Application of Fourier Transform in Signal Processing

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Abstract: Fourier transform is a special integral transformation. The Fourier transform transforms the signal from the time domain to the frequency domain, and the frequency domain method has many outstanding advantages compared with the classical time domain method. Although Fourier analysis is not the only transform domain method in the field of information science and technology, it has to be recognized that Fourier transform analysis has always been widely used in this field. Fourier transform is used to realize the filtering, modulation and sampling of the signal, which is the most important application of Fourier transform in signal processing. Through the modulation of the signal can be low-frequency components of the signal modulation to high frequency, to achieve the spectrum shift, to reduce inter-code crosstalk, improve the new noise resistance, is conducive to long-distance transmission of the signal. In addition, the signal sampling can be continuous signal discretization, help to use the computer to deal with the signal. In short, Fourier transform has a very important role in signal processing, learning Fourier transform is the basis for learning other frequency domain transformations.

Key words: Fourier transform, time domain, frequency domain, signal processing, information science and technology, filtering, modulation, sampling

1 Introduction

Fourier transform has a long history and wide range of communication systems. The development of modern communication systems is accompanied by the careful application of Fourier transform methods. Especially in signal processing, the typical use of Fourier transform is to decompose the signal into amplitude and phase components. At present, in the field of signal processing and communication, the most active use of MATLAB in the mathematical application of software applications in the numerical calculation of second to none, and the current Fourier transform in the field of communication applications is based on this mathematical software , and in addition to digital signal processing, the excellent graphics processing capabilities enable it to solve the problem of specific types of Fourier transforms in these applications in digital image processing techniques, so that Fourier leaf transformation in the communication to be better application and development.

Filtering, modulating and sampling, digitizing the analog signal; processing the signal are used to improve the signal performance, resulting in a new ideal signal. In addition, through the modulation, so that different frequencies, different time domain signals can be sent at the same time, so as to achieve the purpose of saving the band, the so-called time division multiplexing and frequency division multiplexing. Telephone and television are also related to the transformation of Fourier. The establishment of the Fourier analysis method has gone through a long history, involving many people's work and many physical phenomena. Fourier transform plays an important role in different fields, such as modern acoustics, voice communication, sonar, earthquake, nuclear science, and even biomedical engineering. Today's Fourier analysis has become an indispensable tool for signal analysis and systems.

2. Fourier transform

2.1 The development and development of Fourier transform

In 1804, the French scientist J.-B.-J. Fourier was engaged in the study of heat flow due to the need for industrial processing of metals at that time. In his article entitled 'The Analytic Theory of Heat', he developed the heat flow equation and pointed out how to solve it. In the process of solving, he presents the idea that any periodic function can be represented by a triangular series. Although the lack of rigorous argument, the modern mathematics and physics, engineering technology have had a profound impact, as the origin of Fourier transform. From the perspective of modern mathematics, Fourier transform is a special integral transformation. It can meet a certain condition of a function expressed as a sinusoidal basis of the linear combination or integral. In different fields of study, Fourier transforms have many different variants, such as continuous Fourier transform and discrete Fourier transform. Fourier transform through the analysis of the function to achieve a complex understanding of the complex functions and research. Initially, Fourier analysis was a tool for analytical analysis of thermal processes, but its method of thought still had typical features of reductionism and analytics. 'Arbitrary' function through certain decomposition, can be expressed as a linear combination of sine function form, and sine function in physics is fully studied and relatively simple function class. Using this, the Fourier transform can be used to understand complex things by studying relatively simple things, and modern mathematics finds that Fourier transforms have very good properties:

(1) Fourier transform is a linear operator, if given the appropriate norm, it is still unitary operator.

(2) The inverse transformation of the Fourier transform is easy to find, and the form is very similar to the positive transformation.

(3) The sinusoidal basis function is an Eigen function of the differential operation, so that the solution of the linear differential equation can be transformed into the algebraic equation of the constant coefficient. In a linear time-invariant physical system, the frequency is invariant in nature, so that the response of the system to complex stimuli can be obtained by combining its response to different frequency sinusoidal signals.

