International Journal of Advanced Structures and Geotechnical Engineering
ISSN 2319-5347, Vol. 03, No. 03, July 2014

Optimal Design of Simply Supported Prestressed Concrete Girders using Direct Optimization Technique

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Abstract: In this work an algorithm is developed for the optimal design of simply supported prestressed concrete girders subjected to the stress and deflection constraints of the ACI code specifications. The objective is to minimize the total cost of the symmetrical I-section girder, considering cost of concrete, prestressing and mild steel. The design variables considered are, width and thickness of flange, width and depth of web, tendon layout, and area of prestressing and mild steel. Explicit constraints on the design variables are developed on the basis of geometric requirements, practical conditions for construction, and code restrictions. The technique for optimization used is modified Box Complex method; applied for the first time to prestressed concrete girders and has proven as an efficient tool for finding the optimal design of simply supported prestressed concrete girders. In order to perform analysis, design and optimization of simply supported prestressed concrete girders a program is formulated in Visual Basics .NET (VB.Net) programming language. The technique resulted in optimum solution for design variables having different sensitivities. The designs of two prestressed concrete girders are optimized in order to observe the behavior of this technique. The result demonstrates encouraging design improvements and rate of convergence.

Keywords: *Optimization; Box Complex Method; Prestressed concrete girders; Minimum cost; ACI-Code.*

1. Introduction:

In Structural Engineering generally, we strive to achieve a structure which is not only safe for a given set of loads but also serviceable and economical. This is achieved by trial and error, that is; an initial design is assumed and then analyzed to evaluate its performance. The design is modified on the basis of information provided by the analysis, subsequently the designer reanalyzes the design and this process continues. The design-analysis-redesign steps are repeated until no further significant improvement is possible. This trial and error method is very tiresome if it is done manually by designer. Also the time consumption is very large and the designer usually gives up after few tries.

It is theoretically possible to solve the entire structural optimization problem using mathematical programming techniques. However, in practice, difficulties emerge due to computational in-efficiencies, when the dimensions of design space either become large, or contain variables of different sensitivities, or both. In order to overcome these difficulties the variable cross-sectional dimensions of I-shaped prestressed concrete girder i.e flange width, web thickness, flange thickness, depth of inclined portion and prestressing reinforcement and non-prestressed reinforcement, playing more significant role in the optimal design of member are found by the application of modified version of Complex Method of Box.

The Complex Method of Box [1] has been applied to small academic examples of structural optimization subjected to stress constraints of AISC-ASD specifications in the past [2]. It has also been applied to Concrete structures subjected to stress constraints of ACI specifications (Muhammad Rizwan et.al [3]) and performs well in this study. The present work considers the optimal cost design of simply supported prestressed concrete girders under different uniformly distributed loads, spans and material strengths subjected to Ultimate Strength Design and Allowable Stress Design criteria stipulated in ACI code. The results show favorable design improvements and rate of convergence.

Researchers have used various optimization techniques to the optimal design problems of prestressed concrete girders. Raquib Ahsan (et.al) [4] used evolutionary operation (EVOP), for post-tensioned I-girders. Gene F. SircaJr (et.al)[5] presented cost optimization of prestressed concrete bridges using robust neural dynamics technique. Cohn, M. Z. (et.al)[6] developed optimal design of structural concrete bridge system using projected lagrangian technique. Jones, H. L.[7] presented integer programming formulation for minimum cost prestressed concrete beam design using optimization approach.

2. Optimum Design Problem and its Formulation:

The optimum design of simply supported prestressed concrete girders with stress constraints can be stated as follows. The problem is formulated in Visual Basic.Net programming language. Find the cross-sectional dimensions, flange width (b), depth of inclined portion of flange (d), web thickness (w), flange thickness (t), and depth of girder (h) of symmetrical I-shaped girders and amount of mild steel (A_s) and prestressing steel (A_{ps}) so that girder is able to carry safely a set of external loads and, at same time, attain the minimum cost among all feasible designs. The problem can be formulated as a mathematical programming as follows. Find a design vector.

$$\boldsymbol{Z = Zi = (b, d, w, t, h, A_s, A_{ps})}$$

(1)

$$i = 1, 2, 3 \ldots \ldots, N$$

So that the objective function

$$F = \sum_{i=1}^{M} R_i V_i$$

Where

F = Total cost of the Prestressed concrete I-girders

V_i = The Volume of concrete, prestressing steel, and mild steel respectively

R_i = are the rates of concrete, Prestressing steel, and mild steel respectively

attains a minimum value among all feasible designs that satisfy the explicit constraints.

$$b^L \le b \ge b^U$$
$$d^L \le d \ge d^U$$
$$w^L \le w \ge w^U$$
$$t^L \le t \ge t^U$$
$$h^L \le h \ge h^U$$

And U implicit constraints.

$$g_{i(z)} \le 0 \quad (i = 1, 2, 3 \ldots \ldots, u)$$

In which Z_i represents the variables of design vector, N is the dimension of the design space, M is the number of implicit constraints. The superscripts L and U denote the lower and the upper bounds on the design variables. The implicit constraints impose restrictions on the stresses as governed by the ACI specifications.

