Engineering and Scientific International Journal (ESIJ)
Volume 3, Issue 2, April – June 2016
ISSN 2394-7187(Online)
ISSN 2394 - 7179 (Print)

A Generalized Class of Jack-Knifed Estimator for Population Mean using an Auxiliary Variable and Attribute under Measurement Errors

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Abstract— In this paper, we have considered a generalized class of estimators using auxiliary information in both the form variable and attribute under measurement error. We also suggest a class of unbiased estimators using the Jack-knife technique under measurement error. The bias and mean square error (MSE) for the proposed classes are obtained under measurement error. Finally, some concluding remarks are made clearly demonstrating the importance of the proposed classes.

Keywords— Mean Square Error; Non-Response Errors; Generalized Estimator.

1. Introduction

We know that efficiency of an estimator is always increased when we use the auxiliary information in sampling. Such kind of auxiliary information may be in the form of a variable or an attribute which is known in advance or being collected while the survey is conducted without increasing the cost of the survey or at a little cost to improve the efficiency of the estimators. For an example height is closely related to the weight factor and it is also dissimilar to male and female showing that sex is a helpful attribute while estimating height, Amount of milk produced by the cow is associated on the breed as well as on the diet, yield of the crop depends on the variety of the seed and manure used as well etc. In such situation we can use the advantage of auxiliary information available in the form of variable and attribute to improve the efficiency of the estimator. Many of the statistician have utilizes the supplementary information either in the form of variable or in the form of attribute at a time. Here we have considered both type of information in the present study.

Over the past few decades, statisticians were given their interest towards the estimation of parameters in the occurrence of measurement errors. In survey sampling, the possessions of data typically assume that the observations are the right measurements on distinctiveness being studied. However this supposition is not pleased in many functions and data is contaminated with measurement errors, like non-response errors, treating errors and calculating errors. These measured errors make the result unacceptable, which meant for no quantity error case. If measured errors are considerably very small and so we can neglect it, then the statistical inference based on data observed continue to remain valid. On the contrary, when they are not substantially small and insignificant, the inferences might not be invalid and imprecise but may often lead to unforeseen, undesirable and unlucky consequences in Srivastava and Shalabh, (2001). Some important sources of quantity errors in survey data are argued in Cochran (1968), Shalabh (1997), and Singh and Karpe (2008, 2010) discussed some determinants of population mean under measurement errors.

For a simple random sample of size *n*, let (ϕ_j, x_j, y_j) be the pair of values instead of the true values (Φ_j, X_j, Y_j) on the characteristics (Φ, X, Y) respectively. Let the error of the observation or measurement is defined as

$$u_j = y_j - Y_j, \quad j = 1, 2, \ldots, n \tag{1.1}$$

$$v_j = x_j - X_j$$

$$w_j = \phi_j - \Phi_j \tag{1.2}$$

2. Proposed Class of Generalized Estimator

The generalized estimator of population mean μ_Y, using mean μ_X of auxiliary variable X and the population proportion P of auxiliary attribute Φ is given by

$$\bar{y}_{at} = f\left(\bar{y}, \frac{\bar{x}}{\mu_x}, \frac{p}{P}\right) \tag{2.1}$$

where $f(.)$ is a function of $\left(\bar{y}, \frac{\bar{x}}{\mu_x}, \frac{p}{P}\right)$ satisfying the following regularity conditions:

(i) $f(\mu_Y, 1, 1) = 1$,

(ii) The function is continuous and bounded in the closed interval **R** of real line,

(iii) The second order of partial derivatives of the function $f\left(\bar{y}, \frac{\bar{x}}{\mu_x}, \frac{p}{P}\right)$ are exist and are continuous and bounded in R also the first order partial derivatives are zero;

where p , $\bar{x}$ and $\bar{y}$ are the sample proportion, sample means of Φ , X and Y respectively for a simple random variable of size *n*.

