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## Research Paper

# A fuzzy solution approach with genetic algorithm for a new fuzzy queue multi-objective locating model with the possibility of creating congestion for health centers

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### ABSTRACT

Health authorities have always faced the challenge of organizing health-related processes. One of the most important processes is locating health centers, clinics, hospitals, etc. Is. With increasing demand, efforts have been made to increase the efficiency of existing systems and the use of new efficient systems (their type of facilities and location) by health managers. Therefore, the use of new and efficient scientific methods in the direction of health planning has always been of interest to managers and researchers in this field. Considering the huge costs of establishing health centers and their importance, this important issue has been taken into consideration. Accordingly, in this paper, we have presented a multi-objective locating model with maximum coverage. In this model, the distance of centers from demand points, demand rates, and service rates are expressed as triangular fuzzy numbers and also queue theory has been used to improve the quality of the system.

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## 1. Introduction

One of the most important issues in strategic health planning is the location of emergency service centers, hospitals, and clinics (the same is true for other centers such as police stations, fire brigades, and other emergency services). This place has always been considered by researchers due to its importance and many effects in reducing costs. The location of the models can be classified as follows: cover set, maximum cover, center p, scatter p, middle p, and hub.

In the maximum coverage model, a set of customers with a specific demand for a service or product must be provided by some service facility to maximize the covered population or demand. In many systems, facilities are designed to meet average demand. In this situation, if we meet the maximum demand, the system will

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not be able to meet all the demands and the so-called congestion system will be created. The response to this congestion can be defined as a lost demand or queue in the system. With the advancement of science and its development in industry, researchers and users tend to provide solutions to issues of different dimensions simultaneously, so the use of multi-objective functions has become very important in recent years. On the other hand, there are always different and conflicting goals in the real world when it comes to system performance.

Therefore, this paper presents a multi-purpose model for locating maximum coverage medical centers using the fuzzy queue model. The order of the other parts of this article is as follows:

In the second part, a review of the literature will be presented, in the third part, the necessary definitions are presented and in the fourth part, the mathematical model is described. In the fifth part, the solution of the model to the fuzzy approach is described using a genetic algorithm and the sixth part has presented the conclusions and suggestions of the future.

## **2. Literature review**

Determining the location of public service provider facilities has been of interest to researchers for decades. So far, many models have been proposed for locating problems that can be classified into eight basic models. In the meantime, maximum coverage location, due to the wide and sensitive users (such as applications in the field of emergency services locating services) has become very important in recent years.

Most of the studies on the location of the coating have been done conclusively. However, there are some models that have considered parameters such as node demand, coverage radius, travel time, the distance between demand node and facilities, service capacity, and so on. There are two major solutions for modeling uncertainty, probability distributions or using fuzzy set theory. Since we need to record and use information history and expenditure to use possible distributions, in this paper, the fuzzy method is used for this important issue.

Increasing the quality of services is always one of the concerns of service providers. This improvement occurs when a measure of system performance is available to process owners, so quality measurement is one of the most important parts of any system. For example, Al-Mahdawi et al. (2012) presented an innovative application of queuing theory to reduce the latency of ambulances. They considered patients who were taken to the emergency room by ambulance as priority one and other patients as priority two, and they modeled the flow of patients in one department of over glance with two Markov chains for two groups of patients, the model presented by them with several service providers and two priority classes using Markov chain to calculate waiting for time and queue length for both groups.

Nowadays, the importance of investigating cases from different dimensions is not covered by anyone, so in modeling problems, it is tried to pay attention to the contradictory dimensions of the problem in order to provide more practical results.

Since the maximum coverage has been studied in this paper, we will provide an overview of the maximum coverage problem with two definite and probable methods considering the queuing parameters, and finally, the fuzzy studies in this field have been introduced and finally, models with multiple objective functions have been investigated.

