

Motive Control for Two Axes of a Radar Station

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Abstract: Radars are primarily used to detect and identify moving objects, so the multi-axis drive system is one of the most important parts that directly affect the quality and versatility of the radar station. In the problem of control of the two axes of the radar station, we always need to consider many things at the same time such as variable dynamic parameters, the movement of one axis affected by the other, the requirement for accurate traction, the need for wave interference. An adaptive robust control designed to satisfy the quality requirement, with flexible structure which reduces the amount of computation is proposed in this paper.

Keywords: Radar, MIMO, Adaptive Control, Robust Control, Adaptive Robust Control.

I. INTRODUCTION

The development of microelectronics and computer science has created new avenues for the development of radar. The latest radars in the world have advanced features, remote detection distance, high target resolution, compact device, automated signal processing which is very convenient for the users. Radars are primarily used to detect and identify moving objects, so the multi-axis drive system is one of the most important parts that directly affect the quality and versatility of the radar station.

II. AUTOMATIC TARGET TRACKING SYSTEM BASED ON CONICAL SCANNING METHOD LAYOUT

Conical scanning method is commonly used in automatic target tracking according to the radar angles.

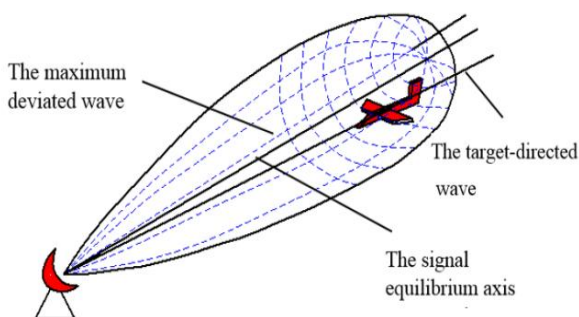


Fig. 1. Radar Waves

In these radars, the same antenna is usually used as the receiver and transmitter with high orientation characteristics, the pencil-shaped waveform (Fig. 1). Rotate the wave around the OO' axis (signal equilibrium axis) with the angular velocity Ω_a (Fig. 2).

The maximum deviated wave and the signal equilibrium axis make the angle ψ . The signal equilibrium axis and the target-directed wave make an angle $\Delta\varphi$.

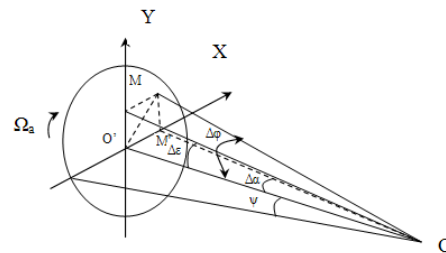


Fig. 2. Signal Modulation Principle

The trajectory of the waves is a cone in space, with the peak at the antenna. In this case, the target amplitude modulation principle will bring the information about the bias of the azimuth angle ($\Delta\alpha$) and the altitude angle ($\Delta\epsilon$) made by the target-directed wave and the signal equilibrium axis OO' . In the direction of the signal equilibrium axis, the intensity of the radiation and the signal received when the wave is rotated are constant, so the signal modulation is not applied.

III. MATHEMATICAL DESCRIPTION OF THE TWO AXES OF THE RADAR STATION

3.1. Characteristics of Movement of the Two Axes of the Radar Station:

The two-axis motion requirements of the radar station are divided as the followings:

Azimuth axis: Rotational movement.

