MEASUREMENT UNCERTAINTY ON THE CORRECTION MATRIX OF THE COORDINATE MEASURING MACHINE

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ABSTRACT

Evaluate an uncertainty of measurement, it defines the measurand with accuracy, identify the different factors that contribute, quantify them in the form of a variance and calculate the uncertainty of measure from the square root of this variance. Then, this uncertainty was expanded by a coefficient depending on the confidence interval. The estimation of this measurement uncertainty of the calibration of a machine with a three-dimensional measure (MMT) is a rather delicate procedure counts held of the diversity of the parameters involved. The purpose of this article is to quantify the uncertainty associated with the GUM method and subsequently, validate this method by the method of Monte Carlo (MCM).

Keywords: CMM, GUM, MCM, uncertainty of measurement.

1. INTRODUCTION

The guide for the expression of measurement uncertainty (GUM) is based on the law of variance propagation for the calculation of the combined standard measurement. This is generated from Taylor development applied to the measurement process model; the calculation therefore requires the validity of a number of mathematical assumptions. First, the model must not show significant non-linearity. The observed dispersions must be small for each of the variables of the measurement process, comparable in terms of their order of magnitude and must have symmetrical distributions. In the end, the distribution of the model output quantity must have a Gaussian profile in order to easily calculate and interpret the value of the expansion factor k.

From a strictly operational point of view, the application of the law of uncertainty propagation requires the derivation of the measurement process model with respect to each of

its variables in order to qualify its sensitivity coefficients. In addition to the mathematical hypotheses of model derivability, its application can be difficult in an industrial context. It can be an obstacle to the progress of the calculation and a potential source of error. An alternative to this calculation is now described in the first GUM supplement, to be published in 2007. The metrologist suggested to use numerical simulation tools specially the Monte Carlo method to propagate not only two statistics (mean and variance) but also the distributions of variables describing the measurement process.

As an illustration, the two methods of calculating the combined standard measurement uncertainty has been applied to the measurement process in order to identify the strengths of each method and the difficulties that may arise in their implementation. This is the determination of the measurement uncertainty for the correction matrix previously determined according to the iso 10360-2 version 2012 standard.

2. CALCULATION OF UNCERTAINTY BY GUM METHOD:

A well-established approach to evaluate the uncertainty of measurement is defined in the "Guide to the Expression of Uncertainty in Measurement (GUM)." Published by ISO [3]. The GUM approach is recommended by the International Bureau of Weights and Measures (BIMP) [4]. She is followed by the NIST guidelines [5]and the rules of laboratory Gov-Chemist [6].

The GUM approach estimates the combined uncertainty through uncertainty propagation in four steps: specification, identification, quantification, and combining bottom-up **[3.8]**. In many cases, a measurand Y is not calculated directly but is determined from N other quantities x1, x2, …, xn through a functional relationship f: y=f (x_1, x_2, …, x_n).

According to GUM, uncertainty uc is obtained from the uncertainty propagation law linking the standard uncertainties u_{xi} than u_c by:

$$u_c^{\,2}(Y) = \sum_{i=1}^{n}\left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=1+1}^{n}\left[\frac{\partial f}{\partial x_i}\right]\left[\frac{\partial f}{\partial x_j}\right]*r(x_i,x_j)*u(x_i)u(x_j)$$

(1)

With: r (xi; xj) is the correlation between xi and xj.

u(xi) is the uncertainty of xi.

2.1. Modeling of the problem:

The result of the correction matrix to be applied to records of measures of the CMM is given as below. For each of the seven directions, the correction has been modeled by a straight-line least square. According to the GUM method, the uncertainty associated with this matrix is calculated by applying the law of propagation of uncertainty to the equations of the following matrix: Vci=Ai*Vlue+Bi.