Equation 1 is a general description of the design vector. In practice, not all elements of the design vector are independent variables of design space. Some of the variables may be linked in order to satisfy symmetry. Thus, the dimensions of the design vector in a particular case may be smaller than suggested for more general case.

3. Solution Procedure:

To solve the stated optimization problem, a computational methodology is developed consisting of three logically separable phases: the optimization phases, the structural analysis phase, and the design evaluation phase. During the optimization phase, attempts are made to improve by finding feasible points that are successively closer to an optimum. In structural analysis phase, the structure, provisionally obtained in the optimization phase, is analyzed and, finally, the feasibility of structure is checked in the design evaluation phase. An overview of the Complex Method of Box is given below.

A. The Complex Method of Box [1]

The Complex Method is a mathematical programming procedure for finding an optimal solution of non-linear, constrained optimization problems. This method derives its acronym COMPLEX from two words, Constrained and Simplex. The Complex method was proposed originally by M.J.Box in 1965, where he demonstrated efficacy of the method in finding near optimal solution to non-linear, constrained optimization problems. It is a Zero-order method optimization method; that is, it does not require either the gradient of objective function, or that of constraints. The choice of Complex Method was made for its ability to span large portions of the design space , thereby providing a better chance of finding the global optimum, and for its ability to deal with constrained optimization problems.

The method attempt to find a design vector

$$X = x_{ii} = 1,2,3, \ldots \ldots \ldots, N$$

Which will minimized the function

$$F(xi)$$

For following N explicit constraints

$$X_i^L \le x_i \le X_i^U \qquad i = 1,2,3, \ldots \ldots, N$$

And **M** Implicit Constraints

$$g_{j}(x_j) \le 0 \qquad j = 1,2,3, \ldots \ldots, M$$
$$N = Number\ of\ design\ variables$$
$$M = Number\ of\ Implicit\ constraints$$

$$x_i^L = Lower\ bound\ limit\ for\ design\ variables$$

$$x_i^U = Upper\ bound\ limit\ for\ design\ variable$$

The Complex Method optimizes a provisional design by reflecting the worst point design (design) through the centroid to find the best point (design). The optimization process is divided into two stages. In the first phase a set of feasible points (*k = 2N*) (satisfying all constraints) are randomly generated. After generating the initial complex, the algorithm moves to reflection phase. In this phase, the methods call for improvements of the worst point in the complex. The method continues in this manner until convergence criteria are met, or the maximum number of iterations is reached. Details are given.

4. Proposed Modifications and implementation of Complex Method:

The modifications to the complex Method as used in this study are summarized as follows.

A. The Improvement Procedure:

The improvement procedure has been modified in that at every iteration the worst design is reflected through the centroid of remaining designs in the design space to a new point. Then, when this new point has been optimally sized, its objective function is evaluated and compared with that of worst design in the complex. If the new point is less, it is accepted as a design improvement and termination criteria are checked; if greater, instead of continuously halving α, it is halved only thrice and then centroid is considered as a candidate for improvement. If centroid is still greater than the worst, then a new point is located at the midpoint of a line joining centroid to the best point in the complex. If the objective function is still greater than the worst, then the worst point is replaced by the best design in the complex.

B. Termination Criteria:

The procedure is repeated until a preset termination criterion is reached. The first termination criterion used in this study is based on the objective function values of all k point in the complex. This convergence criterion is met if the ratio of the difference between the maximum objective function value and the minimum objective function value to maximum objective function value of the points in the complex is less than or equal to the value of $\in$ (a user define variable) i.e.

$$\frac{(f_{max} - f_{\min})}{f_{\max}} \leq \in \quad (2)$$

The second criterion that is checked for the convergence of the solution is a measure of the design space spanned by the vertices of the complex,

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{k}(x_{ij} - c_i)^2}{2\ \sum_{i=1}^{n}(x_i^U - x_i^L)^2} \leq \delta \quad (3)$$

Where

$$c_i = 1/k \sum_{j=1}^{k} x_{ij}$$

Finally, a constraint is placed on the maximum number of iterations that may occur before terminating the optimization. The optimization process is terminated as soon as any of the termination criteria is satisfied.