Altitude angle axis: movement according to the sinusoidal law in the limited surface

Limited angle $\beta=90^0$; The actual angular displacement of the altitude angle axis is $90^0 \leq \alpha \leq 180^0$

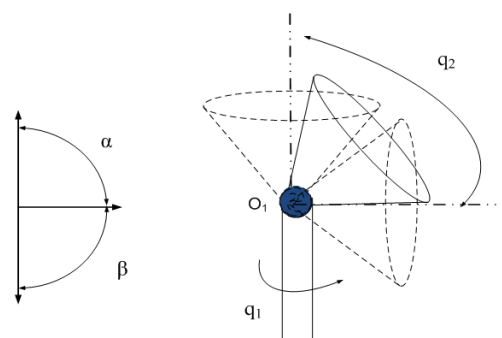


Fig. 3. Two-Axis Motion Requirement

3.2. Dynamic Modelling Sequencedenavit- Hartenberg:

The objects here are the two axes of the radar, one axis rotates along the azimuthal axis, the other rotates along the altitude angle. We can choose the coordinate system on each axis as follows:

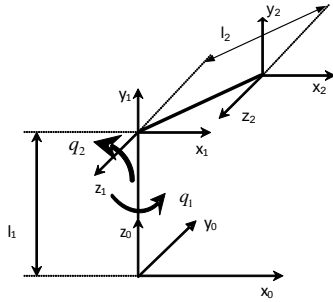


Fig. 4. Radar the Coordinate Mounted on Two Axes

3.3. Dynamics of the Two Axes of the Radar Lagrange Equation:

The two-axis dynamic equation was established from the Lagrange equation:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = \tau \quad (1)$$

In which:

$$q = [q_1, q_2]^T, \dot{q} = [\dot{q}_1, \dot{q}_2]^T$$

$$\tau = [\tau_1, \tau_2]^T \text{ In which: } \tau_i \text{ is the momentum } i.$$

$$L = K - P \quad (2)$$

In which: L is Lagrange function

$$K = \sum_{i=1}^2 K_i \text{ is the sum of kinetic energy,}$$

K_i is the kinetic energy of the bar i.

$$P = \sum_{i=1}^2 P_i \text{ is the sum of potential energy,}$$

P_i is potential energy of the bar i

Combine (1) with (2) we get:

$$\frac{d}{dt} \left[\frac{\partial K}{\partial \dot{q}} \right] - \frac{\partial K}{\partial q} + \frac{\partial P}{\partial q} = \tau \quad \left(\frac{\partial P}{\partial \dot{q}} = 0 \right) \quad (3)$$

Equivalent form of equation

$$\frac{d}{dt} \left[\frac{\partial K}{\partial \dot{q}_i} \right] - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} = \tau_i \quad (4)$$

In which $i = 1, 2.$

3.4. General Dynamics Equation:

General dynamics equation of the two axes of the radar as follows:

$$\tau = M(q)\ddot{q} + V(q, \dot{q}) + G(q)$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} J_1 + m_2 l_{g2}^2 \cos^2 q_2 & 0 \\ 0 & m_2 l_{g2}^2 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -2m_2 l_{g2}^2 \sin q_2 \cos q_2 \dot{q}_2 \dot{q}_1 \\ -m_2 l_{g2}^2 \cos q_2 \sin q_2 \dot{q}_1^2 \end{bmatrix} + \begin{bmatrix} 0 \\ m_2 g l_{g2} \cos q_2 \end{bmatrix} \quad (5)$$

In which: $M(\theta) \in R^{2 \times 2}$ is symmetrical positive matrix, $G(q) \in R^2, V(q, \dot{q}) \in R^2, \tau \in R^2$

$$q = [q_1, q_2]^T, \dot{q} = [\dot{q}_1, \dot{q}_2]^T, \ddot{q} = [\ddot{q}_1, \ddot{q}_2]^T$$

$M(\dot{q})$: Inertia Component

$V(q, \dot{q})$: Reciprocal Component

$G(q)$: Gravity Component

IV. MAKE THE CONTROL LAW FOR THE TWO AXES OF THE RADAR

4.1. Requirements for Controlling the Radar:

The radar work mode consists of two modes: reconnaissance (realignment) and target tracking (when targeting). There are the following requirements for controlling the two axes of the radar: Accurate traction requirement: allowable error less than 1%. Wave interference: internal interference (caused by the system itself), external interference (caused by wind, air resistance...)