The issue of location coverage, which is a subset of collection issues, was first introduced by Toregas et al. (1971). This model determined the minimum number of nodes to cover all demand nodes in the standard interval of time. The main problem with the model was the assumption of an unconstrained budget, which resulted in full coverage of all nodes. This led to the presentation of the problem of maximizing the location of coverage by Church and Reville (1974). In this model, by imposing a constraint on the number of nodes, not all nodes were covered anymore. But in this model, the congestion system was not examined and the problem was modeled assuming the availability of free nodes. The equipment location model with the

possibility of congestion was first proposed by Larson (1974). Larson developed a model in which the servant occupancy fraction was calculated from a fixed position and used to solve the location of emergency centers. The model introduced by Larson is a hypercube model that is a strong descriptive model for introducing systems with mobile nodes and can be used to evaluate a wide range of performance metrics for different modes of the system. Despite using the queue parameters, the Larson model lacked a possible structure. Daskin (1983) proposed a probabilistic model of maximum coverage location and called it the expected maximum coverage location problem. Berman et al. (1985) developed several models using queue theory for systems with the possibility of congestion. These models were: optimal location of nodes in the network with one node, probabilistic queue model, and the problem of allocation location with the possibility of congestion. Beta et al. (1988) introduced a system of prioritization in location. Viol and Hogan (1989) introduced the Probabilistic Model (MCLP) as the maximum availability location problem. The purpose of this model was to maximize the covered population, in which customer demand was met by a customer within the standard distance or time range with an  $\alpha$  probability. Mariano et al. (1994) proposed models for locating multi-service service centers in which demand is time-varying and follows a possible process. These models assume that the travel time between the two nodes is probable and the deduction of the busy nodes is not more than a fixed value. Mariano et al. (1998) presented the location model of the maximum coverage allocation. This model is created by making the following changes compared to the maximum coverage location model: Demand nodes were assigned to nodes within the standard distance or time range and queue theory was used in the allocation location model. With probability  $\alpha$  no demand waits more than a certain period of time and the number of people in the queue is constrained. Wang et al. (2003) presented a constrained location model. In their model, facilities with a certain probability could be used open or closed, and they proposed three heuristic algorithms to solve it. Shahvandi and Mahlooji (2006) presented a mathematical model for the problem of location-allocation of maximum coverage for emergency service considering congestion. In their proposed model, they have discussed the quality of service provided with the capacity of service providers and service time. The authors have used the fuzzy approach in queue theory to model the inherent uncertainty in such issues, and the demand for nodes, node capacity, the average number of customers in the queue, and the service time of each node is a fuzzy number. Have assumed a triangle. In the proposed model, they consider only one type of demand, one type of node, and one queue length. The queue length is used to ensure the quality of the model. Finally, they turned the resulting fuzzy model into an integer linear programming problem and used a genetic algorithm to solve it. Shahvandi et al. (2006) introduced a maximum location-allocation model for the congestion system. The proposed model has a hierarchical structure. This includes only two levels, the upper level, and the lower level. In their model, it is assumed that users are for both levels and can go to a higher level if referred from a low-level node. The fuzzy queue approach has been used to model system congestion. Based on this, the capacity of the node, the demand of the nodes, the service time of the service providers, the average number of customers in the queue, and the entry rate of the demand for receiving the service are expressed in triangular fuzzy numbers. The entry rate is modeled by a Poisson process. The service time is approximated and a fuzzy Markov model is used for each service provider. They transformed the obtained fuzzy model into a complex integer linear programming model and used the branch and constraint method to solve the modeled problem. Siam (2008) proposed a multi-server nonlinear allocation location model with costs associated with other constraints. Zarrinpour et al. (2008) developed a maximum coverage location model with the possibility of creating congestion in a competitive environment. Their proposed model examines a competitive environment based on customer choice and potential demand. The purpose of their model is to maximize the percentage of demand absorbed by service equipment in a competitive environment. To solve their model, they used Lingo optimization software and, on a large scale, a Meta heuristic genetic algorithm. Araz et al. (2009) proposed a multi-objective fuzzy coverage model based on the vehicle location model for use in emergency services. Three objectives are presented to measure the quality of service as follows: 1- Maximum population covered by a vehicle, 2- Maximum population covered by support, 3- Minimizing the sum of maximum distances from the standard