(4) The well-known convolution theorem states that Fourier transform can make complex convolution operations a simple product operation, which provides a simple means of calculating convolution.

(5) Discrete form of Fourier transform can be quickly calculated using a digital computer (the algorithm is called Fast Fourier Transform Algorithm (FFT)). It is because of the good nature of the above, Fourier transform in physics, number theory, combinatorial mathematics, signal processing, probability, statistics, cryptography, acoustics, optics and other fields have a wide range of applications.

2.2 Fourier transform definition

If f (t) satisfies the Dirichlet condition on any finite interval, and f (t) is absolutely integrable on $(-\infty, +\infty)$ (the integral is converged as follows)

(1)

Then the following Fourier transform holds:

(2)

Fourier transform:

(3)

Among them, F (ω) is called f (t) of the image function, f (t) is called F (ω) of the original function.

2.3 Classification of Fourier transform

Continuous Fourier Transform: In general, if the word 'Fourier transform' is preceded by any qualifier, it refers to 'continuous Fourier transform'. 'Continuous Fourier transform' represents the square integrable function f (t) as the integral or series of complex exponential functions, as in Equation 3.

This equation actually represents a continuous Fourier transform, that is, the function f (t) of the time domain is expressed as the integral of the function F (ω) of the frequency domain. In turn, the positive transform happens to represent the function F (ω) of the frequency domain as an integral form of the function f (t) of the time domain. Generally, the function f (t) is the original function, and the function F (ω) is the Fourier transform image function. The original function and the image function form a Fourier transform pair. A generalization of continuous Fourier transform is called fractional Fourier transform (Fractional Fourier Transform). When f (t) is an odd function (or even function), the cosine (or sine) component will die, and it can be said that the transformation is cosine transform or sine transform. Note that F (-ω) = F (ω) holds when f (t) is a pure real function.

Discrete Fourier Transform: In order to use the computer for Fourier transform in the fields of scientific computing and digital signal processing, the function x (n) must be defined in discrete rather than continuous domains, and the finite or periodic conditions must be satisfied. In this case, using the discrete Fourier transform, the function x (n) is expressed as the following summation:

(4)

Where X (k) is a discrete Fourier transform. Directly using this formula to calculate, and fast Fourier transform (FFT) can greatly reduce the complexity. The reduction of computational complexity and the development of digital circuit computing power make DFT a very practical and important method in the field of signal processing.

3.Application of Fourier Transform in Filtering Technology

3.1 The concept of filtering

The use of circuit capacitive reactance or resistance with the frequency of the characteristics of the different frequency of the input signal to produce a different response to the need for a certain frequency of the signal through the smooth, and inhibit the unwanted other frequency signals, this process is Filtering, the system that implements this process is called a filter.

Set the filter input and output, then the filter system input relationship is as follows:

(5)

From the time domain convolution theorem, Equation 5 can be converted to

(6)

among them: , ,

From the formula 6, with the help of Fourier transform not only to simplify the operation, but also from the frequency domain on the signal research, spectrum analysis provides the possibility. Also known from the formula 6

(7)

Which is called a system function, can fully represent the nature and characteristics of the system. Therefore, if the input and the required output are known, the Fourier transform is carried out separately, and the appropriate filtering system can be designed according to the need, so as to meet the practical needs.

3.2 Ideal Selective Filter

Ideal for filtering the frequency characteristics, with a frequency range within a complex exponential signal or sinusoidal signal can be passed without distortion, in addition to the frequency range is given complete suppression. Usually the frequency range through which the signal can pass is called the passband of the filter, and the frequency range through which the signal is passed is called the stopband. The boundary frequency of the passband is called the cutoff frequency. According to the filter through, the stopband in different locations, can be divided into low-pass filter, high-pass filter, band-pass filter and band-stop filter and other basic filters, they are signal and system analysis of the important basic system The

1. The ideal low-pass filter

Ideal low-pass filter is able to make a signal within a frequency range without distortion, and higher than a certain frequency of the signal completely suppressed filter, the system function is

1,

(8)

0,

Which is the cutoff frequency of the ideal low-pass filter. The spectrum is shown in Fig.