4.2. Analysis of Controlling Requirement:

From the dynamic equation of the radar (5) we will analyze this transfer function to indicate the uncertainty of the control object.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} J_1 + m_2 l_{g2}^2 \cos^2 q_2 & 0 \\ 0 & m_2 l_{g2}^2 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -2m_2 l_{g2}^2 \sin q_2 \cos q_2 \dot{q}_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -m_2 l_{g2}^2 \cos q_2 \sin q_2 \dot{q}_1^2 \end{bmatrix} + \begin{bmatrix} 0 \\ m_2 g l_{g2} \cos q_2 \end{bmatrix}$$

in which:

$$\begin{bmatrix} J_1 + m_2 l_{g2}^2 \cos^2 q_2 & 0 \\ 0 & m_2 l_{g2}^2 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} \text{ Typical for Effective inertias}$$

$$\begin{bmatrix} 0 & 0 \\ -2m_2 l_{g2}^2 \sin q_2 \cos q_2 \dot{q}_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{Typical for Coriolis effect}$$

$$\begin{bmatrix} 0 \\ -m_2 l_{g2}^2 \cos q_2 \sin q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \end{bmatrix} \quad \text{Typical for Centripetal effect}$$

$$\begin{bmatrix} 0 \\ m_2 g l_{g2} \cos q_2 \end{bmatrix} \quad \text{Typical for Gravity}$$

Here we can minimize the dynamic equation as follows:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} J_1 + m_2 l_{g2}^2 \cos^2 q_2 & 0 \\ 0 & m_2 l_{g2}^2 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -2m_2 l_{g2}^2 \sin q_2 \cos q_2 \dot{q}_2 \dot{q}_1 \\ -m_2 l_{g2}^2 \cos q_2 \sin q_2 \dot{q}_1^2 \end{bmatrix} + \begin{bmatrix} 0 \\ m_2 g l_{g2} \cos q_2 \end{bmatrix}$$

$$\tau = M(q)\ddot{q} + V(q, \dot{q}) + G(q) \quad (6)$$

Providing the interference effect, the dynamic equation is rewritten as follows.

$$\tau = M(q)\ddot{q} + V(q, \dot{q}) + G(q) + F_d \quad (7)$$

4.3. Synthesis of the two-Axis Control for the Radar According to the Adaptive Robust Control:

The dynamic equation of the control object

$$\tau = M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F_d \dot{q} + F_s(\dot{q}) \quad (8)$$

In which:

F_d : the matrix of the positive defining diagonal line $n \times n$, describe dynamic friction

F_s : vector $n \times 1$ static friction constant

The adaptive robust control uses the same algorithm as the robust control, in which the supplementary control is used to block indefinite parameter limits. This robust control uses the scalar functions as the combination of the error standard and the positive limit constant. For example, the system has a dynamic model given by:

$$w = M(q)(\ddot{q} + \dot{e}) + V_m(q, \dot{q})(\dot{q} + e) + G(q) + F_d \dot{q} + F_s(\dot{q}) \quad (9)$$

The above dynamic equation is uncertain of load, friction coefficient and external interference. It can be concluded that although the positive-ratio function ρ can be used for uncertainty of the limit, the positive uncertainty is as follows:

$$\rho \geq \|w\| \quad (10)$$

We can limit the physical properties of the system by using equation (10):

$$\rho = \delta_0 + \delta_1 \|e\| + \delta_2 \|e\|^2 \geq \|w\| \quad (11)$$

In which $e = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$; $\delta_0, \delta_1, \delta_2$ are the positive bound constants calculated on the basis of maximum values of load, bar weight, friction coefficient, interference, ...

The robust control requires the envelope of the constant defined in (11) to have a predetermined form. The adaptive robust control developed here will learn limited constants online when the mechanism moves. While conducting the control, we do not need to know exactly the limited constants. We only require that the limited constants exist according to (11).

