RESEARCH ARTICLE

Principles of controlling the apparatus function in super-resolution problem

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Abstract: When controlling the Apparatus Function (AF), the size of definition domain of the AF O and the sampling step and conditionality of the AF must be chosen so that its inverse function pR=pO-1 obtains a minimum norm. The compensation of the AF O distortions in the measured images is realized point-by-point (without using the Fourier Transform in convolution). The computer of the device uses the resolving function pR, selected by the controlling procedure, for achieving super-resolution in images. Such controlled super-resolution is demonstrated on the Martian images.

Keywords: regularization method; super-resolution; conditionality; invertibility; modulation; transfer function convolution fourier transform

1. Introduction

We will say that the New Device (ND) implements the Principles of Controlling (PC) AF O with optimal pR = pO-1.

Without the PC, ND=D + computer complexes, as a rule, are not workable.

Regularization methods (in such not workable or non-functional NDs) yield results with a large residual error[1] due to a priori information on the smoothness of the solutions.

With the PC AF O ND complexes are able to realize the super resolution without a residual error.

If it turns out that the step of digitization of AF O, cannot be physically reduced, then in the computer, we perform the corresponding recalculation of the measured data (interpolation) under a smaller step of digitization.

The PC AF O in ND implements the maximum resolution beyond the size of the pixel of the original image.

Under the Characteristics of the Adequacy of the Model (CAM) AF O are understood the mutual dependence of the three функций (in XYZ axes): (X) the noise reactions - Nor(pR), (Y) the errors in the assignment of the AF pO - Err(pO) and (Z) the values of the Indicator of Invertibility - II(pR*O)[2–3].

If the Modulation Transfer Function (MTF) M (O) is bounded: diap≤|M(O)|≤1, then 1≤|M(R)|=1/|M(O)| ≤DIAP (shorts for the word Diapason).

The limitation in the frequency domain DIAP = 1/diap is the conditionality parameter of the AF O. If the DIAP value decrease, the model of AF becomes coarser, its conditionality increases.

The most conditioned object is the delta Kronecker symbol with DIAP = diap = 1[2–3].

In papers[2–3], one can see a detailed explanation of the notation and meaning of new mathematical constructions, such as the indicator of computational errors: II(R*O)==M(R)M(O) ≤1.

Comment. In the constructions that claim to solve the inverse problem, there must be concepts such as “Conditionality” of the AF, matrix and, most importantly, “Measure or Indicator of Invertibility”. AF pO and pR*O, which accom-

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pany the obtained results with super resolution imagers, should be presented.

Of course, there must be a construction about the quality of the solution, the inverse problem, which connects the result of the invertibility of the AF with the obtained errors of the type CAM AF O.

To compare the super resolution results, there must be a corresponding numerical value SR that estimates the obtained super resolution in imagers.

If these concepts are not present, then it is unclear what is at stake.

2. Principles of Controlling the AF O, pO in the New Device with AF pR*O

Suppose that there is a linear model of image measuring

$I_\sigma = O*I_x$(1)

The main task of choosing the AF pO = pR-1 is set as a minimum problem:

$$\min_{LO}\{\|pR\| \,|\, Err(pO) \le err\}, \quad LO = \{pO \,|\, [Loc, dx, DIAP]\} \qquad (2)$$

In (2), the set LO - The AF O Lodge for O contains three sets: Loc is the set of lengths of the domain of definition of the O, dx is the set of steps for digitizing the AO in Loc and DIAP is the set of settings of the conditionality in the controlling AF pO.

Note that the invertibility $pO=pR^{-1}$ in frequency domain corresponds to MTF equality: M(pR)=1/M(pO).

With the solution of problem (1) we bind the CAM AF O, pO of the Discrete ND:

$$\{x = \|pR\|, \quad y = Err(pO), \quad z = II(pR*O)\} \qquad (3)$$

It is well known, that the Fourier coefficients in the data square with side ~ 40-50 are computed with the error 10^{-13}–10^{-14}. We will call this small value an instrumental error or instrumental zero: $Iz=10^{-13}$[2–3].

In connection with the limitations in the accuracy of the calculations, we will consider the variant CAM AF O, taking into account the Iz inversion of M(O):

$$M(zR) = \begin{cases} 1/M(O), \ if \ |M(O)| > Iz \\ M(O) \end{cases} \qquad (4)$$

$$\{x = \|zR\|, y = Err(zO), z = II(zR*O)\} \qquad (5)$$

PC AF O with CAM (2,4) Discrete ND allow us to control the settings of AF O.

The ND must measure pIσ again or recalculate Iσ in pIσ and

calculate pR*pIσ as an estimation of Ix.

We note that in the main task to the minimum (2) we realize the minimum error ± Nor (pR) σ or the controlling maximum accuracy of the estimation for the measuring image Ix (1).

