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Designing the Output Feedback LQG Controller Applying for Electrical Drive System

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***Abstract:** Electric drive systems have been widely used and developed in the industry. With the development of science and technology, these systems are applied strongly in clean technologies such as food technology, biotechnology or chemical pumping and blood pumping system in the artificial heart.New control methods are increasingly being studied and applied in place of conventional control methods with the aim of improving system quality.Today, with advanced sensor technology that enables to measure physical signals, the output feedback controller called Linear Quadratic Gaussian–LQG is useful for evaluating system dynamics based on the output signals.The application of this designing method with some electric drive systems will attract more research interests.*

***Keywords:** LQR, The Kalman Filter, LQG, The Non-Contact Drive, The Accurate Tracking Drive.*

I. INTRODUCTION

Automatic control has been developed and widely applied in different aspects. The quality of the system with PID controllers achieves advantages and is calculated for a specific operation mode. However, in many systems, the quality of the controller is affected by unwanted channel interference, although it also has to cover the problem of improving the quality.With the development of sensor technology, it is much easier to do measurements of physical signals. The system quality in the industry is being improved by new control methods.A new method considered as centralized methods consisting of: LQR, LQG, H∞, µ - synthesis, … has good performances. There have been many applications using the state feedback controller LQR. The state observer is a useful solution in evaluating system dynamics in order to eliminate the disturbances and optimize observed state. This method combines a state observer (Kalman filter) and a state feedback controller to obtain an output feedback LQG controller and generate the most optimal control signals. The research results will be applied to some current drive systems as well as non-contact drive system.

II. THE DESIGNING METHOD

2.1. The State Feedback Controller – LQR Controller:

The state equations of the proposed system can be expressed in the equation (1)

$$\begin{cases} \dot{\underline{x}} = A\underline{x}+B\underline{u} \\ \underline{y} = C\underline{x} \end{cases} \qquad (1)$$

$$\underline{x}\in R^n; \underline{u},\underline{y}\in R^m; A\in R^{n\times n}; B\in R^{n\times m}; C\in R^{m\times n}$$

Designing an optimal controller is to find u control signal for system (1) according to the orbit of state variable x(t) to minimize the quality square.

$$J = \int_{t_0}^{t_1}(x^TQx+u^TRu)dt \qquad (2)$$

Where: Q is the weighting matrix is the state and is semi-positive.

R is the weighting matrix of the control signal and is positive.

X is the state vector.

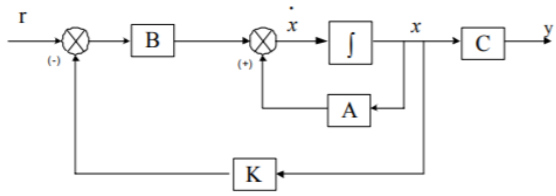

Fig. 1. The block diagram of a system using LQR

The LQR controller is constructed under the assumption that all states are measured completely; the dynamic behaviour of each controlled variable can be checked and the closed – loop poles can be evaluated [6].

The Hamilton function may be expressed as:

$$\frac{\partial f}{\partial t} = -\min\left[x^TQx+u^TRu+\left(\frac{\partial f}{\partial t}\right)^T(Ax+Bu)\right] \qquad (3)$$

Where

$$f(x,t) = x^Tpx \qquad (4)$$

If P is a symmetric matrix, then

$$\frac{\partial f}{\partial t} = x^T\frac{\partial p}{\partial t}x \qquad (5)$$

And

$$\left[\frac{\partial f}{\partial x}\right]^T = 2x^Tp$$
$$\frac{\partial f}{\partial x} = 2px \qquad (6)$$

Replacing equations (5) and (6) into (3) as follows:

$$x^T \frac{\partial p}{\partial t} x = -\min\left[x^T Q x + u^T R u + 2x^T P(Ax + Bu)\right] \quad (7)$$

In order to minimize u, equation (7) is transferred into equation (8)

$$\frac{\partial(\partial f / \partial t)}{\partial u} = 2u^T R + 2x^T PB = 0 \quad (8)$$

Equation (8) can be re-arranged based on the optimal control law

$$u_{opt} = -R^{-1}B^T P x \quad (9)$$

or: $u_{opt} = -Kx$ (10)