Figure 1 Ideal low-pass filter spectrum

2. The ideal high-pass filter

The ideal high-pass filter corresponds to the ideal low-pass filter, which means that the signal above a certain frequency is passed without distortion and the signal below this frequency is completely suppressed. The system function is

1,

(9)

0,

Which is the cutoff frequency of the ideal high-pass filter. The spectrum is shown in Figure 2.

Figure 2 ideal high-pass filter spectrum

3. The ideal band-pass filter

The ideal bandpass filter is a filter that allows a particular band of signal waves to be shielded by other bands at the same time. The system function is

1 ,

(Ten)

0, or

Among them, called the bandpass filter low-pass cut-off frequency, said bandpass filter high-pass cut-off frequency. The frequency response is shown in Fig.

Figure 3 ideal bandpass filter spectrum

4. The ideal band-stop filter

The ideal band-stop filter corresponds to the ideal band-pass filter, which refers to a filter that attenuates or suppresses a signal within a certain frequency range and allows a signal outside the frequency range to pass. The system function is

0

(11)

1, or

The frequency response is shown in Fig.

Figure 4 ideal band-stop filter spectrum

3.3 Physical realizability of the system

For simplicity, the ideal filter is usually defined as a frequency response with a real and unit amplitude in the frequency domain with zero phase characteristics. In fact, the frequency response of all the ideal filters is multiplied, allowing the signal in the passband to pass without distortion and to completely suppress the signal outside the pass band. According to the Fourier transform of the time shift property, multiplied by the linear phase shift factor, only to make the signal produce a time lag, they are still ideal filters. In order to distinguish it from the above-described zero-phase ideal filter, a linear phase ideal filter may be used.

But in fact, there is no real meaning for the ideal filter. The actual filter can not completely filter out the wave designed to allow frequencies to pass outside the frequency range. For example, in the ideal passband boundary there is a part of the frequency attenuation of the region, can not be completely filtered, this curve is called the roll-off slope (roll-off). The roll-off slope is usually expressed in dB degrees to indicate the degree of attenuation of the frequency. In general, the filter is designed so that the transition zone is as narrow as possible so that the filter can be closest to the ideal passband design.

In terms of time domain characteristics, a physically achievable system must be causal, that is, its unit impulse response must be zero at t <0. From the frequency domain characteristics, if the square can be satisfied with the conditions, that is,

(12)

Figure 5 Actual bandpass filter amplitude characteristics

4. Application of Fourier Transform in Modulation and Demodulation Technology

In many engineering problems, the concept of modulation and demodulation plays a very important role, and has a wide range of applications. The so-called modulation is to use a signal to control a signal of another parameter, resulting in modulated signal, its essence is to move the spectrum of various signals, so that they do not overlap each other to occupy a different frequency range. In almost all real communication systems, the signal from the transmitter to the receiver, in order to achieve effective, reliable and long-distance signal transmission, need to be modulated and demodulated. Such as wireless communication. The modulation process moves the spectrum of the signal to any desired higher frequency range, which is easily radiated in the form of electromagnetic waves.

The purpose of modulation is to convert the analog or digital signal to be transmitted into a signal suitable for channel transmission, which means that the baseband signal (source) is converted into a very high frequency signal with respect to the baseband frequency. This signal is called a modulated signal, and the baseband signal is called a modulated signal. Modulation can be achieved by changing the amplitude, phase, or frequency of the carrier by changing the high frequency carrier as the signal amplitude changes. The modulation process is used for the originating of the communication system. At the receiving end, the modulated signal is reduced to the original signal to be transmitted, that is, the process of extracting and understanding the baseband signal from the carrier for the intended recipient (sink) processing and understanding. The process is called demodulation.