$$\bar{y} = \frac{1}{n}\left[\sum_{j=1}^{n} y_j\right]$$

$$= \frac{1}{n}\left[\sum_{j=1}^{n} u_j + Y_j - \mu_Y + \mu_Y\right]$$

$$= \frac{1}{\sqrt{n}}\left[W_u + W_y\right] + \mu_Y \quad (2.2)$$

Proceeding likewise, we have,

$$\bar{x} = \frac{1}{\sqrt{n}}\left[W_v + W_x\right] + \mu_x \quad (2.3)$$

$$p = P + \frac{W_p}{\sqrt{n}} \quad (2.4)$$

Where,

$$W_u = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} u_i ,$$

$$W_y = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(Y_i - \mu_Y) , \qquad W_v = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} v_i ,$$

$W_x = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_i - \mu_X)$ and $W_p = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(\Phi_i - P)$. It is also noteworthy that in most practical situations the attribute is less prone to measurement error than a variable and therefore, we assume that the measurement error $w_j = 0$ and hence $W_w = 0$ so the resultant is (2.4). From generalized estimator (2.3) expanding by Taylor's series and use the results (2.2), (2.3) and (2.4), we have

$$\bar{y}_{at} = f\left[\bar{y}, \frac{\bar{x}}{\mu_X}, \frac{p}{P}\right]$$

$$= \left[f(\mu_Y, 1, 1) + (\bar{y}_j - \mu_Y)\frac{\partial f}{\partial u} + \left(\frac{\bar{x}}{\mu_X} - 1\right)\frac{\partial f}{\partial v} + \left(\frac{p}{P} - 1\right)\frac{\partial f}{\partial w}\right.$$

$$+ \frac{1}{2!}\left\{(\bar{y} - \mu_Y)^2\frac{\partial^2 f}{\partial u^2} + \left(\frac{\bar{x}}{\mu_X} - 1\right)^2\frac{\partial^2 f}{\partial v^2} + \left(\frac{p}{P} - 1\right)^2\frac{\partial^2 f}{\partial w^2} + 2(\bar{y} - \mu_Y)\left(\frac{\bar{x}}{\mu_X} - 1\right)\frac{\partial^2 f}{\partial u \partial v}\right.$$

$$\left.\left. + 2\left(\frac{\bar{x}}{\mu_X} - 1\right)\left(\frac{p}{P} - 1\right)\frac{\partial^2 f}{\partial v \partial w} + 2\left(\frac{p}{P} - 1\right)(\bar{y} - \mu_Y)\frac{\partial^2 f}{\partial w \partial u}\right\} + \ldots\right]$$

$$= \left[\mu_Y + \frac{(W_u + W_y)}{\sqrt{n}} f_1 + \frac{(W_v + W_x)}{\mu_X\sqrt{n}} f_2 + \frac{W_p}{P\sqrt{n}} f_3 + \frac{1}{2!}\left\{\frac{(W_u + W_y)^2}{n} f_{200}\right.\right.$$

$$+ \frac{(W_v + W_x)^2}{\mu_X^2 n} f_{020} + \frac{W_p^2}{P^2 n} f_{002} + 2\frac{(W_u + W_y)(W_v + W_x)}{n\mu_X} f_{110}$$

$$\left.\left. + \frac{(W_v + W_x)(W_p)}{\mu_X P} f_{011} + 2\frac{(W_u + W_y)(W_p)}{P} f_{101}\right\} + \ldots\right]$$

therefore,

$$\bar{y}_{at} - \mu_Y = \left[\frac{(W_u + W_y)}{\sqrt{n}} f_1 + \frac{(W_v + W_x)}{\mu_X\sqrt{n}} f_2 + \frac{W_p}{P\sqrt{n}} f_3 + \frac{1}{2!}\left\{\frac{(W_u + W_y)^2}{n} f_{200}\right.\right.$$

$$+ \frac{(W_v + W_x)^2}{\mu_X^2 n} f_{020} + \frac{W_p^2}{P^2 n} f_{002} + 2\frac{(W_u + W_y)(W_v + W_x)}{n\mu_X} f_{110}$$

$$\left.\left. + \frac{(W_v + W_x)W_p}{\mu_X P} f_{011} + 2\frac{(W_u + W_y)W_p}{P} f_{101}\right\} + \ldots\right] \quad (2.5)$$