Tools of PC AF O

The band path of the Discrete ND will be estimated by the quantities BP=ΣM(pR)*M(O) and BPz = ΣM(zR)*M(O).

The efficiency of (using the Discrete ND) bandwidth will be estimated by the quantities M(pR)M(O) = BP/Σ1 and M(zR)M(O) =BPz/Σ1.

Theorems on invertibility in the bandwidth of Discrete ND. Given AF O, if the computation of the inverse R is realized with an instrumental error Iz, then with this instrumental error the equality and inequality are preserved: II(zR*O) = M(zR)M(O) ≤1.

Inequalities occur in cases with irreversibility II(zR*O)<1. This means that inversion takes place only in the M(zR)M(O) part of the bandwidth of the Discrete ND.

The property of invertibility: For all values of Loc, dx=1 and DIAP in LO, the set AFO is mapped to the set of invertible $pO = pR^{-1}$, M(pR) = 1/M(pO), and the Invertibility Indicator II(pR*pO)=1.

The invertibility theorem: The Iz inversion (3) is given, then the invertibility II(zR*O)=1 implies the we have ordinary invertibility II(R*O) = 1, $zR = R = O^{-1}$ with the use of the full band pass M(zR)M(O)=1 and the identity

M(zR)M(O)==1.

Estimation of the super resolution value

If there is a normalization of the AF O: ΣO = 1, then at the zero of the MTF M(O)(0) = 1. In this case, we estimate the super resolution by value

SR=BP/ΣM(pO)≤Σ1/ΣM(pO)≥1, (5)

i.e. SR≥1. If the AF O = D_K-delta Kronecker symbol, then SR = 1. It is clear that a super resolution problem is raised for the real Devices with AF O≠D_K.

The value of SR does not depend on the size of the AF O domain of definition and sizes of images.

Errors of AF pO

The error in the AF pO was estimated as Err (pO) = SD(O-pO)/max(O). SD is the Standard Deviation of the value O-pO or SD(O-pO) = sqrt(Σ(O-pO)^2)/Σ1, where Σ1 is the number of points in the domain of definition of O, pO.

The convolution theorem

For even A and B, there are equalities in the spatial domain A*B = B*A = F^{-1}(M(A)F(B)) = F^{-1}(M(B)F(A)) and in the frequency domain F(A*B) = F(B*A) = M(A)F(B) = F(A)M(B). M(A) and M(B) are the spectra or MTF of AF A and B, M(A) and M(B) are even in the frequency domain, the functions F() is Discrete FT and F^{-1}() is Inverse Discrete FT.

Theorem: Suppose that we have an even O, dependent on the difference of the arguments (O moves). Let A be given, containing even and odd parts. Then in the spatial domain the convolution O*A = F^{-1}(M(O)F(A)), in the frequency domain F(O*A) = M(O)F(A).

In applications, the last result is almost impossible to use because of the boundary effects in F(A), see the examples at the end of the paper. Therefore, the convolution operation must be used point-by-point, without applying the FT.

MTF in convolution

MTF M (O) from AF O is obtained by solving the eigenvalue problem: O*H = M(O)H, where the orthonormal Fourier harmonics are located in the rows of the matrix H. For an even AF O the MTF M (O) is even, for odd O, M (O) is odd[2–3].

For the delta Kronecker symbol, D_K MTF M (D_K)==1 is even. For D_K we will use the other notation D_C (c from the word cos). The fact is that there is an odd analogue of the Kronecker D_S symbol (s from the word sin). The following result holds: the convolution of two odd D_S*D_S = D_C = D_C-1/ Σ1 is a delta Kronecker symbol without constant.

The finite-dimensional sampling theorem for interpolation

Sampling Theorem FDST: Given a two-dimensional array of samples D = f(x0,y0), two-dimensional matrices of discrete Fourier harmonics H(kx,x0), x0 = 0:N-1, H(ky,y0), y0 = 0:M-1, and the "continuous" Fourier harmonics H(kx, x), x = 0:dx:N-dx, H(ky,y), y0 = 0:dy:M-dy, dx <1, dy <1,

$$f(x,y)=\sum_{k_x,k_y=1}^{N,M} c_{k_x k_y} * H(k_x,x) * H(k_y,y) \quad (6)$$

$$c_{k_x k_y} = (f(x0,y0), H(k_x,x0) * H(k_y,y0)) =$$

$$= \sum_{x0,y0=1}^{N,M} f(x0,y0) * H(k_x,x0) * H(k_y,y0),\ k_x = 1:N,\ k_y = 1:M \quad (7)$$

then "continuous function" passes through the sample points f (x0, y0). The scalar products (6) realize a Direct Discrete FT, and the Fourier series (5) is an Inverse Discrete FT with interpolation if dx <1, dy <1. Such interpolation gives good result, if there are no differences in values at the boundary of the arrays data. Otherwise, the interpolation should be performed in a special protected mode[2–3].