Where : $K = -R^{-1}B^T P$ (11)

(K is the feedback variable matrix of the optimal control rule)

Substituting equation (9) into (7)

$$x^T \dot{P} x = -x^T(Q + 2PA - PBR^{-1}B^T P)x \quad (12)$$

since: $2x^T PAx = x^T(A^T P + PA)x$ (13)

then

$$\dot{P} = -PA - A^T P - Q + PBR^{-1}B^T P \quad (14)$$

Equation (14) is a class of non – linear equations known as the Riccati matrix equations. The coefficients of P(t) are found by the integration in reverse time starting the boundary condition

$$x^T(t_1)P(t_1)x(t_1) = 0 \quad (15)$$

Kalman demonstrated that when integrated in reverse time, the solutions of P(t) converge to constant values. If t_1 is far larger than t_0, the Riccati matrix equations will reduce to a set of equations given by:

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \quad (16)$$

Where P is the positive solutions of the equation (16). After that, the controller K in (11) will be determined. The control signal u is to be found according to equation (17)

$$u_{opt} = -Kx \quad (17)$$

Therefore, if the system behaviour is modified, the weighting matrices Q and R can be adjusted to obtain different values of the optimal feedback matrix K. Choosing Q and R matrices is given by Bryson's rule [5]

The state feedback LQR controller is used without disturbances and with the measured states. Reality, the system always is effected by disturbances and the states are not observed. Therefore, the filter Kalman (for filter and estimate observer of x is optimal) will be combined with the state feedback LQR controller into the output feedback optimal LQG controller**.**

2.2. Kalman Filter:

In fact, with the effects of noise, the system are expressed as the following state space :

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} + w, \quad \underline{y} = C\underline{x} + v \quad (18)$$

Where w and v are measured noises. The problem is to estimate x state that minimizing the error square

$$E\left[\left|x(t) - \hat{x}(t)\right|^2\right] \text{khi as } t \to \infty \quad (19)$$

Theorem 4.1.1[2] : under the assumptions [2], the best optimal state variables can be generated by the Kalman filter under the least square criteria.

$$\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly \quad (20)$$

$$AP + PA^T - PC^T V^{-1} CP + W = 0 \quad (21)$$

Where the matrix $L = PC^T V^{-1}$ (22)

L is called the filter gain matrix and P is the solution of the equation (21)

Note : the output estimation $\hat{y}$ is given by $\hat{y} = Cx$

The error between the measured output y(t) and predicted output $C\hat{x}$ is given by : $e = y - C\hat{x}$ where $\hat{x}$ is determined from equation (9)

2.3.The Output Feedback Optimal LQG Controller:

As the conclusion in section 2.1, the problem is to design the output feedback optimal LQG controller when appearing disturbances described by the following model.

$$\dot{x} = A\underline{x} + B\underline{u} + w, \quad y = Cx + v \quad (23)$$

$$\underline{x} \in R^n; \underline{u}, \underline{y} \in R^m; A \in R^{n \times n}; B \in R^{n \times m}; C \in R^{m \times n}$$

With the quality square functions

$$J_{LQG} = \lim_{T \to \infty} \frac{1}{2T} E\left[\int_{-\infty}^{t} (x^T Q x + u^T R u)dt\right] \quad (24)$$

The weighting matrices Q and R are semi-positive and positive. An assumption is that the w and v noises are both Gaussian, white noise, zero – mean and positive. The problem is to find the optimal control signal (u) minimizing the average values.

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The control vector u for the LQG problem is given by:

$$u = -K\hat{x} \tag{25}$$

Where

1.The K matrix is feedback matrix associated to the LQR problem, that is : $K = R^{-1}B^{T}P$

Where P satisfies

$$PA + A^{T}P + Q - PBR^{-1}B^{T}P = 0$$

2. Vector $\hat{x}$ is generated by the Kalman filter

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) = (A - LC)\hat{x} + Bu + Ly \tag{26}$$

the feedback parameter matrix $L = PC^{T}V^{-1}$ (27)

Where P satisfies

$$AP + PA^{T} - PC^{T}V^{-1}CP + W = 0 \tag{28}$$

The LQG separation principle.