On the other hand, if the transmitted signal is not radiated directly without modulation, the frequency of the signals sent by the stations will be the same, and they will be mixed together and the recipient will not be able to select the signal to be accepted. Demodulation is the opposite process, that is, from the modulated signal to restore the original signal, the essence is to move the spectrum of various signals, so that they do not overlap each other to occupy a different frequency range, that is, the signal is attached to different frequencies Carrier, the receiver can be separated from the required frequency of the signal, do not interfere with each other.

4.1 Modulation and demodulation principle

In the radio technology, a parameter (amplitude, frequency, phase) of a high frequency electrical oscillation (current, voltage) called a carrier is changed in accordance with the characteristic change of the signal to be transmitted. Low-frequency signal (that is, the signal to be transmitted) is low in radiation efficiency and can not be used directly for transmission. The purpose of modulation is to transmit low-frequency signals by means of high-frequency electrical oscillation. Different low-frequency signals can be simultaneously carried on different frequencies of high-frequency electrical oscillations at the same time, so that you can make full use of the radio spectrum while transmitting many broadcast signals, and they will not interfere with each other. According to the characteristics of the amplitude, frequency or phase of the high frequency carrier with the low frequency signal, the modulation is divided into amplitude modulation, FM or phase modulation. In addition, if the signal is used to modulate the parameters of the pulse train (pulse amplitude, pulse width or pulse position, etc.), then this set of modulated pulse sequence to modulate a high-frequency sine wave carrier, this modulation is called pulse modulation The

Demodulation is the reverse of modulation, which refers to the process of restoring a modulated signal to an original signal. The demodulation methods currently used are coherently demodulated and non-coherently demodulated. These methods can effectively demodulate all the characteristics (amplitude, frequency, initial phase) of the modulated signal.

4.2 Sinusoidal modulation process

The carrier signal is its Fourier transform is

F[]=[] (13)

The modulated signal is a finite frequency band with G (), occupying-to-bound, and the modulated signal is obtained by multiplying the time domain, as shown in Eq. (14)

= (14)

Fourier transform of the above formula and Fourier transform of the time domain convolution properties:

F[]==(1/2)*[]

=1/2[] (15)

As a result, the spectrum of the signal is moved to the vicinity, and the movement of the spectrum is realized. Spectrum removal process to achieve the process shown in Figure 6:

Figure 6 Modulation principle box and its spectrum

4.3 Coherent demodulation

Let the signal be the local carrier signal of the receiving end, which is in phase with the carrier of the transmitting end. And multiplied by the results of the spectrum to the left and right, respectively (and multiplied by the coefficient of 1/2), the spectrum can also be derived from the time domain multiplication is explained:

=[]

=1/2+1/2 (16)

F[]==1/2+1/4[] (17)

And then use a suitable low-pass filter, filter in the frequency near the component, you can remove, complete the demodulation. The demodulation process is shown in Fig.

Figure 7 Coherent demodulation block diagram and spectrum

This demodulator is called coherent demodulation (or synchronous demodulation) and requires a local carrier with the same frequency as the transmit end at the receiver, which complicates the receiver. In order to save the local carrier at the receiving end, the following method can be used. If the A is large enough, for all t, there is A +> 0, then the envelope detector of the modulated signal can be the same, if the carrier signal of the transmitting end is added to the transmitted signal. Extraction envelope, recovery, no need for local carrier. This method can reduce the cost of the receiver, but at the expense of the use of expensive transmitters because of the need to provide a sufficiently strong signal A additional power. In such a modulation method, the amplitude of the carrier varies proportionally with the signal and is therefore referred to as 'amplitude modulation' or 'amplitude modulation (AM).' The frequency or phase of the carrier can also be controlled so that they vary proportionally with the signal The principle is also to move the spectrum.