The approach developed in this study to associate uncertainty is based on the analytical method recommended by the GUM. It is possible to divide the calculation of uncertainty into four steps:

2.1.1. Specify the measurand:

The definition of the measurand is based on writing a mathematical model based on input factors. Each factor is calculated and the value of the result (measurand) is determined. Indeed, the measurand is

$V_{ci}=A_i*V_{lue}+B_i$ (2)

2.1.2. Identify sources of errors:

A measured value is therefore not an exact or certain value: it is derived from a result with a certain dispersion, or variation. This measurement result may be affected by the Material or its physicochemical properties, the environment in which the measurement is made, the Means or equipment for monitoring and measurement, the Measurement Method or procedure and the manpower or personnel responsible for the measurement.

2.1.3. Quantify the sources:

The uncertainty associated with this matrix depends on the coefficients of the calibration lines A1; A2; A3; A4; B1; B2; B3; B4; a1; a2; a3; a3; a4; b1; b2; b3; b4. The combined uncertainty according to the 7 directions is written for each of the directions as:

$$u^2_{vc_i} = V_{lue_i}{}^2 * u^2_{Ai} + A_i^2 * u^2_{Vlue_i} + u^2_{B_i} + 2 * V_{lue_i} * U(A_i, V_{lue_i}) + 2 * V_{lue_i} * U(A_i, B_i) + 2 * A_i * U(B_i, V_{lue_i})$$

(3)

  -**Uv$_{ci}$** is the uncertainty of the adjusted value in the direction (i).

  - **Uv$_{lue, i}$** is the uncertainty about the read value.

  - A$_i$ and B$_i$ are respectively the coefficients and the origins of interpolations right direction i.

  -U (A, Vlue), U (A, B), and U (B, Vlue) is the uncertainty on the correlation coefficient between the parameters.

  ⊕ Uncertainty due to the read value: Uvlue

U$_{vlue}$ essentially depends on the machine resolution and is assumed to be triangular since the measurement is made twice between the plane and the point, so: Uvlue=R/√6

  ⊕ Uncertainty on coefficients A and B: U$_A$ and U$_B$

U$_A$ and U$_B$ are assumed to be normal laws and calculated by the difference between the maximum and minimum of the calibration coefficients of each of the 3 tests divided by 6.

  U$_A$= [max (A$_{i1}$, A$_{i2}$, A$_{i3}$) -Min (A$_{i1}$, A$_{i2}$, A$_{i3}$)] /1.693

  U$_B$= [max (B$_{i1}$, B$_{i2}$, B$_{i3}$) -Min (B$_{i1}$, B$_{i2}$, B$_{i3}$)] /1.693

2.1.4. Combination of standard uncertainties:

After having calculated the different standard uncertainties, we found the standard uncertainties on the coefficients Ai which are in the order of 10^-6, and those of the coefficients Bi which are in the order of 10-4, on the other hand, the uncertainty associated with the correlation coefficients between the 3 parameters is in the order of 10-8. And then, the sum of the different standard uncertainties is calculated to obtain the standard uncertainty on adjusted value according to the 7 directions and for each of the directions measured according to the iso 10360-2 standard:

Calculation of standard uncertainties on Vc					
Vref(mm)	125	150	175	200	250
Uvc$_x$(µm)	1.13	1.19	1.27	1.35	1.53
Uvc$_y$ (µm)	1.29	1.39	1.51	1.63	1.89
Uvc$_z$(µm)	0.94	1.07	1.20	1.34	1.62
Uvc$_{xy}$(µm)	0.11	1.19	1.27	1.35	1.53
Uvc$_{xz}$(µm)	0.94	1.07	1.20	1.34	1.62
Uvc$_{yz}$(µm)	1.13	1.19	1.27	1.35	1.53
Uvc$_{xyz}$(µm)	0.078	0,078	0,079	0,079	0,081
Maximum uncertainty (µm)			1.89		

2.2. Determination of the expanded uncertainty

In the GUM uncertainty assessment method, the value assigned to measurand Y must be announced, as well as an expanded uncertainty U = k. u(y), which allows an interval to be defined around the value assigned to the measurand [y -U; y + U]. There is a consensus to choose k = 2 for the majority of uncertainty calculations because it is the numerical value that provides a 95% confidence level in the case of a normal distribution. This is justified by an interpretation of a statistical theorem called "central limit theorem" which allows us to affirm that in the case of a large number of independent and small effects that are added, the output quantity follows approximately a normal distribution. The expanded uncertainty on the adjusted value is then:

Vref (mm)	125 mm	150 mm	175 mm	200 mm	250 mm
Uvcx (µm)	2.25	2.39	2.54	2.70	3.06
Uvcy (µm)	2.58	2.79	3.02	3.26	3.78
Uvcz (µm)	1.89	2.14	2.40	2.67	3.23
Uvcxy (µm)	2.25	2.39	2.54	2.70	3.06
Uvcyz (µm)	1.89	2.14	2.40	2.67	3.23
Uvcxz (µm)	2.25	2.39	2.54	2.70	3.06
Uvcxyz (µm)	1.56	1.57	1.58	1.59	1.62

During the uncertainty assessment on the adjusted value by the GUM approach. We have tried to integrate uncertainty on probing errors while using the text of the iso 10360-2 standard and to make a reasonable stimulation ensure that the way of reporting does not give an erroneous impression of uncertainty. However, to validate our proposed correction model, we will apply the Monte Carlo method to compare the two methods following GUM Supplement 1 and then conclude on the validity of the model.

3. CALCULATION OF UNCERTAINTY BY THE METHOD OF MONTE CARLO (MCM):

The Monte Carlo method is a general method. It can artificially reconstruct a random phenomenon simulating a fictitious sample of accomplishment from the input variables. We must therefore define the probability densities of each input variable that will be propagated to obtain the probability density of the output variable. This method can be used to validate the approach outlined by the law of propagation of uncertainty by making a comparison of results. The approach suggested by the Supplement 1 of the GUM on numerical methods of propagation of distributions is whether the confidence intervals, obtained by the method of Monte Carlo and the uncertainty propagation law, agree to within a stipulated numerical tolerance. $\zeta=0.5 \times 10^l$.

3.1. Principle of calculation:

The principle of this method is quite simple **[8.10]**; the analytical model is replaced by perfectly deterministic mathematical description. The calculation is made by the Mc-Ed software that is primarily a graphical interface to control the software Pack Monte Carlo using the Windows environment. The calculation module used simulates the process step by step. In fact, simulation, each input is disturbed using random number distributions (uniform probability density laws, Gaussian laws, uniform laws, arcsin…) for an assessment of the studied measurands. Then uses a random generator to generate prints of each influencing factor. The number of prints is a key issue: it should be infinite, which is impossible: we will seek a compromise between the computation time and the quality of the response (quality of

the description of the probability density based on output variable). From these drawings, it calculates the corresponding output values, and summarizes the results.

Conventionally, the selected values are the mean, standard deviation and confidence interval for a given probability level (95.45% for example).

3.2. Selection of distributions for the input and output variables:

The choice of these laws generally depends on the nature of the distribution. When a parameter is under the influence of a random distribution and a large number of sampling, so the overall effect is normally distributed. While, the uniform law states intuitively that all values of the interval have the same chances of appearing, that means the probability density is constant. The function is sought to simulate the uncertainty of the correction matrix according to7 directions expressed by: $Vci=Ai*Vlue+Bi$. This function depends on the random variable Vlue that will affect a normal law, however, the change in the coefficients straight Ai and Bi calibration is uniform.

