotropy and homogeneity of the Universe. Thus cosmological principle consists in taking for granted the concept that the Universe is isotropic arise as to how far this assumption may be considered valid on the basis of observations?

It turns out that the answer to this basic question is fortunately not baset with much complexity. Observations do really tend to confirm that the large-scale aspect of the Universe is isotropic and homogeneous. We have already seen that the Hubble's law of expansion of no matter which direction of the Universe we look to, the type and number density of galaxies are essentially the same. Of course, the colors of galaxies become increasingly redder as we look to greater distance on account of increasing red shifts, but this changed in colors' is not at all dependent on direction. In other words, the Universe is isotropic. No direction is preferred so far as observable aspect of the universe are concerned.

We next consider the case of the homogeneity of the Universe. This means that the picture of the Universe that we observe locally does not deffer from the picture at large distances. In other words, the conditions and environments are independent of the locality of the Universe. Of course, we must remember that the reshifts of objects are different at different distances. But what we like to emphasize is that in the absence of the effects of recessional velocities that cause redshifts. The distant parts of the universe would not have been different from our local environment. This observation aspect underlies the basic postulate that the Universe is homogeneous.

All evolving cosmological models are based on these basic postulates of isotropy and homogeneity of the Universe at any chosen time. The picture can be different at different times only. The basic idea of the cosmological principle is analogous to that of Copernican principle which states that in the system of the heavens, the Earth does not enjoy any preferred position and we would not consider ourselves favored observers. The cosmological principle may be considered. It asserts that the physical laws which are valid locally, are also valid in any arbitrary region of the Universe.

Although the cosmological principle turns out to be a necessary and sufficient hypothesis for the construction of the evolving models of the Universe, an entirely different type of model, these steady state model makes a more general postulate that the universe looks the same from any arbitrary position and at all times. This implies that the picture of the Universe is not only independent of the distance and direction, but also of time. An aspect of the Universe as observed by a local observer at the present epoch will be found unchanged by any other observer situated at an arbitrary location of the Universe atman arbitrary time. This principle, which is vastly more general and is of far-reaching importance, constitutes the foundation on which the Steady state model of the Universe has been built. This has been called the Perfect Cosmological Principle.

### **3- OBSERVATIONAL WORKS**

In this section, we shall discuss the basic equations which are used to construct cosmological models, Although such models are now-a-days constructed exclusively on the basic of Einstein's theory of General Relativity, attempts were originally made to construct models on the basis of Newton's theory of gravitation. Newtonian Cosmology has, therefore, a place of its own in the history of the development of the subject. The early attempts to tackle cosmological problems with Newtonian theory failed because,

over and above in the cosmological principle, the Universe was assumed to be static. The last assumption implies that besides the small random motions, the matter in the Universe does not possess any large-scale motions. It was found that under the combined effects of these assumptions, Newtonian theory did not yield any valid solution. The theory itself was therefore discarded for being incapable of correctly tackling the problem. It was later discovered, particularly through the works of E.A. Milne and W.H. Mc Crea in 1934, that not the theory itself but the assumption of a static Universe was wrong. By that time, the observational works of astronomers such as V.M. Slipher, E.P. Hubble and M.L. Humason had established the large recessional velocities of distant galaxies which laid the basis of the theory of Expanding Universe. The concept of the static Universe had to be abandoned altogether. In this background, the works of Milne and of Mc Crea and Milne revived the interest in Newtonian Cosmology by showing that with suitable interpretation of the terms Newtonian theory could represent many essential aspects of the relativistic cosmology. Not only was that, the two theories of cosmology demonstrated to be at par in many respects. We shall, therefore, first present briefly the essential mathematical steps of Newtonian cosmology.

Suppose that an observer moving with the origin of a system of coordinates observes the physical properties at an arbitrary point A, where  $OA = r$ . It can be established that in an expanding Universe in which the cosmological principle strictly holds, the velocity  $v$  of the point P relative to the observer O and the density and pressure at P are given by the relations

$$v = F(t) r$$

$$\rho = \rho(t)$$

$$p = p(t)$$

It may be noted that under the assumptions we have made, the velocity  $v$  is a function of the position of the point observed as

well as of the time of observation, while the pressure and density depend only on the time, which can be looked upon as the equation of motion of the particle occupying the position P can be integrated to yield

$$r = R(t) r_0'$$

Where  $r = r_0$  at  $t = t_0$ , so that  $R(t_0) = 1$  and  $R(t)$  is a time-dependent scale factor satisfying the relation

$$R(t) = F(t) R(t)$$

$R(t)$  thus represents the expansion parameter and Eq. shows that, if cosmological principle is assumed to hold strictly in the universe, its expansion (or contraction) must be uniform. Equations describe the character of motion of the particle at P. Such a motion must also satisfy the conservation laws of mass (equation of continuity) and momentum