3. An Example of the Implementation of An ND with A PC AF

In the low-contrast fragment of the Martian byte image, the relative errors in small brightness are of the order of 3–10%, due to the fact, that we use one byte for brightness storage or we have luminance in the range 0–255.

Therefore, the reaction to noise Nor (pR) should be of the order of ~10. It is precisely this that determined the coarsening of the AF pO model by the conditionality parameter DIAP = 15.

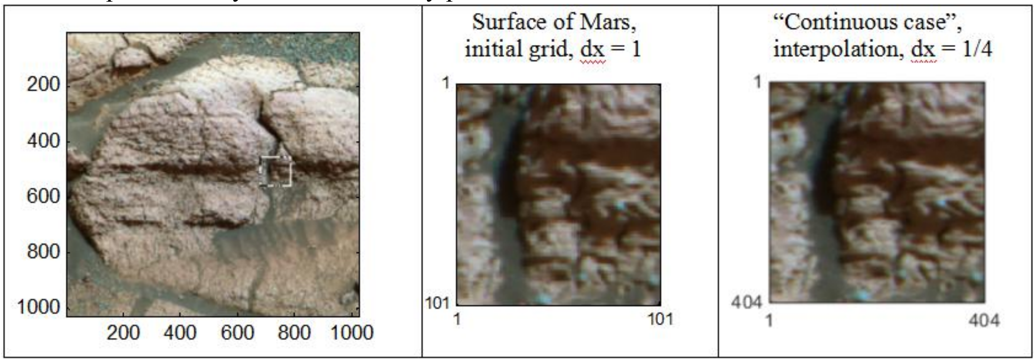

Figure 1. Fragment of the image of the Martian surface presented in the original grid dx = 1 and recalculated with help the FDST to "continuous case" with a grid dx = 1/4.

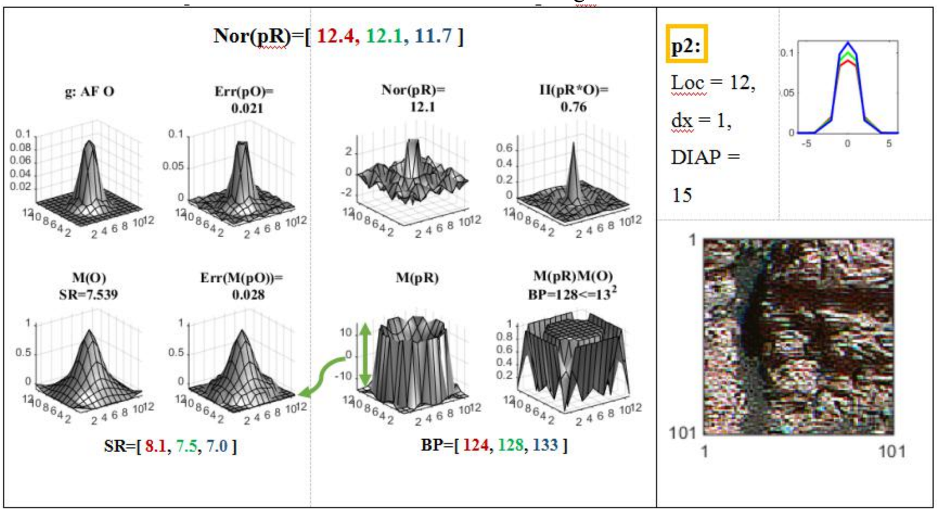

Figure 2. Image with super-resolution is in the original grid with Loc = 12 with DIAP = 15 for AF O.

The detailed graphic data in **Figure 2**, 3 are given in green g color. By using three rgb colors, we show values: noise reactions - Nor (pR), SR-super-resolutions and BP-band paths.