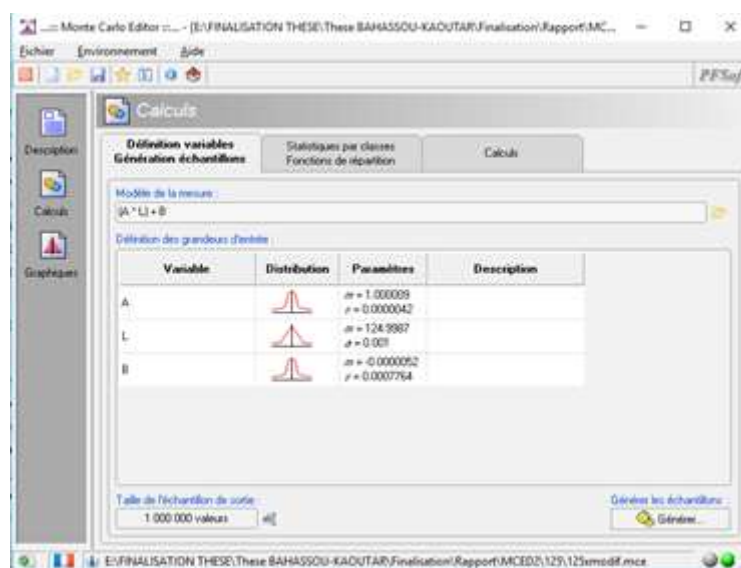**Figure 1**: Attribution of laws for input variables

3.3. Random number generator:

When all distributions of input variables were defined, we can then generate $M=10^6$ accomplishment of each size input by drawing in their probability density, it is necessary to have efficient enough random number generator. In the end, he chose the numerical value of each term in the model, its associated uncertainty and probability function.

3.4. Results and interpretation:

The Monte Carlo method makes it possible to avoid a large part of the calculation of partial drifts. In particular, when the analytical expressions are complex, it also permits to associate distribution laws to each of the measured quantities.
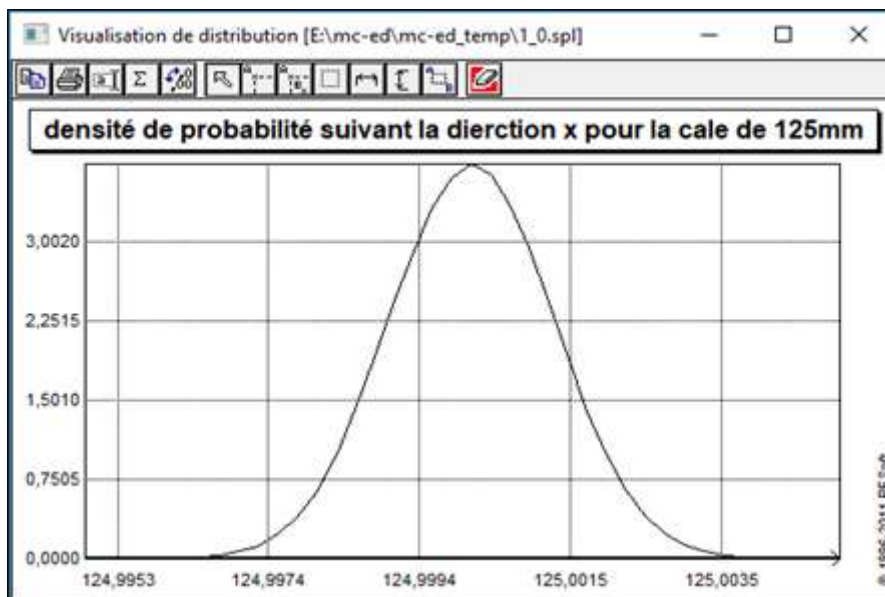
Figure 2: simulation of the uncertainty of measurement by the Monte Carlo method according to x direction and the wedge 125mm

3.4.1. Interpretation:

In Figure 2, the solid line is a probability density of average equal to the value calculated by the analytical method. We can see the excellent consistency of the two methods and although several inputs are laws of normal probability densities, the final result is Gaussian in appearance. As long as the validity range of the Monte Carlo approach is broader than that of the GUM uncertainty approach, it is recommended to compare the two methods. If the comparison is favorable, the GUM uncertainty framework could be used on this occasion and for sufficient similar problems in the future. Otherwise, consideration should be given to using the Monte Carlo approach

5. COMPARISON OF TWO METHODS GUM AND MCM:

The purpose of this work is to compare the principles on which the GUM and Monte-Carlo approaches implement the propagation of distributions. The two methods are then compared with a confidence interval of 95%. This part also provides a motivation for the use of Monte Carlo in circumstances where it is doubtful that the application of the uncertainty of the GUM framework is valid, the recommendations are also presented in the context of the assessment of the uncertainties of the measurements of the adjusted value.