The adaptive robust control is as follows:

$$\tau = K_v r + v_R \quad (12)$$

In which:

K_v : Diagonal Matrix, Positive $n \times n$

r : Trajectory Error

v_R : Vector ($n \times 1$) Supplementary Control

Supplementary control v_R is defined by:

$$v_R = \frac{r \hat{\rho}^2}{\hat{\rho} \|r\| + \epsilon} \quad (13)$$

In which $\dot{\epsilon} = -k_\epsilon \epsilon$, $\epsilon(0) > 0$

k_ϵ : a positive ratio control parameter

$\hat{\rho}$: is a proportional function defined by

$$\hat{\rho} = \hat{\delta}_0 + \hat{\delta}_1 \|e\| + \hat{\delta}_2 \|e\|^2 \quad (14)$$

and dynamic estimates of the restricted kinetic parameters defined in (15). These limited estimates marked by “ $\hat{\cdot}$ ” are updated online based on a new updated adaptive law, written in (14) as follows:

$$\hat{\rho} = \hat{S} \hat{\theta} \quad (15)$$

In which:

$$S = [1 \|e\| \|e\|^2] \text{ v\grave{a}} \hat{\theta} = [\hat{\delta}_0 \hat{\delta}_1 \hat{\delta}_2]^T \quad (16)$$

The limit function ρ given by (3-42) can be written in matrix form

$$\rho = S \theta \quad (17)$$

In which $\theta = [\delta_0 \delta_1 \delta_2]^T$

There is a similarity between the formula of the regression matrix in the adaptive control method and the formula given by (17). In particular, the matrix S (1×3) made by “the regression matrix” and vector $\hat{\theta}$ create a parametric estimation vector.

The limited estimates defined in (17) are updated online by the relation:

$$\frac{d}{dt} \hat{\theta} = \gamma S^T \|r\| \quad (18)$$

In which:

r : trajectory error with $r = e + \dot{e}$

γ : a positive control constant

Because $\delta_0, \delta_1, \delta_2$ are constants, (13) can be rewritten as:

$$\frac{d}{dt} \tilde{\theta} = -\gamma S^T \|r\|; \tilde{\theta} = \theta - \hat{\theta} \quad (19)$$

Paying attention to the stability analysis of the bias system for the control.

$$M(q)\dot{r} = -V_m(q, \dot{q})r - K_v r + w - v_R \quad (20)$$

In which w is defined by (9)

We again analyze the stability of the system error given by (20) with the Lyapunov function:

$$V = \frac{1}{2} r^T M(q)r + \frac{1}{2} \tilde{\theta}^T \gamma^{-1} \tilde{\theta} + k_\epsilon^{-1} \quad (21)$$

Derived from time, we have

$$\dot{V} = \frac{1}{2} (r^T \dot{M}(q)r + r^T M(q)\dot{r}) + \tilde{\theta}^T \gamma^{-1} \frac{d}{dt} \tilde{\theta} + k_\epsilon^{-1} \dot{\epsilon}$$

From (19) and (20) we have

$$\dot{V} = -r^T K_v r - \frac{1}{2} S \tilde{\theta} \|r\| + \frac{1}{2} r^T (w - v_R) + k_\epsilon^{-1} \dot{\epsilon} + \frac{1}{2} r^T (\dot{M}(q) + V_m(q, \dot{q})) r \quad (22)$$

Because $(\dot{M}(q) + V_m(q, \dot{q}))$ is a associated matrix so $\frac{1}{2} r^T (\dot{M}(q) + V_m(q, \dot{q})) r = 0$.

From (10, 17, 22)

$$\dot{V} \leq -r^T K_v r - \frac{1}{2} S \tilde{\theta} \|r\| + \frac{1}{2} S \tilde{\theta} \|r\| - \frac{1}{2} r^T v_R + k_\epsilon^{-1} \dot{\epsilon} \quad (23)$$