distance of each area. Using linguistic terms, they defined an ideal level for goals and used four different approaches to modeling and problem solving based on the literature on multi-objective linear programming and fuzzy ideal programming. Abolian et al. (2009) presented a location model with multiple service centers with the closest location constraints. Batanovic et al. (2009) presented a problem of locating maximum application coverage in networks with uncertain environments. They considered coverage to be inaccurate for a situation where the boundary between a subset of covered nodes and a subset of uncovered nodes is acceptable if a gradual coverage of demand points is given the distance or travel time between nodes and the facility is intended to exist. They presented three different algorithms for locating the best node for the facility. Their hypotheses in this article were: 1- The same importance of demand in all nodes, 2- The amount of demand in each node is different. Each demand node is assigned a definite weight, which depends on the amount of demand generated in that node. 3- Demand weight is inaccurate and is expressed in linguistic terms and triangular fuzzy numbers. The goal in all methods is to identify places that maximize the belief that more coverage is achieved at demand points. The algorithms developed by them are based on searching at potential points for the location of facilities and by comparing separate fuzzy sets. The authors believe that the proposed models are particularly suitable for modeling logistics activities in the real world, given that inaccuracies in travel time and weight of demand points are common. Giannikos (2010) proposed three different formulations of fuzzy ideal programming for demand coverage analysis. He used fuzzy sets to model the uncertainty of the quality of demand point coverage. More specifically, for each location  $j$  and each demand point  $i$ , define the membership function  $(i, j) \mu$ , which expresses the level or quality of server coverage  $j$  at the demand point  $i$ . In each formulation, he considered three goals, which are: 1- Maximizing coverage, 2- Maximizing minimum coverage, 3- Minimizing the total distance of employees from uncovered demand points. The problem is modeled on a geographic plane in which the continuous surface is divided into a set of points by placing a grid that covers the entire map. The midpoint of each component is the demand point, and the Euclidean metric is used to calculate the distance between these points and the facility. The models have been used for the real problem of bank branch network and have been solved by accurate linear programming method. Ding (2011) in this paper presents a location-allocation model of maximum fuzzy coverage based on queue theory. In this article, he has considered the uncertainty in the quality and reliability of the congestion system. In this model, Ding is considered one type of demand and one type of service provider. The queue length constraint considered in the problem ensures that none of the customers with the minimum  $\alpha$  probability will spend the maximum waiting time in the queue. The maximum number of customers per server (server capacity), node demand, and service rate for each server is expressed in triangular fuzzy numbers. To solve the model, Ding used a different evolutionary algorithm. Zarandi et al. (2011) defined the problem of locating the post office. They used fuzzy ideal geographic information systems and planning to model and solve their model. Six different types of service facilities with a high degree of interaction with the post office were considered. With the help of experts, they set a breathing level for the service facility. The proposed solution method by sadeghi et al (2021) consists of three steps: defining breathing levels, searching for candidate locations, and reaching optimal locations. Lou et al. (2010) proposed a location-allocation model of maximum coverage to determine the location of large-scale emergency relief facilities. They used a fair approach to modeling. To model geographical areas, demand areas are used, which are classified based on factors: population density, economic importance, geographical features, and the like. For each demand area, there is a possibility of an emergency and a level of contact. In order to obtain the total coverage, in addition to the above, the capacity of the servant, the demand of the region, the rate of service by the servant have been considered, and all these cases were expressed by triangular fuzzy numbers. They used the ant colony optimization (ACO) algorithm to solve the model. They tested their model with a large benchmark problem. Davari et al. (2011) presented a fuzzy version of MCLP. They considered travel time to be fuzzy and expressed it as triangular fuzzy numbers. They proposed a formulation based on the odds approach in which feasibility measurements are performed to calculate distances between locations and estimate expected coverage for needs. They used SA to solve their model. Pahlavani and Mehrabad (2011)