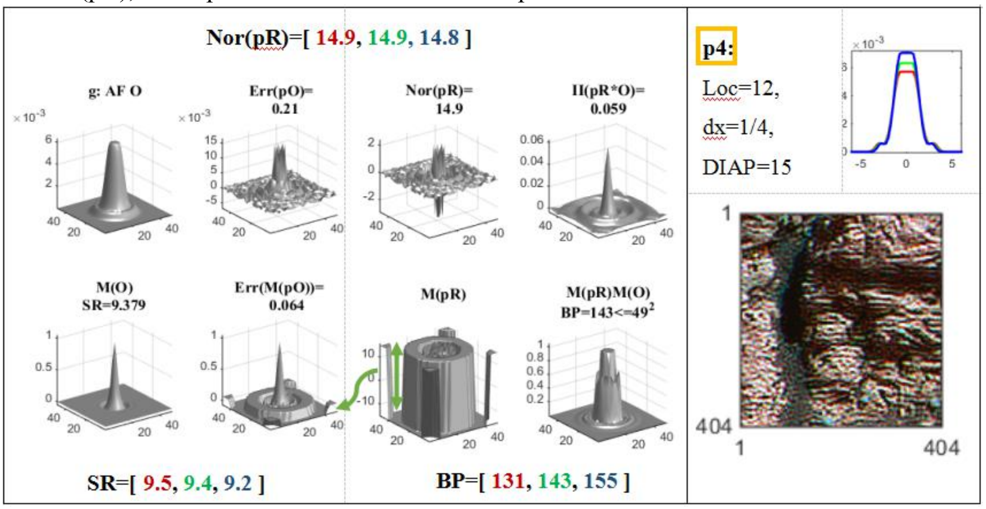

Figure 3. "Continuous" case with Loc = 12, dx = 1/4, DIAP = 15.

Conditionality with DIAP = 15 gives us an acceptable result, see images in **Figure 2**, 3.

The super-resolution in the "continuous case" is approximately on 70% higher than in the discrete case, compare SR values in **Figure 2**, **3.**

Examples of CAM AF O, pO for ND

In this section, we will present CAM AF O, depending on the values of the parameters from [Loc, dx, DIAP] (1).

AF O with Loc = 4 corresponds to the length of the side of the square of the domain of definition of AF O with sides [-2, 2] and on the graphs of **Figure 4**, 5 this is the left points of the graphs, which are labeled with the number 1.

Similarly, for the length Loc = 12 for squares with sides [-6, 6], the right-hand points of the graphs are labeled with the number 2, see **Figure 4.**

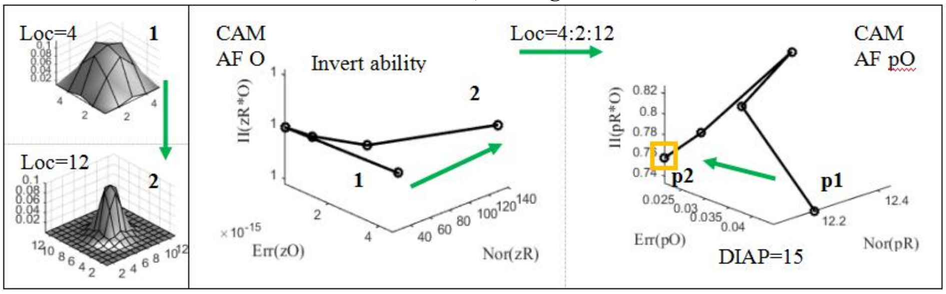

Figure 4. The effect of DIAP = 15 conditionality on the noise reaction Nor(zR), Nor(pR), invert ability II(zR*O), II(pR*O) and errors Err(zO), Err(pO) depending on different lengths of the Loc = 4:2:12 definition domains in the original grid dx = 1.

The lengths Loc = [4, 6, 8] in the original grid are associated with a noise reaction from ~7 to~18. This is acceptable for low-precision measurements. The lengths Loc = [10, 12] are associated with a noise reaction from ~45 to~140. This is acceptable for high-precision measurements.

With the DIAP = 15 conditionality, the noise reaction are down to~12.3, but this decreases the invert ability and increases errors, see **Figure 4.**

In a "continuous case" with the changing in the lengths Loc = 4: dx : 12, dx = 1/4 on the graph CAM AF O, the left point with Loc = 4 is labeled with the number 3, and the right-hand point with Loc = 12 is labeled with the digit 4, see **Figure 5.**

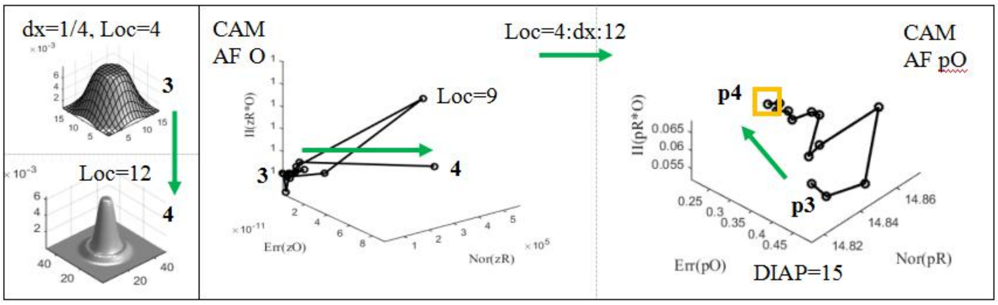

Figure 5. The effect of DIAP = 15 conditionality on the noise reaction, invert ability and errors, depending on the different lengths of the definition domains Loc = 4: dx: 12 in the "continuous case" with dx = 1/4

The prefix "p" before numbers 2 and 3 in **Figure 4**, 5 means that conditionality with DIAP = 15 is activated. In **Figure 2**, 3, the results are presented by the super resolved images in the cases of p2 and p4.