Replace (13), (14), (15), (19) into (23), we have:

$$\dot{V} \leq -r^T K_v r - \epsilon + S \hat{\theta} \|r\| - \frac{r^T r (S \hat{\theta})^2}{S \hat{\theta} \|r\| + \epsilon} \quad (24)$$

$$\dot{V} \leq -r^T K_v r - \epsilon + S \hat{\theta} \|r\| - \frac{\|r\|^2 (S \hat{\theta})^2}{S \hat{\theta} \|r\| + \epsilon} \quad (25)$$

$$\dot{V} \leq -r^T K_v r - \epsilon + \frac{\epsilon S \hat{\theta} \|r\|}{S \hat{\theta} \|r\| + \epsilon} \quad (26)$$

Because the sum of the last two terms of (26) is always less than 0, we can set the new limit of \dot{V} .

$$\dot{V} \leq -r^T K_v r \quad (27)$$

Consider the sustainability of the trajectory error : First, notice from (27), we can change the new upper limit of \dot{V} :

$$\dot{V} \leq -\lambda_{\min} \{K_v\} \|r\|^2 \quad (28)$$

We see all the limited signals $r \in L_2^n$. So we can use $r = e + \dot{e}$ to show that the position error (e) is related with the trajectory error filter r by the transfer function:

$$e(s) = G(s)r(s) \quad (29)$$

From the above results, we can see that the position error e is the asymptotically stable state. This corresponds to the extension rule described in this section, we can only set the speed error e and estimate the limit θ in the area.

In general, the torque control is designed as follows:

$$\tau = K_v r + v_R = K_v r + \frac{r \hat{p}^2}{\hat{p} \|r\| + \epsilon} \quad (30)$$

In which:

$K_v r$: component of sustainability

$\frac{r \hat{p}^2}{\hat{p} \|r\| + \epsilon}$: (the supplementary control), is the adaptive component. The estimated value of \hat{p} is designed as follows :

$$\hat{p} = S \hat{\theta} = \begin{bmatrix} 1 & \left\| \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \right\| & \left\| \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \right\|^2 \end{bmatrix} [\hat{\delta}_0 \hat{\delta}_1 \hat{\delta}_2]^T$$

$$r = e + \dot{e} \text{ v\grave{a}} \dot{\epsilon} = -k_\epsilon \epsilon$$

The updated law estimates the limit of the parameters $\theta = [\delta_0 \delta_1 \delta_2]^T$; $\frac{d}{dt} \hat{\theta} = \gamma S^T \|r\|$

The position error e is in the asymptotically stable state. Limit estimate $\hat{\theta}$ and velocity error are limited. The adaptive robust control with compensation of wave interference but without change.

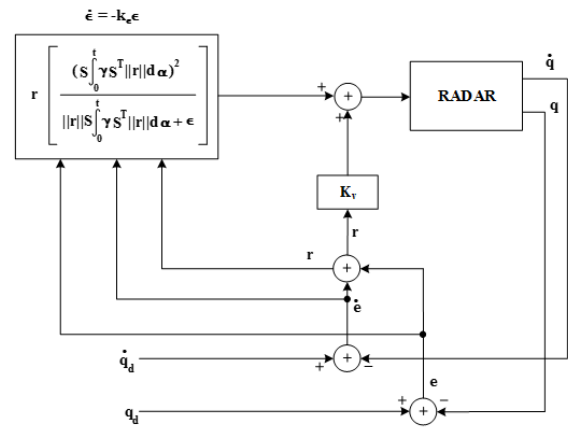


Fig. 5. The Adaptive Robust Control Block

V. DEMONSTRATION OF CONTROL SYSTEM

5.1. Building Two-Axis Model of he Radar Station Using Simmechanics:

To consider the effect of external interference on the model we need to add the external interference model to the two-axis model of the radar. The external interferences in this case are static friction and dynamic

friction. Supposed that the external interference has the dynamic model:

$$F_d \dot{q}_i + F_s(\dot{q}_i) = \dot{q}_i + \text{sign}(\dot{q}_i)$$

$$[\delta_0 \delta_1 \delta_2]^T = [8 \ 9 \ 10]^T$$

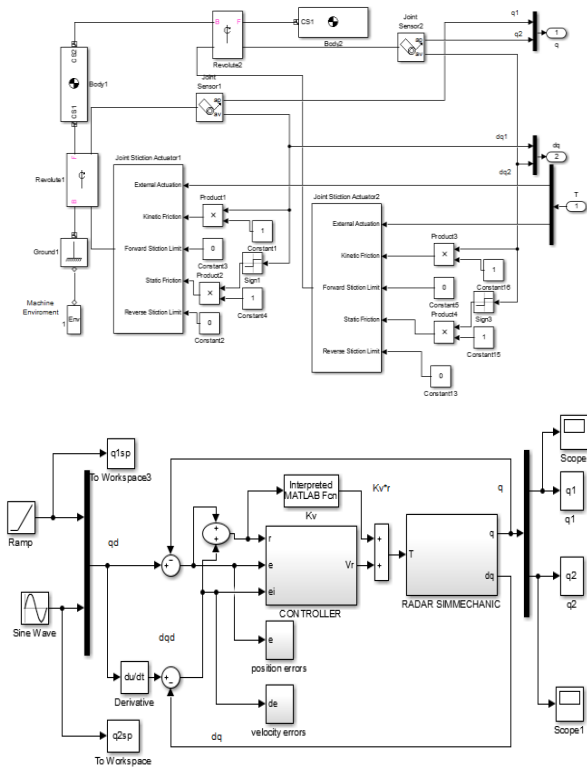


Fig. 6. Two-Axis Model of the Radar when Considering the Effect of Friction on Joints

5.2. System Simulation with the Adaptive Robust Control:

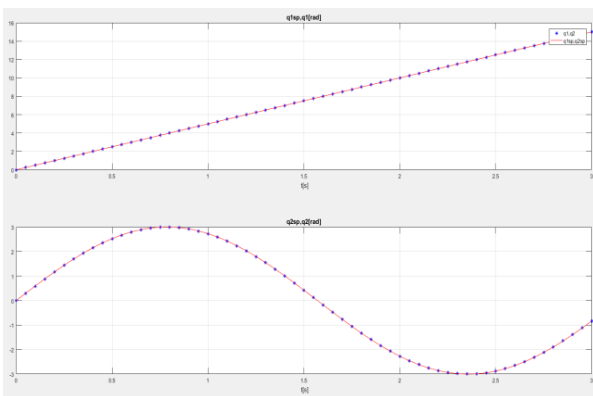


Fig. 7. Dynamic Trajectory when there is Friction

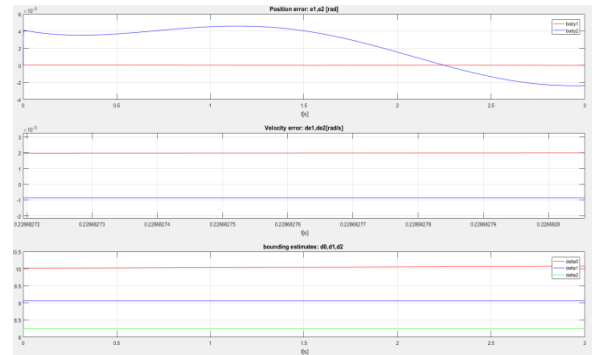


Fig. 8. Responses to Displacement, Velocity Deviation, Bounding Estimates in the Presence of Friction

Comment on the response of the system:

Position Error:

The azimuth axis : Position error $\leq 0,0002(\text{Rad})$.

The altitude angle axis : Position error $\leq 0,004(\text{Rad})$

Velocity Error:

The azimuth axis : Position error $\leq 0,002 (\text{Rad/s})$

The altitude angle axis : Position error $\leq 0,00085 (\text{Rad/s})$

Bounding estimates: fast convergence, in which the change in the delta 0 bounding estimates is greatest at 0,25.

VI. CONCLUSIONS

With the adaptive robust control algorithm applied for the two axes of the radar, it is found that the position error and velocity error are not stable, the limit estimate is within limited area. In all cases, the position error and velocity error are always around the zero point. This indicates that the position error is asymptotically stable when all other signals remain within limited area.

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