Formulation of implicit constraints is given below:

1) Top Fibers Stresses Constraint at Transfer (G1)

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{e c_t}{r^2}\right) - \frac{M_D}{S^t} \quad (4)$$

$$G_1 = -ABS\left(\frac{f^t}{f_{ti}}\right) + 1 \quad (5)$$

2) Bottom Fibers Stresses Constraint at Transfer (G2)

$$f^b = -\frac{P_i}{A_c}\left(1 - \frac{e c_b}{r^2}\right) + \frac{M_D}{S^b} \quad (6)$$

$$G_2 = -ABS\left(\frac{f^b}{f_{ci}}\right) + 1 \quad (7)$$

3) Top Fibers Stresses Constraint at Service Load (G3)

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{e c_t}{r^2}\right) - \frac{M_T}{S^t} \quad (8)$$

$$G_3 = -ABS\left(\frac{f^t}{f_c}\right) + 1 \quad (9)$$

4) Bottom Fibers Stresses Constraint at Service Load (G4)

$$f^b = -\frac{P_e}{A_c}\left(1 - \frac{e c_b}{r^2}\right) + \frac{M_T}{S^b} \quad (10)$$

$$G_4 = -ABS\left(\frac{f^t}{f_t}\right) + 1 \quad (11)$$

5) Minimum mild Reinforcement Constraint (G5)

$$G_5 = -ABS\left(\frac{Min\ A_s}{A_S}\right) + 1 \quad (12)$$

$$Min\ A_S = 0.004$$

6) f) Ultimate moment Constraint (G6)

$$G_6 = -ABS\left(\frac{M_U}{\emptyset M_n}\right) + 1 \quad (13)$$

For rectangular beam analysis:

$$\emptyset M_n = \emptyset\left\{A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - \frac{a}{2}\right) + A_s' f_y\left(\frac{a}{2} - d'\right)\right\} \quad (14)$$

For T- beam analysis:

$$\emptyset M_n = \emptyset\{A_{pw} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - d_p\right) + 0.85 f_c'(b - b_w) h_f\left(d_p - \frac{h_f}{2}\right)\} \quad (15)$$

For over-reinforced sections

i) Rectangular beam analysis

$$\emptyset M_n = \emptyset\{f'_c b d^2_p (0.36\beta_1 - 0.08\beta_1{}^2)\} \quad (16)$$

ii) T-beam analysis

$$\emptyset M_n = \emptyset\{f'_c b_w d^2_p (0.36\beta_1 - 0.08\beta_1{}^2) + 0.85 f'_c (b - b_w) h_f (d_p - 0.5 h_f)\} \quad (17)$$

7) *Reinforcement Limits (G7)*

$$G_7 = -ABS\left(\frac{1.2\, M_{cr}}{M_u}\right) + 1 \quad (18)$$

$$M_{cr} = f_r S_b + P\_e\left(e + \frac{r^2}{c_b}\right) \quad (19)$$

8) *Shear Capacity Constraint (G8)*

$$G_8 = -ABS\left(\frac{V_u}{\emptyset(V_c + 8\sqrt{f_c'}\, b_w d_p)}\right) + 1 \quad (20)$$

$$V_c = \left(0.60\sqrt{f_c'} + 700\,\frac{V_u d_p}{M_u}\right) b_w d_p \quad (21)$$