5. Application of Fourier Transform in Sampling Technique

The rapid development of digital electronic technology, especially the computer in automatic control, automatic detection and many other areas of the extensive application, making the use of digital technology to deal with analog signals is also more common. In the communication system, the use of existing digital technology to deal with analog signals, not only can make analog signal transmission more simplified, but also to ensure the accuracy of transmission. The use of digital technology to deal with analog signals, the first digital signal will be digital. The analog signal can be digitized using sampling.

By Fourier transform, it can be seen that under certain conditions a continuous time signal or discrete sequence can be uniquely represented by its equally spaced sample values, which are complete and sufficient. In other words, this set of equally spaced sample values contains all the information of the original signal or sequence, and the original signal can be fully recovered from this set of sample values.

5.1 Ideal sampling

In general, without any additional conditions, it can not be expected that a continuous function can uniquely characterize its set of equally spaced sample values because there are infinite number of signals at a given interval Produce a set of identical samples. However, if it is a finite continuous time signal and the sample is sufficiently dense, then the signal can uniquely be characterized by its sample value, and the original signal can be completely recovered from these sample values.

Let the original continuous time signal be a limited period of time limit signal, ie

F[] = , 且 = 0， (18)

If the sampling interval is met:

< 或 = > 2 (19)

Is uniquely determined by its sample values {x (n), n = 0, 1, 2, ...}.

The sampling pulse signal is an impulse signal, ie

(20)

The time-domain waveform and spectrum are shown in Figure 5.1.2.

The sampled signal is also an impulse string, and the intensity of each impulse is equal to the sample value that is the interval. which is

(21)

It is achieved by the ideal sampling shown in Figure 8. The band-limited signal is multiplied by the impulse string of the period to obtain the sampled signal

(22)

Figure 8 Block diagram of ideal sampling system

Figure 9 (a) shows a time-domain waveform for an ideal sample. The ideal sampling process can be observed intuitively in the frequency domain using Fourier transform. Figure 9 (b) shows the spectrum of the above process.

The spectrum of the sampled pulse signal is

(23)

Using the frequency domain convolution properties, the available spectrum is

(24)

The spectrum shown above is the period of reproduction and multiplied by (1 /).

Figure 9 (a) Signal waveform at the time of impulse string sampling (b) Spectrum of the corresponding signal

5.2 Sampling recovery

As can be seen from Fig. 9, if the sampling frequency is not less than 2, the spectrum of the sampled signal is repeated without overlap. As long as the conditions to meet the 19 type, from the frequency domain point of view, faithfully at the integer frequency of the sampling frequency to reproduce, so you can use a low-pass filter, from which to fully recover or rebuild out. The frequency response of the low-pass filter is

,

= (25)

0,

Which is the cutoff frequency of the ideal low-pass filter. The frequency response is shown in Fig.

For the convenience of discussion, the phase characteristic is zero and Ts is the period of the sampling pulse sequence.

Figure 10 Low-pass filter H (w) spectrum

The filter impulse response expression is

=Sa() (26)

If the sampled signal s (t) is

s(t)= (27)

Using the time domain convolution relationship can be obtained output signal, that is, the original continuous time signal (t)

= (t)= s(t)*

=* Sa()

= (28)

Equation 28 shows that the continuous time signal can be expanded into the infinite series of Sa functions, and the coefficients of the series are equal to the sampling value (nTs). It can also be said that at each sampling value of the sampled signal s (t), a Sa function waveform with a peak of (nTs) is drawn, and the synthesized signal is (t). In accordance with the superposition of linear time-invariant systems, s (t) pass through the ideal low-pass filter, each impulse signal of the sampling sequence produces a response, and these responses are superimposed to restore (t), so that s (T) to restore (t) for the purpose.

5.3 Zero order sample hold

Let the original continuous time signal, for the sampling pulse sequence, is the sampled signal, their waveform shown in Figure 11. At the sampling moment, the pulse sequence pair is sampled and the sample value is maintained until the next sampling instant, thereby obtaining the output signal as a sampled signal with a stepped shape.