presented a fuzzy multi-objective planning model for locating a hierarchical service center with a nested structure. In a nesting system, high-level servers also provide services offered by lower-level servers. They used triangular fuzzy numbers to model uncertainty in installation costs and travel time between nodes and facilities. The total cost function they presented consisted of fixed and variable facility costs that depended on the level of the hierarchy. They expressed their model with a fuzzy ideal planning approach. Pahlavani and Mehrabad tried to maximize the degree of satisfaction set by the decision-makers for each goal. The objectives of their model were to minimize the average travel time, maximize the covered demand, and minimize the total cost associated with the facility. To integrate the model solution, they integrated all targets with different aggregation operators and used Meta heuristic algorithms such as genetic algorithms, taboo search, and SA to solve the problem. They did not compare the results of these methods. Ghahremani et al. (2020) conducted their study on the design of a closed-loop distribution network with forward and reverse currents. The network they studied included a set of suppliers, a manufacturing plant, a warehouse, retailers, and a customer. Their purpose was to determine the optimal location and number and capacity levels of factories and warehouses to transfer products to retailers, as well as the return of products by retailers to the warehouse at the lowest possible cost and taking into account their desired level of service. They introduced a fuzzy ideal planning approach with feedback on the problem and used the membership function to define ideal levels for decision-makers for each goal. They presented three different models. In one of these models, both directions of current were considered, while in the other two models, only one of the currents was considered. They solved the proposed models with precise solution methods. The authors of this paper pointed to the problems of accurate solutions to large problems and used a practical Meta heuristic algorithm to solve the problem on a large scale. Takaci et al. (2012) proposed a fuzzy skin coverage location model. In their model, they calculated the travel time once with a triangular fuzzy number and again with a trapezoidal fuzzy number, which they also considered as the radius of coverage of the right fuzzy number. Their innovation was to use different fuzzy numbers. Using the coverage radius of a fuzzy set I find that a level is defined for the demand level. The nodes closest to the facility have a larger coverage than nodes with longer distances. Therefore, there is no mathematical cut of the mean set of covered and uncovered nodes. They solved their model with a particle swarm optimization (PSO) method. Davari et al. (2013) presented the MCLP model in large dimensions and with a fuzzy radius. In their model, each node has a specific demand that must be met. Several distances between nodes were considered as a symmetric matrix and the radius of the facility cover as a triangular fuzzy number. To solve the model, a variable neighborhood heuristic search method has been used. Davari and his colleagues created the application of this model in the installation of wireless networks. It should be noted that in this case, the coverage radius will depend on factors such as network traffic, weather conditions, the topography of the area, and such factors. Xu et al. (2013) introduced a set of fuzzy applications in the transport of hazardous materials. They have created a two-tier problem for locating emergency services involved in accidents involving dangerous goods vehicles. At the first level, the design level, they have developed a transport network with the aim of minimizing the risk in transporting hazardous materials. At the second level, they sought to use the coverage of emergency service units to respond to accidents in the network. The risk in the hazardous materials transportation network is estimated according to the probability of occurrence and increase of the consequences of the accident. Given that the risk has reached you is not a piece, the authors have made this piece in a mathematical model. Thus, the probability of an accident with the linguistic origin and fuzzy sets is expressed and the definite consequences (mortality, damage to the road network, impact on the population, etc.) are interpreted as economic losses and are expressed with triangular fuzzy numbers. The proposed model has two objective functions, the first objective function minimizes the risk and the second objective function can cover the road network. Zanjirani et al. used an artificial bee cloning algorithm and an extended genetic algorithm to solve the model and purchased these two methods. Hajipour et al. (2016) with the help of queue theory presented a multi-objective, multi-objective location-allocation model for a multi-objective congestion system. In this article, they discussed the number of dedicated facilities and services in each layer. The objective functions of their

proposed model were: 1- Minimizing the total travel and waiting time, 2- Reducing the cost of loading the facility, 3- Reducing the possibility of unemployment of the facility. The proposed model was correct as a nonlinear multi-objective programming model and four Pareto-based Meta heuristic algorithms were used to solve it.

In none of the models presented so far, the location of over-allocation of coverage with the possibility of multiple marriages has been investigated.

Point

### Definitions

Prof. Zadeh (1965) for the first time introduced the theory of fuzzy sets and provided the basis for modeling with inaccurate information and approximate reasoning with mathematical equations, which caused a great change in mathematics and classical logic. We are satisfied with the triangular fuzzy numbers and fuzzy decision-making [8].

Definition 1- Fuzzy set: If  $X$  is a set of elements denoted by  $x$ , then the fuzzy set  $\tilde{A}$  in  $X$  is a set of ordered pairs as follows:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}.$$

Definition 2- Triangular fuzzy number: Fuzzy number is defined as follows:  $\tilde{A}$

$$\tilde{A} = (a^p, a^m, a^o).$$

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{a^m - x}{a^m - a^p} & a^p \leq x \leq a^m \\ 1 - \frac{x - a^m}{a^o - a^m} & a^m \leq x \leq a^o \\ 0, & otherwise \end{cases}$$