3. Bias and Mean Square Error of the Proposed Class of Generalized Estimator

Taking expectation on both side of (2.5), we have

$$Bias(\bar{y}_{at}) = E(\bar{y}_{at} - \mu_y)$$

$$= E\left[\frac{(W_u + W_y)}{\sqrt{n}} f_1 + \frac{(W_v + W_x)}{\mu_X\sqrt{n}} f_2 + \frac{W_p}{P\sqrt{n}} f_3 + \frac{1}{2!}\left\{\frac{(W_u + W_y)^2}{n} f_{200}\right.\right.$$

$$+ \frac{(W_v + W_x)^2}{\mu_X^2 n} f_{020} + \frac{W_p^2}{P^2 n} f_{002} + 2\frac{(W_u + W_y)(W_v + W_x)}{n\mu_X} f_{110}$$

$$\left.\left. + \frac{(W_v + W_x)W_p}{\mu_X P} f_{011} + 2\frac{(W_u + W_y)W_p}{P} f_{101}\right\} + \ldots\right]$$

$$= \frac{1}{2n}\left[\frac{(\sigma_v^2 + \sigma_x^2)}{\mu_X^2} f_{020} + \frac{\sigma_p^2}{P^2} f_{002}\right] + \left[\frac{\sigma_{(y,x)}}{\mu_X} f_{100} + \frac{\sigma_{(x_1,p)} f_{011}}{\mu_X P} + \frac{\sigma_{(y,p)}}{P} f_{101}\right] \quad (3.1)$$

Now, squaring (2.5) on both sides and then take expectation, we have

$$MSE(\bar{y}_{at}) = E(\bar{y}_{at} - \mu_Y)^2$$

$$= E\left[\frac{(W_u + W_y)}{\sqrt{n}} f_1 + \frac{(W_v + W_x)}{\mu_X\sqrt{n}} f_2 + \frac{W_P}{P\sqrt{n}} f_3\right]^2$$

$$= E\left[\frac{(W_u + W_y)^2}{n} f_1^2 + \frac{(W_v + W_x)^2}{\mu_X^2 n} f_2^2 + \frac{(W_{v_2})^2}{P^2 n} f_3^2\right.$$

$$\left. + 2\left\{\frac{(W_u + W_y)}{\sqrt{n}}\frac{(W_v + W_x)}{\mu_X\sqrt{n}} f_1 f_2 + \frac{(W_v + W_x)}{\mu_X\sqrt{n}}\frac{W_p}{P\sqrt{n}} f_2 f_3 + \frac{W_p}{P\sqrt{n}}\frac{(W_u + W_Y)}{\sqrt{n}} f_3 f_1\right\}\right]$$

$$MSE(\bar{y}_{at}) = \frac{1}{n}\left[\sigma_y^2 f_1^2 + \frac{\sigma_x^2}{\mu_X^2} f_2^2 + \frac{\sigma_p^2}{P^2} f_3^2 + 2\left\{\frac{\sigma_{(y,x)}}{\mu_X} f_1 f_2 + \frac{\sigma_{(x,p)}}{\mu_X P} f_2 f_3 + \frac{\sigma_{(y,p)}}{P} f_3 f_1\right\}\right]$$

$$+ \frac{1}{n}\left[\sigma_u^2 f_1^2 + \frac{\sigma_{v_1}^2}{\mu_X^2} f_2^2\right] \quad (3.2)$$

ISSN 2394-7187(Online)
ISSN 2394 - 7179 (Print)