After the transmission reaches the receiving end need to restore the signal,

= (29)

=1/ (30)

Where is the sampling period, = 2 / is the repetition angle frequency, which is the spectrum of (t).

Figure 11 Zero-order sample hold box

Figure 12 Zero-order sample hold waveform

Let the system function of the zero order hold system be

=u(t)-u(t-) (31)

The waveform diagram shown in Figure 13.

Figure 13 Waveform of the system function h (t)

The output signal can be expressed as follows:

= s(t)*ho(t) (32)

In the formula, the Fourier transform is

F[]=Sa(ωTs/2) (33)

By frequency domain relation: = F []

= F[]

=F[]

=∑F(w-n) Sa(ω/2) (34)

It can be seen that the basic characteristic of the spectrum of the zero-order sample-hold signal is still that the F (w) spectrum is repeated in cycles, but multiplied by the Sa (ω / 2) function and the delay factor is added. When the F (w) band is limited and the sampling theorem is satisfied, a low-pass filter with the following compensation characteristics is introduced at the receiver

/Sa(ω/2), (|w|≤/2)

Hor(w)= (35)

0 , (|w|≥/2)

Figure 14 Compensates low-pass characteristics

Its amplitude-frequency characteristic | Hor (w) | and phase-frequency characteristic curves are shown in Fig. When the signal through the compensation filter, you can restore the original signal (t). From the frequency domain, it will be multiplied by Hor (w) to get F (w).

Under normal circumstances, in the communication system, only the amplitude-frequency characteristics as required to meet the compensation requirements, and phase frequency characteristics as long as the linear phase shift characteristics can be met.

6. Frequency division multiplexing and time division multiplexing

Will be a number of road signals in some way convergence, unified transmission in the same channel called multiplexing. Reuse technology has penetrated into our daily lives. Like a mobile phone, it can accept audio, video and other different frequency signals, can not be separated from the application of technology. In the modern communication system commonly used in multiplexing technology. Multiplexing techniques are frequency division multiplexing and time division multiplexing.

Frequency division multiplexing refers to the use of sine amplitude modulation to move the spectrum of various signals, so that they do not overlap each other to occupy a different frequency range. The signal is attached to different frequencies of the carrier, so that you can use the same channel transmission. In the receiver using a number of filters can separate the signal, and then by demodulation can be reduced to the original signal, Figure 15 shows the principle of frequency division multiplexing block diagram. Normally, the addition signal (t) is also subjected to a second modulation, demodulating the signal at the receiver and then demodulating it via a bandpass filter.

The theoretical basis of time division multiplexing is the sampling theorem. From the sampling theorem, it can be seen that the frequency band is limited by - m ~ + m, which can be uniquely determined by the interval of the sample value. From these instantaneous sample values, the original continuous signal can be restored. Thus, only the sampling values are allowed to be transmitted, the channel is occupied only at the sampling moment, and the remaining idle time is used for the transmission of the second and third channels. Timing multiplexing can be achieved by arranging the sampled values of the respective signals sequentially. At the receiving end, the sampled signal values are separated by the appropriate synchronization detector. Of course, the actual transmission of the signal is not impulsive sampling, can occupy a period of time. Fig. 16 shows a waveform in which two sampling signals are sequentially arranged in the same channel transmission (time division multiplexing).

For frequency division multiplexing systems, each signal is present in the channel at all times and mixed together. However, each signal occupies a different frequency range, this interval is not occupied by other signals. In a time division multiplexing system, each signal occupies a different time interval, which is not occupied by other signals, but the spectrum of all signals may have any component of the same frequency interval. Essentially, the frequency division multiplexed signal retains the personality of the spectrum and retains the personality of the waveform in the time division multiplexed signal. Since the signal is completely determined by its time domain characteristics or completely by its frequency domain characteristics, it is always possible to recover the multiplexed signal in the receiver by applying the appropriate technique in the corresponding domain.

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