The equation of continuity

$$\frac{d\rho}{dt} + \rho \nabla \cdot v = 0$$

Reduces to

$$\frac{d\rho}{dt} + 3\rho(t) F(t) = 0$$

In this case by virtue of Eq. since  $\text{div } r = 3$ . Using can be integrated to

$$\rho(t) r(t_0) = 1/R^3(t)$$

Which states that as the linear dimensions of the Universe increase by a factor  $R(t)$ , the density diminishes inversely as the cube of this factor. This is in accordance with the properties of the Euclidian space which only is appropriate for Newtonian Cosmology.

The vector form of the equations of motion is

$$\frac{dv}{dt} + \frac{1}{\rho} \nabla p - F = 0$$

Where  $F$  is the gravitational force per unit mass. With Eqs. and using Poisson's equation

$$\nabla \cdot F = -4\pi G \rho(t)$$

And taking the divergence of Eq., we get finally

$$3 \left[ \frac{d^2 F(t)}{dt^2} F^2(t) \right] = 4\pi G \rho(t)$$

$G$  being constant of gravitation.

Eliminating  $F(t)$  and its derivative and also  $r(t)$  between Eqs. and we have.

$$R^2(t) \ddot{R}(t) + \frac{4}{3} 4\pi G \rho(t_0) = 0$$

For static universe,  $R(t_0 = R(t_0)) = 1$  and the only solution yielded by Eq. is the trivial one  $\rho(t_0) = 0$  that is, the case of an empty Universe. In order to render the Newtonian Cosmology more general so as to be competitive with the Relativistic Cosmology, an additional term has been arbitrarily added to the right-hand side of Poisson's equation, which can therefore be represented as

$$\nabla \cdot F = 4\pi G \rho(t) +$$

Whence,

$$F = -\frac{4}{3} 4\pi G \rho(t) r + \frac{1}{3} \Lambda r$$

$\Lambda$  used here is the analogue of the cosmological constant which is a very important entity in the relativistic theory.  $\Lambda$  The introduction of actually ushers in a change in the Newtonian law of gravitation whose effect is supposed to be left only at very great distances. With this introduction Eq. becomes

$$R^2(t) \ddot{R}(t) + \frac{4}{3} 4\pi G \rho(t_0) - \frac{1}{3} \Lambda R^3(t) = 0$$

which can be integrated to yield

$$\frac{1}{3} \Lambda R^3(t) + \frac{8}{3} \pi G \frac{\rho(t_0)}{R(t)} - R^2(t) = E$$

$E$  being the constant of integration, which actually represents the total energy of the moving particle under consideration.

Equation which is a differential equation in  $R(t)$  and  $t$  represents the basic relationship with which the cosmological models can be constructed. The parameters  $E$  and  $\Lambda$  are arbitrary each of which may be positive, zero or negative independently of the other. The nature of the models is determined by the actual relationships that hold between  $R(t)$  and  $t$  which again are dependent on the choice of the values of  $E$  and  $\Lambda$ . All the models obtained from Eq. have, however, one feature in common: they all just represent the dynamical picture of the Universe in the classical sense. This is inherent in Eq. which does not contain  $c$ , the speed of light, as an entity. But since all our knowledge about

the structure and dynamics of the Universe are obtained from light signals, the inclusion of  $c$  is relevant in any comprehensive, mathematical formulation of the World models. The limitations of Newtonian cosmology are imposed by the fact that the latter artificially divides the applicability of gravitational and electromagnetic fields into two completely separated mains. so Newtonian dynamical system can predict nothing about the nature of propagation of light which is essential for a World model. The theory has therefore been rightly superseded by Einstein's theory of General Relativity which combines gravitation with the propagation of light. Since both gravitation and electromagnetism play their respective roles united in the theory of General Relativity, the latter has proved vastly more comprehensive in the study of cosmological problems. Almost all of the more recent cosmological models have been built on the basis of this theory.

According to the theory of General Relativity, the space has a curvature and any events characterized by four general coordinates in space-time labeled by  $x^i, i = 0, 1, 2, 3$ . In such a four-dimension space-time, two neighboring events are separated by interval  $ds$ .

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