By substituting 25 into 26, we obtain

$$\dot{\hat{x}} = (A - BK - LC)\hat{x} + LCx \qquad u = -K\hat{x} \tag{29}$$

It is easy to see that the closed –loop matrix satisfying the differential equation

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix} \tag{30}$$

To the convenient for the system stable check that for the error vetor $e = x - \hat{x}$ substitute for state estimation $\hat{x}$

Ta có:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} I & 0 \\ I & L \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix} \tag{31}$$

Furthermore, if A and B are controllable ; and A and C are observable, then both the matrix A-BK and A-LC are stable. Matrix A-BK-LC is not necessarily stable.

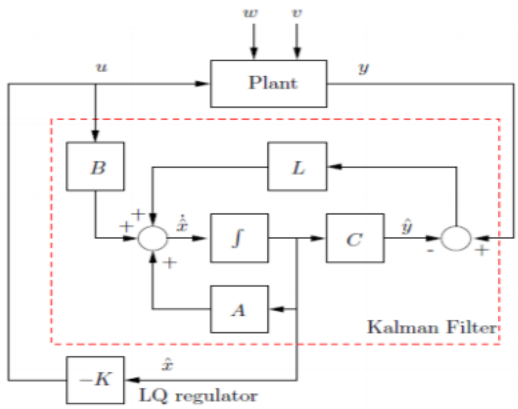

Fig. 2. The Closed Loop using the LQG Controller

III. Applications In The Electric Drive

3.1. Non – Contact Driver System:

3.1.1 The System Model: Electric drive systems are being used in clean industries such as food technology, chemical technology, biotechnology (chemical pumps, artificial heart pumps, etc.). However, the problem of these systems is the requirement of maintenance and replacement of mechanical bearings that require high level experts or high costs.In addition, lubricate oil can not be used in vacuum environment, very high or very low temperature or in very clean environment such as pharmaceutical processing, material technology and etc.

The alternative method studies systems which the rotating shaft of the electric motor does not contact to the static part, so there are no dust particles generated by the abrasive nor the use of lubricants.The result is no pollution to the surrounding environment.To satisfy these requirements, the non-contact drive system using synchronous motors is one of the most suitable choices and is being developed in many countries in the world.

The object of this research is a drive system including two magnetic bearings mounted in both shaft poles. The AMB1 (Magnetic Bearing) provides radial forces according to the x1, y1 directions. The AMB2 provides radial forces in terms of x2, y2 directions [1].

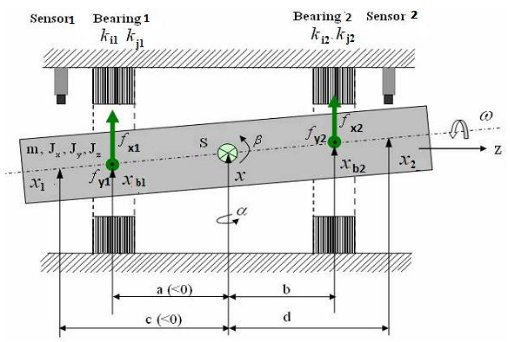

Fig. 3. The System in the Horizontal Axis [3]

The state – space model of 4- DOF AMB system yields [1,3,4]

$$\begin{cases} \dot{X} = AX+BU \\ Y = CX + DU \end{cases} \qquad (32)$$

$$X = \begin{bmatrix} q & \dot{q} \end{bmatrix}^T$$

$$U = \begin{bmatrix} i_{x1} & i_{x2} & i_{y1} & i_{y2} \end{bmatrix}$$

$$Y = [x_1 \quad x_2 \quad y_1 \quad y_2]$$

$$A = \begin{bmatrix} O_{4\times 4} & I_{4\times 4} \\ M^{-1}B_f K_s T_s & M^{-1}G \end{bmatrix}, B = \begin{bmatrix} O_{4\times 4} \\ M^{-1}B_f K_i \end{bmatrix}$$