$$Min.V_c = 2\sqrt{f_c'}\, b_w d_p \quad (22)$$

$$Max.V_c = 5\sqrt{f_c'}\, b_w d_p \quad (23)$$

$V_u = Factored\ shear\ at\ a\ distance\ 1/2h$

9) *Deflection Constraint (G9)*

$$G_9 = -ABS\left(\frac{\Delta_{p+d}}{\Delta_a}\right) + 1 \quad (24)$$

$$\Delta_p = \frac{ML^2}{8EI} \quad (25)$$

$$\Delta_d = \frac{5}{384}\frac{wL^4}{EI} \quad (26)$$

$$\Delta_a = \frac{l}{360} \quad (27)$$

Δ_p=Due to prestress

Δ_d=Due to self-weight

Δ_a=Allowable deflection

5. Examples

Two numerical examples are solved to demonstrate the versatility of the proposed procedure. The results of the examples were generated with Visual basic 2010 Express edition computer program executed on Core(TM) 2 Duo, 2.26 GHz laptop computer with 2.00 GB of RAM. This program is in three interacting modules, performs search for optimum, structural analysis and structural design. In the development of optimization routine guidance is taken from Muhammad Rizwan (et.al)[3], Bashir (et.al)[2] in order to produce modified version of complex method. The structural analysis and structural design routine is developed by using fifth edition of prestressed concrete by Edward G.Nawy [9]. The decision-making and the computational assignments are carried out by separate subroutine in each phase. This feature is highly desirable, especially when the need for modification to the design code may arise in the future. However, different modules representing the provision of other commonly used design codes can be appended to the existing design routine with relative ease.

The material specified for girder is concrete with crushing strength of 5000 psi (normal- weight concrete), prestressing steel with ultimate strength of 270,000 psi and non-prestressed steel with yielding stress of 60,000 psi. The implicit constraints are to meet the relevant provisions of ACI specification for both allowable stress design and ultimate strength design.

A. Example 1

Consider a prestressed symmetrical I-section to carry a sustained uniformly distributed per foot superimposed dead load of 0.1 kip/ft and a sustained service live load of 1.1 kip/ft on a 50 ft simply supported span. The aim of this design exercise is to minimize the cost of the girder by selecting a set of cross-sectional dimensions, prestressed and non-prestressed steel so that the girder can carry externally applied loads fulfilling stress requirements as per ACI code provisions. The design variables (*b, d, w, t* and *h*, A_s, A_{ps})defines the width of half portion of flange, depth of inclined portion of flange, thickness of web, thickness of flange and depth of girder, respectively. The top flange is linked with bottom flange and the top inclined portions are linked with bottom inclined portions of I-section in order to impose symmetry. The histories of the constraints showing capacity versus demand curves are shown in figure 2 and 3 which clearly depicts that there is no violation of constraints.

The program performs 44 iterations. The initial complex randomly generated, consist of nine designs with girder cost ranging from Rs. 91967.3 to Rs.237673.63. The girder cost at iteration 0 and 5 are 48.79 and 8.23% respectively, more than the cost of the final design. Figure 1 shows the maximum and minimum objective function values in complex with successive iterations. The optimal cost obtained based on ACI specifications for this problem is Rs. 61811.92.