Definition 3- Fuzzy multi-objective decision making: Suppose a fuzzy target ( $\tilde{G}$ ) is  $\tilde{a}$  and a C phase constraint is given a space  $X$ . Then the combination ( $\tilde{C}$ ) and ( $\tilde{G}$ ) forming  $\tilde{D}$  decision, which is a fuzzy set of subscriptions ( $\tilde{C}$ ) and ( $\tilde{G}$ ) i.e., we have [8]:

$$\tilde{D} = \tilde{G} \wedge \tilde{C}.$$

$$\tilde{\mu} = \min \{ \mu_{\tilde{C}}, \mu_{\tilde{G}} \}.$$

### 3. Model Description

In this paper, we have presented a multi-objective fuzzy queue locating model with the possibility of creating congestion for medical centers, in this study, mariano-serra model (1998) and shahvandi and Mahlooji model (2006) have been developed and our problem is among the maximum coverage issues in these issues, the aim is to achieve maximum coverage of demand points with constrained number of facilities. Usually, in these matters, the distance or physical distance is introduced as the radius of the cover, if the demand and facilitation of the distance is less than this (which is called the standard distance), the request is covered, in many cases there is no accurate recorded information for decisions. In this situation, there are two ways to collect samples by spending time and cost and based on the society obtained by statistical and probable methods, investigate the model or use fuzzy set theory and estimate the parameters approximately. In this paper, it is assumed that the distance of facilities (health and service centers) from the demand points is a triangular fuzzy number, Mariano and Serra (1998) in the model of locating their maximum coverage, two constraints with the queuing theory literature to determine the maximum congestion in the system. Added:

$$\sum_i f_i X_{ij} \leq \sqrt[2]{1 - \alpha \mu_j} \rightarrow p(\text{People in line} \leq b) \geq \alpha.$$

$$\sum_i f_i X_{ij} \leq \mu_j + \frac{1}{t} \ln(1 - \alpha). \quad \rightarrow \quad p(\text{Waiting time in line} \leq t) \geq \alpha.$$

Where the demand rate in node  $f_i$  is based on the Poisson process the service rate in the  $\mu_j$  node is based on the Poisson process.

shahvandi and Mahlooji (2006) presented a fuzzy queue locating model, and in the Mariano and Serra (1998) model, they considered the distance to be fuzzy, adding the constraint  $T(N^{Sj} \leq b) \geq \alpha$  to it, in which the average number of customers was in the  $J N^{Sj}$  service, thus obtaining:

$$\tilde{\omega}_j = \sum_{i=1}^n \tilde{f}_i X_{ij}.$$

$$\tilde{\omega}_j = \left( \sum_{i=1}^n f_i^p X_{ij} \sum_{i=1}^n f_i^m X_{ij} \sum_{i=1}^n f_i^o X_{ij} \right).$$

$$\tilde{N}_j^S = \frac{\tilde{\omega}_j}{\tilde{\mu}_j - \tilde{\omega}_j}.$$

$$\tilde{N}_j^S = \frac{\sum_{i=1}^n \tilde{f}_i X_{ij}}{\mu_j - \sum_{i=1}^n \tilde{f}_i X_{ij}}.$$

They proved that the above phrase can  $\sum_{j=1} \beta_i X_{ij} \leq \gamma_j$  be replaced by  $\beta$  and  $\gamma$ :

$$\beta_i = f_i^m + b^o f_i^m - (\alpha)(b^o - b^m) f_i^m.$$

$$\gamma_j = b^o \mu_j^m - (\alpha)(b^o - b^m) \mu_j^m.$$

In this model, the demand and service rates are also 05 according to the Poisson process and a triangular fuzzy number. In this model, our objective functions are maximizing demand coverage, minimizing the cost of deployment and minimizing the unemployment time of centers.

### 3.1. Model Assumptions

$j$  set are demand points and  $j = 1 \dots N$ .  $I$  set points are candidates for the establishment of facilities and  $i = 1 \dots n$ . Since there is only one service provider in each center and each request is given by a service center, the service providers are independent and the queue model of each service provider is  $j, m/m/1$ . The demand rate, service rate and node distance are considered fuzzy.

### 3.2. Parameters and Indexes

- $I$  : Set demand points and  $j = 1 \dots n$ .
- $i$  : Set of candidate points for the establishment of facilities (emergency service centers)  $i = 1 \dots n$ .
- $a_i$  : The population is the node  $i$ .
- $k_j$  : The cost of using  $j$  facilitation.