4. Proposed Class of Generalized Unbiased Estimator

Now, let us consider a simple random sample of size *n=2m*be split randomly into two sub-samples each of size *m* each. Let $(\overline{y}_{2m}, \overline{x}_{2m}, p_{2m})$ be the sample mean of values(Y, X, Φ) respectively for the entire sample of size 2*m* and $\left(\overline{y}_m^{(i)}, \overline{x}_m^{(i)}, p_m^{(i)}\right)$ be the sample means of values of (Y, X, Φ) respectively for i^{th} $(i = 1,2)$ sub-sample of size *m*. Let $\overline{y}_a^{(3)}$ is a generalized estimator for the entire sample of size *2m*. $\overline{y}_a^{(1)}$ and $\overline{y}_a^{(2)}$ be the generalized estimators for the two randomly splited sub-samples of size *m* each so that,

$$\overline{y}_a^{(3)} = f\left[\overline{y}_{2m}, \frac{\overline{x}_{2m}}{\mu_X}, \frac{p_{2m}}{P}\right] \tag{4.1}$$

$$\overline{y}_a^{(1)} = f\left[\overline{y}_m^{(1)}, \frac{\overline{x}_m^{(1)}}{\mu_X}, \frac{p_m^{(1)}}{P}\right] \tag{4.2}$$

and

$$\overline{y}_a^{(2)} = f\left[\overline{y}_m^{(2)}, \frac{\overline{x}_m^{(2)}}{\mu_X}, \frac{p_m^{(2)}}{P}\right] \tag{4.3}$$

satisfying the regularity conditions mentioned in (2.1). Also, we can easily define the following terms mentioned in (2.1), (2.3) and (2.4) on similar lines so that

$$\overline{y}_{2m} = \frac{1}{\sqrt{2m}}\left[W_u + W_y\right] + \mu_Y \tag{4.4}$$

$$\overline{x}_{2m} = \frac{1}{\sqrt{2m}}\left[W_v + W_x\right] + \mu_X \tag{4.5}$$

$$p_{2m} = \frac{1}{\sqrt{2m}} W_P + P \tag{4.6}$$

$$\overline{y}_m^{(1)} = \frac{1}{\sqrt{m}}\left[W_u^{(1)} + W_y^{(1)}\right] + \mu_Y \tag{4.7}$$

$$\overline{x}_m^{(1)} = \frac{1}{\sqrt{m}}\left[W_v^{(1)} + W_x^{(1)}\right] + \mu_X \tag{4.8}$$

$$p_m^{(1)} = \frac{1}{\sqrt{m}} W_P^{(1)} + P \tag{4.9}$$

$$\overline{y}_m^{(2)} = \frac{1}{\sqrt{m}}\left[W_u^{(2)} + W_y^{(2)}\right] + \mu_Y \tag{4.10}$$

$$\overline{x}_m^{(2)} = \frac{1}{\sqrt{m}}\left[W_v^{(2)} + W_x^{(2)}\right] + \mu_X \tag{4.11}$$

$$p_m^{(2)} = \frac{1}{\sqrt{m}} W_P^{(2)} + P \tag{4.12}$$

On expanding (4.1), (4.2) and (4.3) in third order Taylor's series similarly as expand (2.1), we get

$$\overline{y}_{at,2m}^{(3)} = \left[\mu_Y + \frac{(W_u + W_y)}{\sqrt{2m}} f_1 + \frac{(W_v + W_x)}{\mu_X\sqrt{2m}} f_2 + \frac{W_p}{P\sqrt{2m}} f_3 + \frac{1}{2!}\left\{\frac{(W_u + W_y)^2}{2m} f_{200} + \frac{(W_v + W_x)^2}{2\mu_X^2 m} f_{020} + \frac{W_p^2}{2P^2 m} f_{002} + 2\frac{(W_u + W_y)(W_v + W_x)}{2m\mu_X} f_{110} + \frac{(W_v + W_x)W_p}{\mu_X P} f_{011} + 2\frac{(W_u + W_y)W_p}{P} f_{101}\right\} + \ldots\right] \tag{4.13}$$