$$C = [C_{4\times 4} \quad O_{4\times 4}], D = [0_{4\times 4}]$$

3.1.2. The Simulation Results With PID Controller:

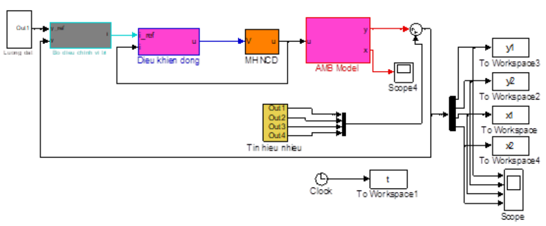

Fig. 4. The Simulation Diagram of the AMB Controller with 02 Closed Loops

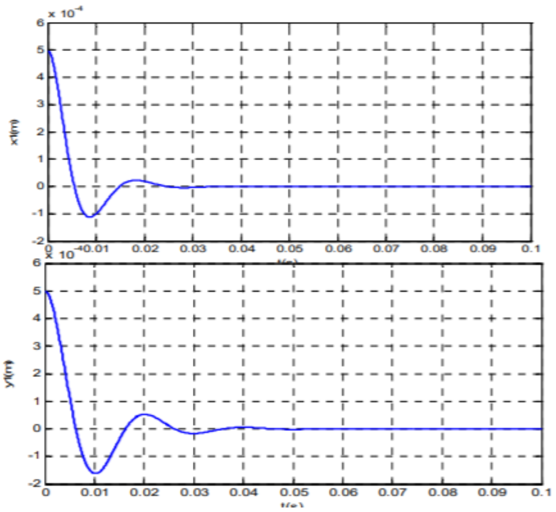

Fig. 5. Output Response of AMB using PID Controller

Comment: When using two PID closed loops that is the position control loop and the speed control loop, we see the trajectory of the AMB1 system oscillates around the center of the rotor with the largest amplitude at about 0.00015m.The mathematical model of the AMB system is not really close to the reality, in which random noises and channel interference are ignored.At the same time, because linearizing around the working points, the working area is narrowed.

Therefore, it is necessary to take a method improving the control quality of the magnetic bearing with the load and the noise.

3.1.3. The Simulation Results with LQG Controller:

With an AMB system containing unwanted channel interference and the limited number of sensors and actuators, the LQG control will be an effective alternative.

When using the LQG controller we have the simulation results as follows:

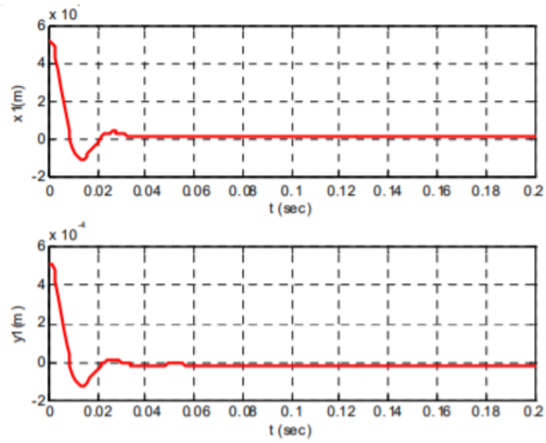

Fig. 6. Output Response of AMB 1 in Terms of x and y Axes using the LQG Controller

We see that the trajectory of the AMB1 system oscillates around the center of the stator and the system quickly gets stable in a short period of 0.01s (the deviations in term of x and y axes rapidly decrease to zero) .

The use of the LQG system has been improved compared to the PID system.

3.2. The Precise Tracking System:

3.2.1. The System Model:

Fig. 7. The Model of the Precise Tracking System

The system model in Fig. 7 is designed from the principle of the printing technology, in which the slider can be moved forward and backward flexibly by the drive of the DC motor through the curoa belt.

In the model, the designers arranged the entire electric motor, scroll bar, slider and curoa belt on a plastic

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frame with the purpose to create the vibration when the slider moves.Model development can be applied in practice as 2D, 3D drawers, CNC machine or other precise position control systems.