- $f_i$  : Demand rate in node i based on Poisson process
- $w_j$  : Demand rate for J nodes based on Poisson process
- $\mu_j$  : Service rate in j node based on Poisson process
- $N_j^S$  : The average number of customers in stable mode is in the serviceman J and is a triangular fuzzy number.
- $X_{ij}$  : The variable is zero and one. If the node i is covered by the node j, the value is one and otherwise the value takes zero.

- A : Preset Reassurance Coefficient
- $Y_j$  : A variable is zero and one if a service provider (facilitation) is embedded in the node j otherwise it is zero.
- $d_{ij}$  : It expresses the distance between nodes i, j.
- S : The standard distance between the center and the demand points.

$\tilde{b} = (b^p, b^m, b^o)$  The maximum number of customers per serviceman is a triangular fuzzy number.

$\tilde{F}_i = (f^p, f^m, f^o)$  The demand rate for service is in node i and the fuzzy number is triangular.

$\tilde{M}_j = (\mu^p, \mu^m, \mu^o)$  The service rate for the serviceman is j and a triangular fuzzy number.

The node i is covered by the j node service provider when their distance is approximately smaller or equal to the standard distance, in this model the distance is expressed fuzzily, in other words, "the distance of node I from the node J is approximately smaller or equal to the standard distance", in other words,  $d_{ij} \lesssim s$  if we consider the sum of the  $N_j$  nodes which includes nodes that are smaller or equal to the standard distance, then the phase set, which includes  $N_j^S$ , the nodes and the degree of membership are defined:

$$\tilde{N}_i = \{(1, \lambda_{1i}), (1, \lambda_{2i}), (1, \lambda_{3i}), \dots, (1, \lambda_{ni})\}.$$

In which it  $\lambda_{ij}$  is defined according to Figure 1 and the following relationship, and u is the acceptable upper constraint for the standard distance.

$$\lambda_{ij} = \begin{cases} 0, & d_{ij} > u \\ \frac{u - d_{ij}}{u - s}, & s \leq d_{ij} \leq u \\ 1, & d_{ij} \leq s \end{cases}.$$

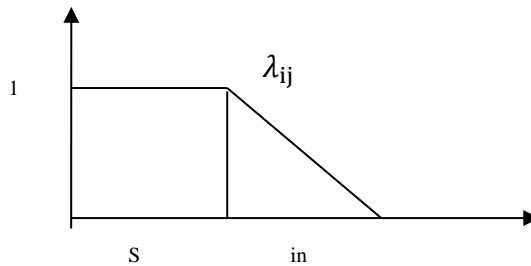


Figure 1- Standard distance membership function



However, considering that the objective function is to maximize population coverage with standard distance, the p phase set consists of members whose distance from node j is less than the standard distance.

$$\tilde{p}_i = \{(a_1, \lambda_{1i}), (a_2, \lambda_{2i}), (a_3, \lambda_{3i}), \dots, (a_n, \lambda_{ni})\}.$$

$c_i$  The variable is exponential and is actually defined as classical, the node I is either covered by the node j or not, this set is defined as follows:

$$c_i = \{(a_1, x_{1i}), (a_2, x_{2i}), (a_3, x_{3i}), \dots, (a_n, x_{ni})\}.$$

$\tilde{p}_j^c$  The fuzzy set is composed of the share of the p fuzzy set and the classic c set, representing the nodes that are coated and their distance from the node j is less than the standard distance.

$$\tilde{p}_j^c = \{(a_i, \min(\lambda_{ij}, x_{ij}))\}.$$

$$I = 1, 2 \dots n$$

Or equivalent:

$$\tilde{p}_j^c = \{(a_i, \lambda_{ij} X_{ij})\}.$$

$$I = 1, 2 \dots n$$

### 3.3. Fuzzy Multi-Objective Model

$$(1) \quad \max z_1 = \sum_{ij} a_i \lambda_{ij} X_{ij}$$

$$(2) \quad \min z_2 = \sum_{ij} k_j \lambda_{ij} X_{ij}$$

$$(3) \quad \min z_3 = \max \left( 1 - \left( \frac{w_j}{\mu_j} \right) \right) Y_j$$