$$\overline{y}_{at,m}^{(1)} = \left[\mu_Y + \frac{\left(W_u^{(1)} + W_y^{(1)}\right)}{\sqrt{m}} f_1 + \frac{\left(W_v^{(1)} + W_x^{(1)}\right)}{\mu_X\sqrt{m}} f_2 + \frac{W_p^{(1)}}{P\sqrt{m}} f_3 + \frac{1}{2!}\left\{\frac{\left(W_u^{(1)} + W_y^{(1)}\right)^2}{m} f_{200} + \frac{\left(W_v^{(1)} + W_x^{(1)}\right)^2}{\mu_X^2 m} f_{020} + \frac{W_p^{(1)2}}{P^2 m} f_{002} + 2\frac{\left(W_u^{(1)} + W_y^{(1)}\right)\left(W_v^{(1)} + W_x^{(1)}\right)}{m\mu_X} f_{110} + \frac{\left(W_v^{(1)} + W_x^{(1)}\right)W_p}{\mu_X P} f_{011} + 2\frac{\left(W_u^{(1)} + W_y^{(1)}\right)W_p^{(1)}}{P} f_{101}\right\} + \ldots\right] \tag{4.14}$$

$$\overline{y}_{at,m}^{(2)} = \left[\mu_Y + \frac{\left(W_u^{(2)} + W_y^{(2)}\right)}{\sqrt{m}} f_1 + \frac{\left(W_v^{(2)} + W_x^{(2)}\right)}{\mu_X\sqrt{m}} f_2 + \frac{W_p^{(2)}}{P\sqrt{m}} f_3 + \frac{1}{2!}\left\{\frac{\left(W_u^{(2)} + W_y^{(2)}\right)^2}{m} f_{200} + \frac{\left(W_v^{(2)} + W_x^{(2)}\right)^2}{\mu_X^2 m} f_{020} + \frac{W_p^{(2)2}}{P^2 m} f_{002} + 2\frac{\left(W_u^{(2)} + W_y^{(2)}\right)\left(W_v^{(2)} + W_x^{(2)}\right)}{m\mu_X} f_{110} + \frac{\left(W_v^{(2)} + W_x^{(2)}\right)W_p}{\mu_X P} f_{011} + 2\frac{\left(W_u^{(2)} + W_y^{(2)}\right)W_p^{(2)}}{P} f_{101}\right\} + \ldots\right] \tag{4.15}$$

Thus, on the lines of Sukhatme and Sukhatme (Chapter IV, page 162), for *N* large, the Jack-knifed generalized estimator $\overline{y}_{atj}$ be defined as

$$\overline{y}_{atj} = 2\overline{y}_{at,2m}^{(3)} - \frac{1}{2}\left(\overline{y}_{at,m}^{(1)} + \overline{y}_{at,m}^{(2)}\right) \tag{4.16}$$

$$= 2\left[\mu_Y + \frac{(W_u + W_y)}{\sqrt{2m}} f_1 + \frac{(W_v + W_x)}{\mu_X\sqrt{2m}} f_2 + \frac{W_p}{P\sqrt{2m}} f_3 + \frac{1}{2!}\left\{\frac{(W_u + W_y)^2}{2m} f_{200} + \frac{(W_v + W_x)^2}{2\mu_X^2 m} f_{020} + \frac{W_p^2}{2P^2 m} f_{002} + 2\frac{(W_u + W_Y)(W_v + W_x)}{2m\mu_X} f_{110} + \frac{(W_v + W_x)W_p}{\mu_X P} f_{011} + 2\frac{(W_u + W_y)W_p}{P} f_{101}\right\} + \ldots\right]$$