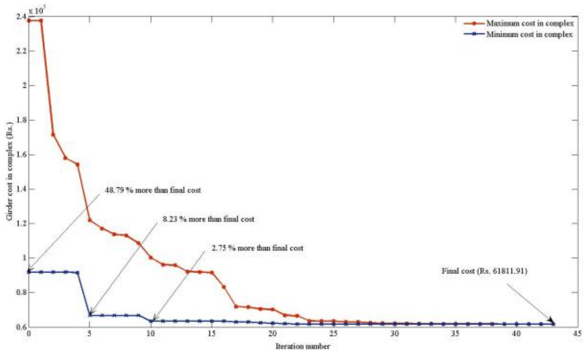

***Figure1**: History of Maximum/Minimum Cost of Prestressed Concrete girders in the Complex*

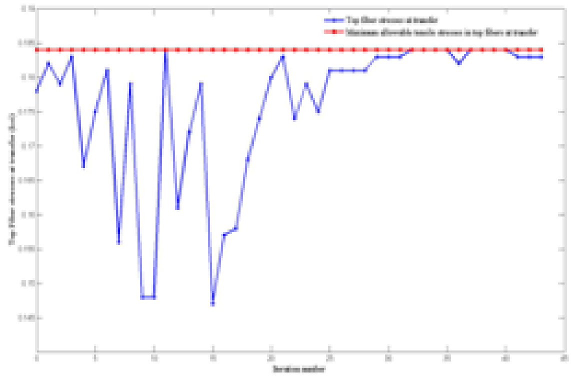

***Figure2:** History of top fiber stresses at transfer*

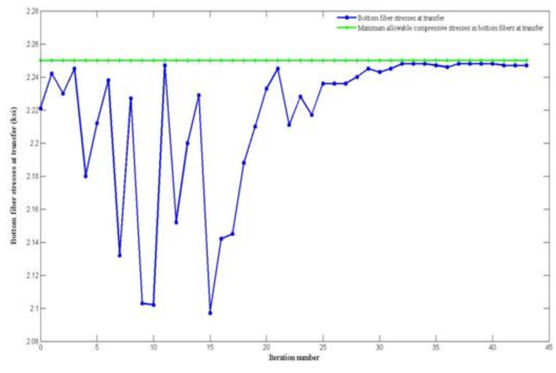

***Figure3:** History of stresses in bottom fibers at transfer*

B. Example 2

The aim of this second design exercise is to minimize the cost of an excessively loaded girder i.e. 0.75 kip/ft and a service live load of 1.8 kip/ft with increased span length of 60 ft than previous example and selecting a set of girder dimensions so that it can carry the externally applied loads while the stress and serviceability requirements are satisfied. This example is presented to demonstrate the robustness of the formulation presented in this work. The design variables *b, d, w ,t* and *h* defines the width of half portion of flange, depth of inclined portion of flange, thickness of web, thickness of flange and depth of girder, respectively. The top flange is linked with bottom flange and the top inclined portions are linked with bottom inclined portions of I-section in order to impose symmetry. The histories of the constraints showing capacity versus demand curves are shown in figure 05 and 06 which clearly depicts that there is no violation of constraints.

Solution of uniformly loaded optimal prestressed concrete girder required 43 iterations. The best point in the complex at this time has a cost of Rs.189024.21.The girder cost at iteration0, 3 and 13 are19.51 %, 5.86 % and 3.13 % respectively, more than the cost of the final design. Plots of histories of maximum and minimum objective functions are shown in Figure 04.

Summary and Conclusions:

In this study, a direct-search optimization algorithm based on the Complex Method with suitable adoptions, refined

Iin this study, is presented to be related to the simply supported prestressed concrete girders Cost optimization problems, utilizing cross sectional dimensions and non-prestressed reinforcement as design parameters subjected to strength considerations. The modified Complex Method has the ability to seek the design space of an assigned prototype and is efficient tool in evaluating the optimal design of the simply supported prestressed concrete girders. The generation of an initial complex of reasonable points is required, that is scattered through the design space and leads for enrichment of the design in the diverse design space. These characteristics afford the prospective for achieving global optimum. The improvement in function value is very rapid in the initial 10-15 iterations after the initial feasible complex is established. It becomes very easy to apply the method when there are more than a few constraints in a problem.

This procedure offers the designer the ability to search more of the design space with a minimal amount of effort, and the ability to optimize complicated models, for which the gradients would be time consuming, if not impossible to calculate. The designer can examine the many different optimum design states and can set some additional goals or constraints.

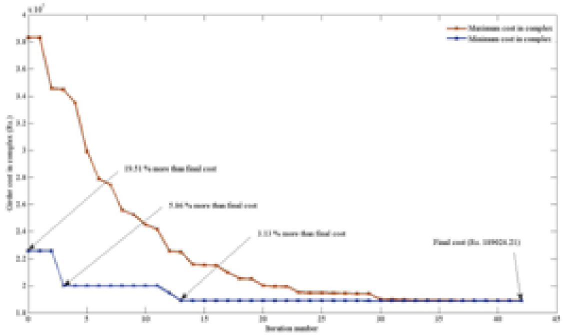

Figure4: *History of Maximum/Minimum Cost of Prestressed Concrete girders in the Complex*

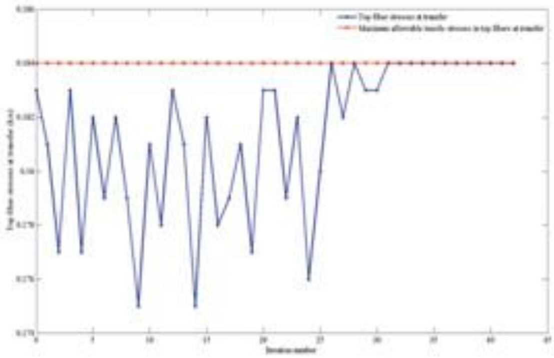

Figure5*: History of in top fibers stresses at transfer*

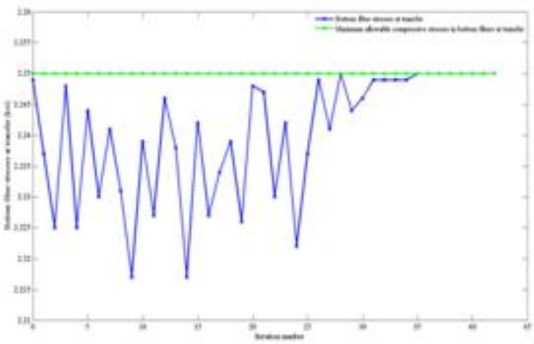

Figure6*: History of bottom fibers stresses at transfer*

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