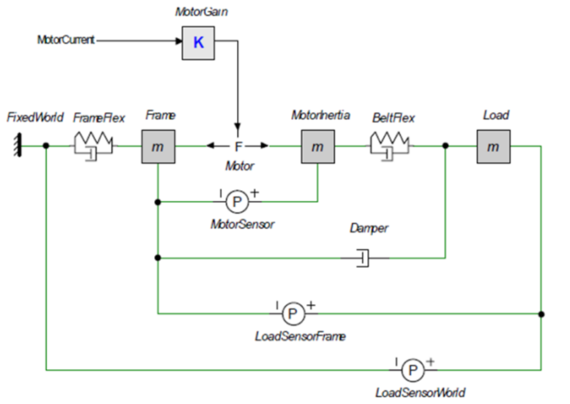

Fig. 8. Simulation of Mechanical Structure

In calculating, by ignoring the nonlinear components of friction, the mathematical model of the object is determined as the linear 6-DOF system [7].

If the belt between the motor and the slider is considered as rigid and ignores the rotor mass of the engine, the object will be a linear 4-DOF system.

If the frame is solid, the object form is represented by 2-DOF by the state equations as follows

$$\begin{cases} \begin{bmatrix} \dot{x}_{loadworld} \\ \dot{v}_{loadworld} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{d}{m_l} \end{bmatrix} \begin{bmatrix} x_{loadworld} \\ v_{loadworld} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m_l} \end{bmatrix} F \\ y = [1 \quad 0] \begin{bmatrix} x_{loadworld} \\ v_{loadworld} \end{bmatrix} \end{cases} \quad (33)$$

Where $v_{loadworld}$ *and* $x_{loadworld}$ are the speed and position of the slider in the original coordinate.

The simulation is performed on the Matlab/Simulink with the input square pulse.With a set value of 25000 pulses and errors, the position signal from the encoder will be sent and compared with the set value. After, the controller creates the control signal to control the motor.

3.2.1. The Simulation Results with the PID Controller:

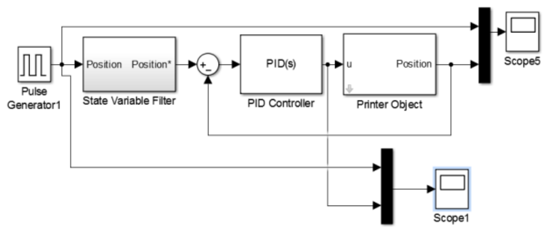

Fig. 9. The Simulation Structure of the PID Controller

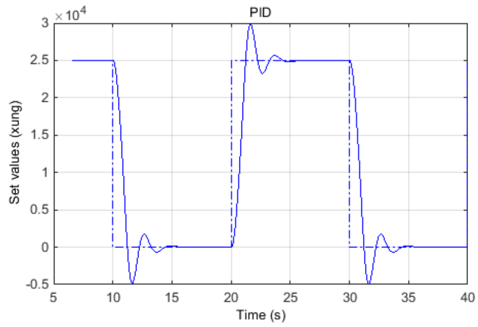

Fig. 10. Simulation Results of the PID Controller

With the PID controller, the simulation results show that the output signal tracks the reference signal after a certain period, and the error goes to zero, but the overshoot and the settling time are too large.

3.2.2. The Simulation Results with the LQG Controller:

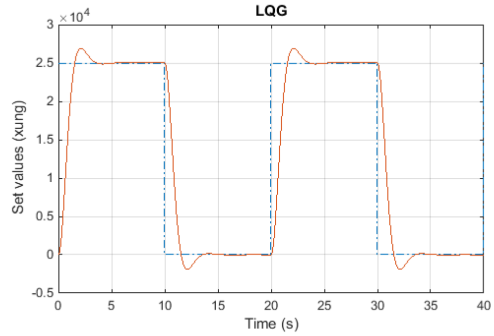

Fig 11. Simulation Results with LQG

Comment:

- It is clear that the system response is fixed, in which the output signal tracks the desired signal after the 3rd second and the error reduces to zero.

- The system response with the LQG controller has a small overshoot and a short settling time.

- This indicates the feasibility of using the LQG into motion control systems with precise control and continuous rotation reverse.

IV. Conclusions

This paper proposes the LQG controller used for some electric drive systems.The simulation results show that the automatic system using the LQG meets the required quality. Moreover, this system is considered as a good premise for other designing approaches in centralized control solutions for the drive systems

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