ISSN 2394-7187(Online)
ISSN 2394 - 7179 (Print)

$$-\frac{1}{2}\left[\left\{\mu_Y+\frac{\left(W_u^{(1)}+W_y^{(1)}\right)}{\sqrt{m}}f_1+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)}{\mu_X\sqrt{m}}f_2+\frac{W_p^{(1)}}{P\sqrt{m}}f_3+\frac{1}{2!}\left\{\frac{\left(W_u^{(1)}+W_y^{(1)}\right)^2}{m}f_{200}\right.\right.\right.$$

$$+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)^2}{\mu_X^2 m}f_{020}+\frac{W_p^{(1)2}}{P^2 m}f_{002}+2\frac{\left(W_u^{(1)}+W_y^{(1)}\right)\left(W_v^{(1)}+W_x^{(1)}\right)}{m\mu_X}f_{110}$$

$$\left.\left.+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u^{(1)}+W_y^{(1)}\right)W_p^{(1)}}{P}f_{101}\right\}+\ldots\right\}$$

$$+\left\{\mu_Y+\frac{\left(W_u^{(2)}+W_y^{(2)}\right)}{\sqrt{m}}f_1+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)}{\mu_X\sqrt{m}}f_2+\frac{W_p^{(2)}}{P\sqrt{m}}f_3+\frac{1}{2!}\left\{\frac{\left(W_u^{(2)}+W_y^{(2)}\right)^2}{m}f_{200}\right.\right.$$

$$+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)^2}{\mu_X^2 m}f_{020}+\frac{W_p^{(2)2}}{P^2 m}f_{002}+2\frac{\left(W_u^{(2)}+W_y^{(2)}\right)\left(W_v^{(2)}+W_x^{(2)}\right)}{m\mu_X}f_{110}$$

$$\left(\bar{y}_{atj}-\mu_Y\right)=2\left[\frac{\left(W_u+W_y\right)}{\sqrt{2m}}f_1+\frac{\left(W_v+W_x\right)}{\mu_X\sqrt{2m}}f_2+\frac{W_p}{P\sqrt{2m}}f_3+\frac{1}{2!}\left\{\frac{\left(W_u+W_y\right)^2}{2m}f_{200}\right.\right.$$

$$+\frac{\left(W_v+W_x\right)^2}{2\mu_X^2 m}f_{020}+\frac{W_p^2}{2P^2 m}f_{002}+2\frac{\left(W_u+W_Y\right)\left(W_v+W_x\right)}{2m\mu_X}f_{110}$$

$$\left.\left.+\frac{\left(W_v+W_x\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u+W_Y\right)W_p}{P}f_{101}\right\}+\ldots\right]$$

$$-\frac{1}{2}\left[\left\{\frac{\left(W_u^{(1)}+W_y^{(1)}\right)}{\sqrt{m}}f_1+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)}{\mu_X\sqrt{m}}f_2+\frac{W_p^{(1)}}{P\sqrt{m}}f_3+\frac{1}{2!}\left\{\frac{\left(W_u^{(1)}+W_y^{(1)}\right)^2}{m}f_{200}\right.\right.\right.$$

$$+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)^2}{\mu_X^2 m}f_{020}+\frac{W_p^{(1)2}}{P^2 m}f_{002}+2\frac{\left(W_u^{(1)}+W_y^{(1)}\right)\left(W_v^{(1)}+W_x^{(1)}\right)}{m\mu_X}f_{110}$$

$$\left.\left.+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u^{(1)}+W_y^{(1)}\right)W_p^{(1)}}{P}f_{101}\right\}+\ldots\right\}$$

$$+\left\{\frac{\left(W_u^{(2)}+W_y^{(2)}\right)}{\sqrt{m}}f_1+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)}{\mu_X\sqrt{m}}f_2+\frac{W_p^{(2)}}{P\sqrt{m}}f_3+\frac{1}{2!}\left\{\frac{\left(W_u^{(2)}+W_y^{(2)}\right)^2}{m}f_{200}\right.\right.$$

$$+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)^2}{\mu_X^2 m}f_{020}+\frac{W_p^{(2)2}}{P^2 m}f_{002}+2\frac{\left(W_u^{(2)}+W_y^{(2)}\right)\left(W_v^{(2)}+W_x^{(2)}\right)}{m\mu_X}f_{110}$$

$$\left.\left.\left.+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u^{(2)}+W_y^{(2)}\right)W_p^{(2)}}{P}f_{101}\right\}+\ldots\right\}\right]$$

$$+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u^{(2)}+W_y^{(2)}\right)W_p^{(2)}}{P}f_{101}\left.\left.\left.\right\}+\ldots\right\}\right] \qquad (4.17)$$