The conditionality parameter DIAP = 15 with Loc = 12 gives us the g: resolving functions pR c Nor(pR)~12.1 and 14.9, with which the super resolved results are obtained in discrete and "continuous" cases.

PC of the AF pR*O

In this section, we will compare CAM AF O with Iz invertibility (3) with CAM AF pO for a given conditionality DIAP = 15 (2) in the "continuous case" with dx = 1/4

In **Figure 6**, we see on the left, marked by double arrows, a very small region of AF O within the main lobe.

In the domain of definition with Loc = 12, the noise reaction is Nor(zR)~6*10^{10}. In order to see this all, we use Iz inversion.

The result is obtained with Nor(pR)~10 if we include conditionality with DIAP = 15. In this case, the invert ability falls and errors are fixed; see **Figure 6** graphics on the right.

It is curious, but in the methods of regularization in this situation, it is proposed to seek smooth solutions. In the regularization method, there is no indicator of invertibility and the conditionality parameter (see Comment).

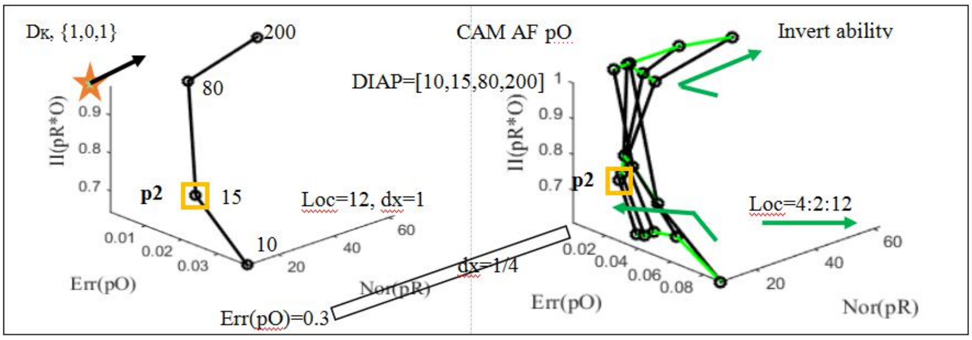

Figure 6. Examples of CAM AF O in the discrete case

It turned out that almost invert ability occurs in four discrete cases out of five from Loc = 4: 2: 12, with a larger error Err (pO) and a smaller noise reaction Nor(pR), see **Figure 6.**

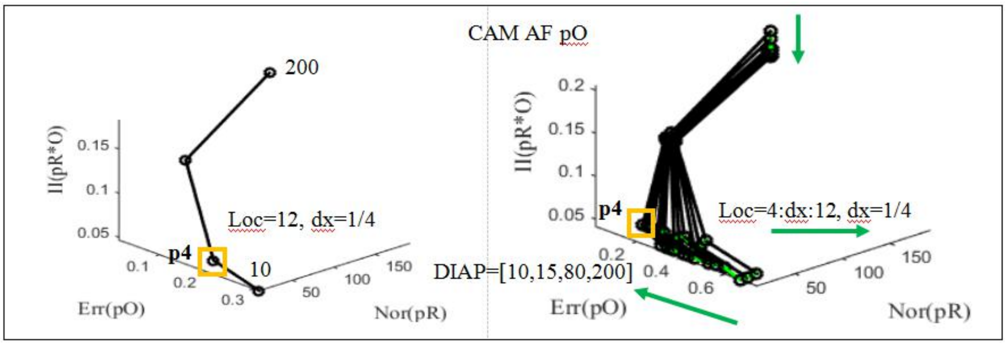

Figure 7. Examples of CAM AF pO in the continuous case

In the continuous case, we noticeably lose the invert ability II (pR*O) and obtain a noticeable increase in the noise reaction Nor(pR), see **Figure 4**,7. If a continuous case is represented together with a discrete one, the continuous case will be located in the place of the long rectangle, see **Figure 6.**

With light squares in **Figure 4**–7, we note the two parametric situations corresponding to the super-resolved images p2, p4 with Loc = 12 in **Figure 2**, 3.