$$(4) \quad X_{ij} \leq Y_j$$

$$(5) \quad \sum_{j=1} X_{ij} \leq 1$$

$$(6) \quad \sum_{j=1} Y_j = p$$

$$(7) \quad \sum_{j=1} \beta_i X_{ij} \leq \gamma_j$$

$$(8) \quad X_{ij} = 0, 1$$

$$(9) \quad Y_j = 0, 1$$

Objective function 1 maximizes the demand for coverage, considering that the target function tries to maximize the trunk of the  $z_1$  number,  $X_{ij}$  which is not zero, and in other words, the distance from node  $\lambda_{ij}I$  to the serviceman is approximately less than the standard distance, the objective function 2, the total cost of placement is minimum. Objective function 3 minimizes the unemployment time of the centers. Constraint 4 ensures that only if a center is located, the constraint 5 ensures that each node (demand) is covered by a maximum service provider. Constraint 6 determines the maximum number of servicemen, constraint 7 ensures that with the possibility of  $\alpha$ , the average number of people in the facility queue is less than 8 and 9 variables are binary decisions.  $\vec{b}$

#### 4. Fuzzy Solving Approach

When an optimization problem consists of more than one objective function, finding one or more optimal answers for it is called multi-objective optimization. The use of classical methods for multi-objective optimization has some drawbacks and complexities, the most important of which are:

- Each time the classical method is implemented, only one optimal pareto answer can be achieved.
- Some classic methods cannot be used for issues with an undressed response space.
- All of these methods require knowledge about the problem. Such as proper weighting

Therefore, in this paper, we have used fuzzy optimization method.

$$\begin{aligned} &\widetilde{\max} f_1(X) \\ &\widetilde{\min} f_2(X) \\ &\widetilde{\min} f_3(X) \end{aligned}$$

$$\begin{aligned} &\widetilde{\max} f_1(X) \\ M_1 &= \max f_1(X) \\ m_1 &= \min f_1(X) \\ m_1 &\leq f_1(X) \leq M_1 \end{aligned}$$

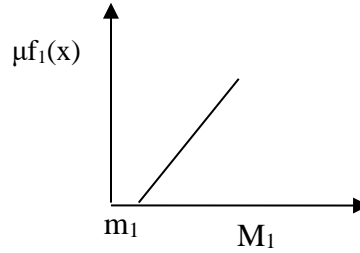


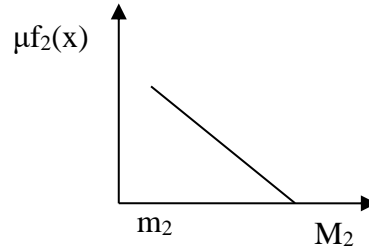
Figure 2 – Membership function  $f_1(X)$

$$\mu_{f_1}(X) = \frac{(f_1(X) - m_1)}{(M_1 - m_1)}$$

$$\begin{aligned} &\widetilde{\min} f_2(X) \\ M_2 &= \max f_2(X) \end{aligned}$$

$$\begin{aligned} m_2 &\leq f_2(X) \leq M_2 \\ \text{Figure 3 – Membership Function } f_2(X) \end{aligned}$$

$$\mu_{f_2}(X) = \frac{(M_2 - f_2(X))}{(M_2 - m_2)}$$

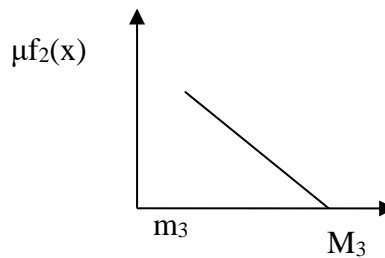


$$m_2 = \min f_2(X)$$

$$\begin{aligned} &\widetilde{\min} f_3(X) \\ M_3 &= \max f_3(X) \\ m_3 &= \min f_3(X) \end{aligned}$$

Figure 4– Membership Function  $f_3(X)$

$$\mu_{f_3}(X) = \frac{(M_3 - f_3(X))}{(M_3 - m_3)}$$



$$m_3 \leq f_3(X) \leq M_3$$

Finally, we have:

$$\text{Max Min } \{f_1(X), -f_2(X), -f_3(X)\}.$$

#### 5. Conclusion

Considering the importance of optimal use of resources, locating problem is one of the most important issues in the field of health care. In this paper, we have presented a multi-objective model for locating health centers. In modeling, queue theory and fuzzy set theory have been used. To solve the model, we proposed fuzzy optimization. In this paper, only one type of customer was considered that in the next studies, several types of demands can be investigated.

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