5. Bias and Mean Square Error of the Proposed Unbiased Class of Generalized Estimator

On taking expectation on both sides of (4.17), we get the bias of $\bar{y}_{atj}$ given by

$$Bias\left(\bar{y}_{atj}\right)=E\left(\bar{y}_{atj}-\mu_Y\right)$$

$$=2E\left[\frac{\left(W_u+W_y\right)}{\sqrt{2m}}f_1+\frac{\left(W_v+W_x\right)}{\mu_X\sqrt{2m}}f_2+\frac{W_p}{P\sqrt{2m}}f_3+\frac{1}{2!}\left\{\frac{\left(W_u+W_y\right)^2}{2m}f_{200}\right.\right.$$

$$+\frac{\left(W_v+W_x\right)^2}{2\mu_X^2 m}f_{020}+\frac{W_p^2}{2P^2 m}f_{002}+2\frac{\left(W_u+W_y\right)\left(W_v+W_x\right)}{2m\mu_X}f_{110}$$

$$\left.\left.+\frac{\left(W_v+W_x\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u+W_y\right)W_p}{P}f_{101}\right\}+\ldots\right]$$

$$-\frac{1}{2}E\left[\left\{\frac{\left(W_u^{(1)}+W_y^{(1)}\right)}{\sqrt{m}}f_1+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)}{\mu_X\sqrt{m}}f_2+\frac{W_p^{(1)}}{P\sqrt{m}}f_3+\frac{1}{2!}\left\{\frac{\left(W_u^{(1)}+W_y^{(1)}\right)^2}{m}f_{200}\right.\right.\right.$$

$$+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)^2}{\mu_X^2 m}f_{020}+\frac{W_p^{(1)2}}{P^2 m}f_{002}+2\frac{\left(W_u^{(1)}+W_y^{(1)}\right)\left(W_v^{(1)}+W_x^{(1)}\right)}{m\mu_X}f_{110}$$

$$\left.\left.+\frac{\left(W_v^{(1)}+W_x^{(1)}\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u^{(1)}+W_y^{(1)}\right)W_p^{(1)}}{P}f_{101}\right\}+\ldots\right\}$$

$$+\left\{\frac{\left(W_u^{(2)}+W_y^{(2)}\right)}{\sqrt{m}}f_1+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)}{\mu_X\sqrt{m}}f_2+\frac{W_p^{(2)}}{P\sqrt{m}}f_3+\frac{1}{2!}\left\{\frac{\left(W_u^{(2)}+W_y^{(2)}\right)^2}{m}f_{200}\right.\right.$$

$$+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)^2}{\mu_X^2 m}f_{020}+\frac{W_p^{(2)2}}{P^2 m}f_{002}+2\frac{\left(W_u^{(2)}+W_y^{(2)}\right)\left(W_v^{(2)}+W_x^{(2)}\right)}{m\mu_X}f_{110}$$

$$\left.\left.\left.+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u^{(2)}+W_y^{(2)}\right)W_p^{(2)}}{P}f_{101}\right\}+\ldots\right\}\right]$$

$$+\frac{\left(W_v^{(2)}+W_x^{(2)}\right)W_p}{\mu_X P}f_{011}+2\frac{\left(W_u^{(2)}+W_y^{(2)}\right)W_p^{(2)}}{P}f_{101}\left.\left.\left.\right\}+\ldots\right\}\right]=0 \qquad (5.1)$$