It is curious, if the AFO O to convert to the delta Kronecker symbol D_K, then all the graphs of CAM AF O will converge in one point D_K, {1, 0, 1}. Near this point, with a small arrow, we indicated parametric situations, when problems of super-resolution are solved with high accuracy by ordinary inversion of AF O, $R=O^{-1}$, see "Star with arrow" in **Figure 6.**

About the Value of Super-resolution SR

In reversible discrete cases, SR (5) is a function of [Loc, dx, DIAP] parameters with saturation in two DIAP and Loc parameters, see SR on the left in **Figure 8.** If the AF pO area is an image larger than small sizes Loc, the function SR enters saturation and thus the values of SR do not depend on the size of the images.

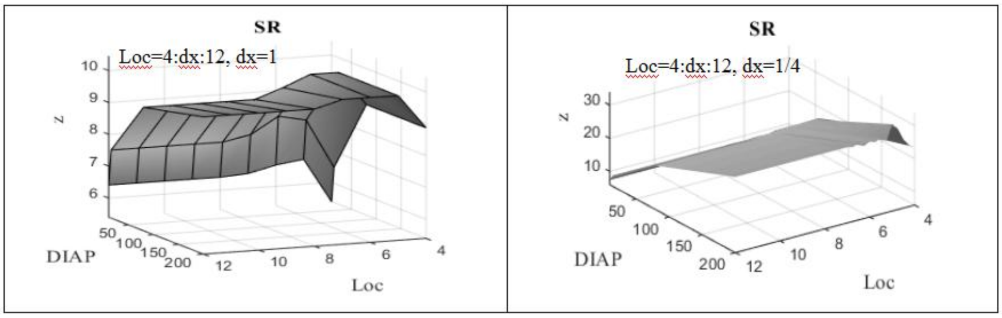

Figure 8. Examples of the behavior of the dependence of SR on LO parameters in discrete and continuous cases.

In the continuous case, in our example, the invert ability by conditionality of DIAP is not reached, saturation with sizes Loc takes place, see SR on the right in **Figure 8.** In the basic continuous case with Loc = 12, dx = 1/4 with increasing

DIAP has a linear section with a noise reaction Nor (pR) from 10 to 30. In the reversible continuous case growth SR goes into saturation, see Figure 9. The situation is clear; in SR in saturation (in the reversible case), all information about AFO be presented.

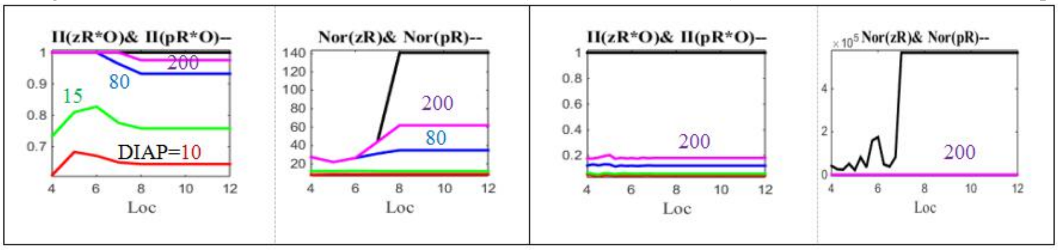

Figure 9. Dependencies of the indicators of invert ability, noise reaction on sizes Loc in discrete and continuous cases.

In the continuous case, zSR allows us to “look beyond the horizon”, i.e. find the super-resolution values SR for very large DIAPs up to Iz^{-1}. If at DIAP = 15 we have pSR = SR~9.5, then for zSR, we will have almost an order of magnitude more - exotic~156 in saturation at Loc = 12, see **Figure 10.**

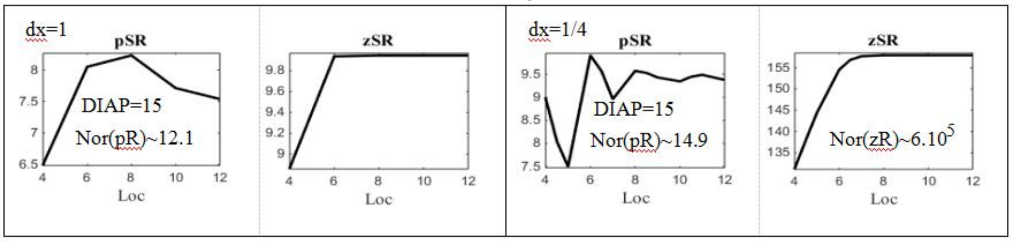

Figure 10. Dependencies of SR on Loc in discrete, continuous cases and in cases of Iz inversions.

It is understood that such an exotic super-resolution unavailable for processing of real experimental data (calculation zR*Iσ (1),4)) due to the very high noise reaction Nor (zR)~6.10^5. This result is essentially the "basis" for conducting modeling PC ND experiments with super-resolution.

For completeness of understanding, we give an example when even modeling experiments on super-resolution have no reason: this is when for wide “AF O” there are large regions of narrow MPF M (O) with min|M(O)|<Iz, i.e. calculations are impossible due to mantissa errors.