We have also applied GUM Supplement 1, which has a broader validity framework for the MCM than the GUM method. First of all, it is necessary to determine the tolerance δ associated with the calculated z result. This is expressed as $z=c*10^l$ where c is the value of the last digit considered significant for z. Then we will write $\delta=0.5.10^l$. Let us take the uncertainty u(y) on the adjusted value, only one significant digit is retained, i. e. ndig=1, $u(y)=2.10^{-3}mm$. The tolerance is then $\delta=0.5.10^{-3}mm$.

The second step in the validation process is to compare the expanded uncertainties obtained by each of the methods. We drill the differences:

Dlow=│YlowGUM-YlowMCM│

Dhigh=│YhighGUM-YhighMCM│

If these differences are all no larger than or equal to the previously determined tolerances δ, then the results obtained by the GUM method will have been validated. Otherwise, the results obtained by the MCM method should be used.

Here is a summary table of the various D_{low} and D_{high} values corresponding to each hold value following each direction:

		125	150	175	200	250
X	Dlow(mm)	0,0002	0,0001	0,0002	0,0002	0,0002
	Dhigh(mm)	0,0002	0,0002	0,0002	0,0003	0,0002
Y	Dlow(mm)	0,0002	0,0001	0,0001	0,0001	0,0001
	Dhigh(mm)	0,0002	0,0003	0,0002	0,0002	0,0003
Z	Dlow(mm)	0,0000	0,0002	0,0002	0,0001	0,0001
	Dhigh(mm)	0,0001	0,0002	0,0002	0,0002	0,0002
XY	Dlow(mm)	0,0002	0,0001	0,0002	0,0002	0,0002
	Dhigh(mm)	0,0002	0,0002	0,0002	0,0003	0,0002
XZ	Dlow(mm)	0,0000	0,0002	0,0002	0,0001	0,0001
	Dhigh(mm)	0,0001	0,0002	0,0002	0,0002	0,0002
YZ	Dlow(mm)	0,0002	0,0001	0,0002	0,0002	0,0002
	Dhigh(mm)	0,0002	0,0002	0,0002	0,0003	0,0002
XYZ	Dlow(mm)	0,0002	0,0002	0,0002	0,0002	0,0001
	Dhigh(mm)	0,0002	0,0002	0,0002	0,0002	0,0002

We can observe that the differences Dlow and Dhigh are all no larger than or equal to δ $=0.5.10^{-3}$ mm.

Thus, the difference between the two confidence intervals obtained is very low and the interval calculated by numerical simulation of Monte Carlo is included in that of the uncertainty propagation.

In addition to this, these results are quite consistent with respect to the other and we can consider that the results given by the numerical simulation of Monte Carlo validate the law of propagation of uncertainty

6. CONCLUSION

The proposed paper presents a comparative study between the methods of assessing the uncertainty of measurement on the calibration correction matrix of the coordinate measuring machines, by different methods of propagation of uncertainties. We began by presenting the GUM method and the MCM method. Thus, we were conducted to the estimation of this uncertainty using both of methods.

The analytical method (GUM) allows a complete uncertainty analysis to be carried out, it allows the uncertainty of the impact factors to be calculated, their sensitivity to variation in the input parameters and the contribution of the uncertainty of the adjusted value to be calculated. On the other hand, Monte Carlo simulation is based only on the distribution laws given to each of the input quantities. The advantage of this method is in the calculation of partial derivatives especially when the measurand has a complex mathematical expression.

The results found were compared, it appears that the uncertainty found by the two methods are almost identical with a difference 10^{-4} and the difference between the two confidence intervals is very low. The criterion described by Supplement 1 GUM is observed, it can be concluded that the model developed for the propagation of uncertainty according to GUM is validated.

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