Therefore, $\bar{y}_{atj}$ is an unbiased estimator of population mean μ_Y. On squaring (4.17) on both sides and then take expectation we get mean square error of $\bar{y}_{atj}$ given by

$$MSE\left(\bar{y}_{atj}\right)=\left[\frac{\left(\sigma_u^2+\sigma_y^2\right)}{2m}f_1^2+\frac{\left(\sigma_v^2+\sigma_x^2\right)}{2m\mu_X^2}f_2^2+\frac{\sigma_p^2}{2mP^2}f_3^2\right]$$

$$+\frac{1}{m}\left[\frac{\sigma_{(y,x)}}{\mu_X}f_1f_2+\frac{\sigma_{(x,p)}}{\mu_XP}f_2f_3+\frac{\sigma_{(y,p)}}{P}f_3f_1\right]$$

$$=\frac{1}{2m}\left[\sigma_y^2f_1^2+\frac{\sigma_x^2}{\mu_X^2}f_2^2+\frac{\sigma_p^2}{P^2}f_3^2+2\left\{\frac{\sigma_{(y,x)}}{\mu_X}f_1f_2+\frac{\sigma_{(x,p)}}{\mu_XP}f_2f_3+\frac{\sigma_{(y,p)}}{P}f_3f_1\right\}\right]$$

$$+\frac{1}{2m}\left[\sigma_u^2f_1^2+\frac{\sigma_{v_1}^2}{\mu_X^2}f_2^2\right] \tag{5.2}$$

6. Conclusion

We can easily see that bias and mean square error of $\bar{y}_{at}$ and $\bar{y}_{atj}$ are

$$Bias\left(\bar{y}_{at}\right)=\frac{1}{2n}\left[\frac{\left(\sigma_v^2+\sigma_x^2\right)}{\mu_X^2}f_{020}+\frac{\sigma_p^2}{P^2}f_{002}\right]+\left[\frac{\sigma_{(y,x)}}{\mu_X}f_{100}+\frac{\sigma_{(x_1,p)}f_{011}}{\mu_XP}+\frac{\sigma_{(y,p)}}{P}f_{101}\right] \tag{6.1}$$

$$MSE(\bar{y}_{at})=\frac{1}{n}\left[\sigma_y^2f_1^2+\frac{\sigma_x^2}{\mu_X^2}f_2^2+\frac{\sigma_p^2}{P^2}f_3^2+2\left\{\frac{\sigma_{(y,x)}}{\mu_X}f_1f_2+\frac{\sigma_{(x,p)}}{\mu_XP}f_2f_3+\frac{\sigma_{(y,p)}}{P}f_3f_1\right\}\right]$$

$$+\frac{1}{n}\left[\sigma_u^2f_1^2+\frac{\sigma_{v_1}^2}{\mu_X^2}f_2^2\right] \tag{6.2}$$

$$Bias\left(\bar{y}_{atj}\right)=0 \tag{6.3}$$

$$MSE\left(\bar{y}_{atj}\right)=\frac{1}{2m}\left[\sigma_y^2f_1^2+\frac{\sigma_x^2}{\mu_X^2}f_2^2+\frac{\sigma_p^2}{P^2}f_3^2+2\left\{\frac{\sigma_{(y,x)}}{\mu_X}f_1f_2+\frac{\sigma_{(x,p)}}{\mu_XP}f_2f_3+\frac{\sigma_{(y,p)}}{P}f_3f_1\right\}\right]$$

$$+\frac{1}{2m}\left[\sigma_u^2f_1^2+\frac{\sigma_{v_1}^2}{\mu_X^2}f_2^2\right] \tag{6.4}$$

From (6.2) and (6.4), we see that both estimators $\bar{y}_{at}$ and $\bar{y}_{atj}$ have the same mean square error but from (6.1) bias of $\bar{y}_{at}$ is not zero whereas from (6.3) bias of the jack-knifed estimator $\bar{y}_{atj}$ is zero. Hence, both the estimators $\bar{y}_{at}$ and $\bar{y}_{atj}$ having the same mean square error, but the jack-knifed estimator $\bar{y}_{atj}$ may be preferred to the estimator $\bar{y}_{at}$ in the sense of unbiasedness under the presence of measurement errors. Furthermore, since the attribute auxiliary information is less prone to measurement error this estimator has more practical utility than other estimators like ratio estimator where such information is lacking.

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