Resolution of Objects Smaller Than a Pixel

In the highlighted area of the stone, an area with a suspicion of sand in the dunes is visible. Below you can see the possibility of selecting individual grains, smaller than a pixel size.

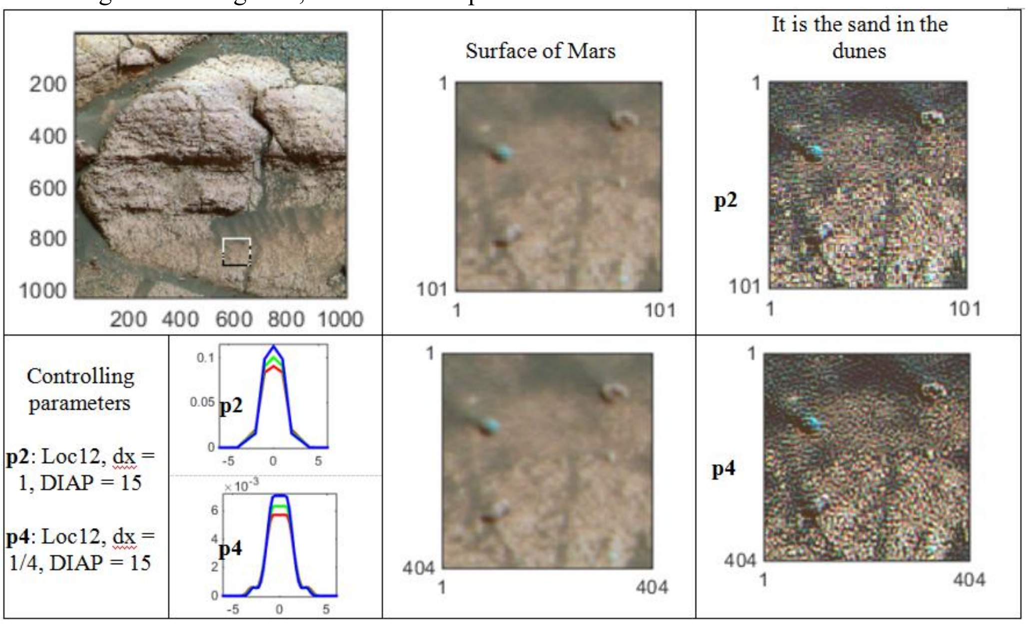

Figure 11. Resolution of individual grains in the discrete case p2 and continuous case p4.

In the “continuous” case, the super-resolution of SR is ~on 15% higher than in the discrete case, compare the values of SR in **Figure 2**–3 and the images in **Figure 6**, 7.

Structure of Stone Similar to Fossilized Coral

In the selected area of the stone, we found structures, in our opinion, similar to the fossilized coral. Of course, specialists must make the final decision.

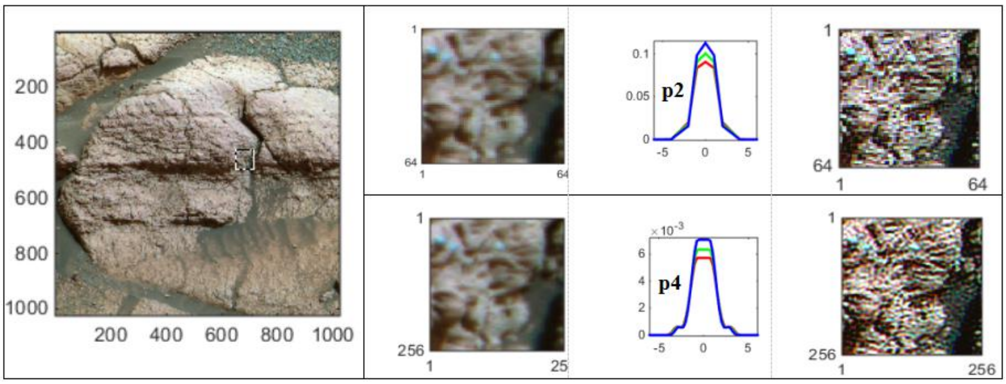

Figure 12. Resolution of the suspected region to organic origin in the discrete case p2 and the continuous case-p4.

The Application of the Convolution Theorem

The convolution operation together with the Fourier Transformation is usually used in many cases.

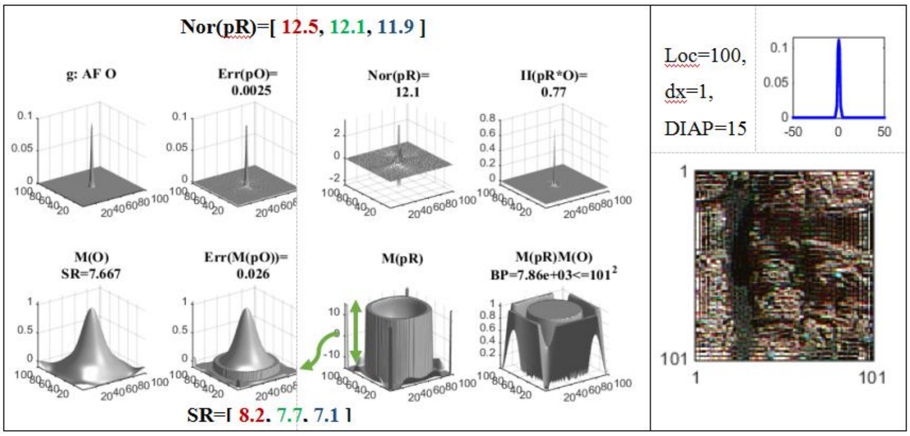

Figure 13. The AF O definition domain is the entire image, discrete case.

The result is completely incomprehensible if we try to realize this through Discrete FT. The well-known result that convolution is the Inverse of the Fourier Transform of the Direct FT functions is valid with certain conditions for even functions.

The fragment of the image is not an even function and brightness jumps occur on the boundaries of the images.

We do not use FT for solving invers problems[5]; see boundary effect in imager in **Figure 13.**

However, if the image is continued evenly on the torus, then the boundary effects are weakened, the convolution theorem will be true. In our papers, an even continuation to the torus was supplemented with special neutralization regions to completely suppress the boundary effects[5], see **Figure 14.**

4. Discussion

For single-byte color images, the SR values vary from [8.1, 7.5, 7.0], dx = 1 is the discrete case to [9.5, 9.4, 9.2], dx = 1/4 is a continuous case.

Transition to a continuous case (for one-byte image) gives us an increase in the SR value by about 15%.

For double-byte images with a similar AF O, the values SR ~ 15–25 are expected in the continuous case.

In super-resolved images, jumps and brightness differences are possible, that is, the solutions of the inverse problem to compensate for the distortion of AF O are not smooth.

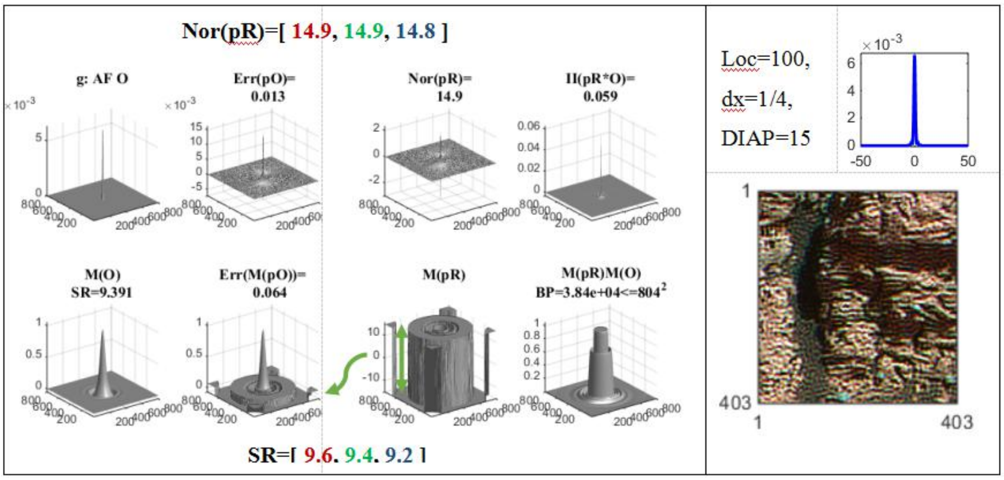

Figure 14. The AF O definition domain is the entire image, continuous case.

The non-smoothness of the solutions allows us to apply this method reasonably and safely, for example, in the reactor control loops.

It is desirable that the ND "can be automatically tuned" according to AF, that is, it can measure with a smaller step and if step reduction is impossible, then the ND should implement interpolation.

If there is a need, then the solutions–super-resolved images–can be "smoothed" according to a priori information.

Controlling the AF of devices with FDST on CAM AF O is the basis for creating a controlling ND with pR*O AF with embedded processors in various fields.

It is possible to use PC AF with interpolation to achieve the maximum resolution in electron microscopy, atomic force microscopes, radar, telescopes, tomography, etc. To increase resolution in tomography requires a very powerful processor.

Interesting modifications of the PC AF O methods are expected in new radar technologies, synthesized aperture, X-ray tomography, telescopes, etc.

In this paper, we considered problems of increasing the resolution only due to the methods of controlling the AF O, mathematical methods proved much more complicated than the methods of classical regularization.

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