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SEASONAL AND PERIODIC AUTOREGRESSIVE TIME SERIES MODELS USED FOR FORECASTING ANALYSIS OF RAINFALL DATA

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ABSTRACT

The amount of rainfall received over an area is an important factor in assessing availability of water to meet various demands for agriculture, industry, irrigation, generation of hydroelectricity and other human activities. In our study, we consider seasonal and periodic time series models for statistical analysis of rainfall data of Punjab, India. In this research paper we apply the Seasonal Autoregressive Integrated Moving Average and Periodic autoregressive model to analyse the rainfall data of Punjab. For evaluation of the model identification and periodic stationarity the statistical tool used are PeACF and PePACF. For model comparison we use Root mean square percentage error and forecast encompassing test. The results of this research will provide local authorities to develop strategic plans and appropriate use of available water resources.

Keywords: Encompassing Test, PAR, PeACF, PePACF, RMSPE, SARIMA

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1. INTRODUCTION

Autoregressive Integrated Moving Average (ARIMA) model is proposed by Box and Jenkins [6] and they gives detail knowledge about ARIMA and Seasonal ARIMA models in their book. Franses H. P. & Paap [13] discussed in their book about periodic time series models. In recent time there has been considerable research in the development of time series models with seasonal or periodic properties in metrological and hydrological area. These periodic models have been introduced earlier in the article of Hannan [15] and mostly spread in geophysics and environmental studies. In the past decades there are various researchers provide literature on univariate periodic time series models includes Gladyshev [14], Jones and Brelsford [19], Pagano [26], Troutman [35], Tiao and Grupe [34] and many more . On evaluating forecasts from PAR models there are several studies like Novales and Flores de Frute [25], Wells [41], Herwartz [16], [17] and Frances and Paap [12] and they yield mixed results.

In periodic modeling we find the various applications such as fourier parameterisation of periodic ARMA model is applied by Lund, Shao and Basawa [24] for daily time series of temperature data. Dudek A. E. [8] et.al find the PARMA methods based on Fourier representation of periodic coefficients. Tesfaye, Meerschaert and Anderson [33] has applied this periodic ARMA modelling on monthly river flows. Recently Ursu E. et.al have used PAR model identification using genetic algorithms and also have done work on application of periodic autoregression process to the modelling of the Garonne river flows[36], [37]. We also find the various application of PAR model on river flow [33] and stream flow [29], [40]. Sarnaglia A. J. Q. et. al have used robust estimation process for the periodic autoregressive in the presence of outliers [28].

Rainfall prediction is an important aspect of climate forecasting. Many researchers have been applied different techniques on accurate rainfall forecasting all over the world. Recently Aftab S et. al [2], B. Kavitha Rani & A. Govardhan [4] applied data mining techniques, Edwin, A. I and Martins O.Y. [9] applied decomposition technique with ARIMA model and periodic autoregressive model scheme, Sopipan N.[31] applied ARIMA model, Exponential Smoothing and Holt Winter model and many more[8],[20]. Most of the prediction of rainfall data analyzed done by SARIMA (Seasonal Autoregressive Integrated Moving Average) model , like Papalaskaris T. et. al [27] , Etuk E. H. and Mohamed T. M [10] in different regions.

There are several studies relating to pattern of rainfall in India. There is no clear trend of increase or decrease in average rainfall in the country studied in [23], [39].There are also lot of research work done on forecasting and analysis of rainfall trends of different regions in India and they have used various statistical techniques and models such as [7],[30],[38]. In 2017 Arvind G. et.al [3] has done research on statistical analysis of Rainfall data and Akash Parmar et al [1] gives a review about machine learning techniques for rainfall prediction. About 80% of the rainfall in India occurs the four monsoon months (June-Sepetember) with large spatial and temporal variations over the country. In India rainfall of July month is very much crucially for the agricultural crop production. The production of seasonal crops particular under rain fed condition is closely linked with amount and distribution of rainfall. Punjab is agricultural state of northern India. It has played a prominent role by self sufficiency in food grains by wheat and paddy. Some research has been carried out on analysis of rainfall in region of Punjab. Singh R. & Pal R.K [32], Kumar R. & Bhardwaj A.[22] and Krishan G. et, al [21] have studied about the analysis of rainfall in Punjab.

In this paper we used seasonal and periodic autoregressive models for forecasting rainfall of Punjab. The study is execute with the following objectives: (i) to develop autoregressive time series models for rainfall of Punjab; (ii) to validate and find future values using developed time series model and finally to assess the change of quarterly rainfall in developed time series. This study aims to provide forecasting information of rainfall data in various districts of Punjab. Autoregressive models is applied to analyze the rainfall of Punjab and for analysing we used R software. For calculating the accuracy of our result on the basis of Root mean square percentage error and forecast encompassing test.

2. SOURCES OF DATA

The source of rainfall data in our study is India Metrological Department (IMD), Chandigarh. IMD provide metrological data for various research and development activities in the fields of like Monsoon, Climate Change and Agricultural Meteorology. The geography and subtropical latitudinal location of Punjab lead to large variations in temperature. There are three main seasons of Punjab i.e. Summer Season, Rainy Season and Winter Season. Punjab's rainy season is start from July as monsoon currents generated in the Bay of Bengal. Our study focus on rainy season of Punjab. We collect our data form IMD, Chandigarh period from 2000 to 2017.

3. METHODOLOGY

Time series model can be divided into two components deterministic and stochastic. The deterministic component use for forecast of time and chance independent future events, while the stochastic component use for determination of the chance and chance dependent effects. Deterministic component are either periodic or non periodic in nature. The periodic nature of a deterministic component is identify by its cyclic pattern, which exhibits an oscillatory movement and it is repeated over a fixed interval of time and the non periodic component is identify by its trend. Although stochastic component consists of irregular oscillation and random effects.

There are different approaches to modelling and forecasting seasonal time series. One approach gives Box and Jenkins [6] and on moving average models for double differenced time series which is known as a Seasonal ARIMA (SARIMA) models. Another approach assumes that seasonal time series can be decomposed into trend, cycle, seasonal and irregular components, it reduced forms of the resultant models and have many similarities with the aforementioned SARIMA models. Another approach is based on seasonal variation and it is described by allowing the parameters in an autoregression to vary with the seasons i.e. called Periodic Autoregression (PAR) model.

3.1. Mathematical formation of SARIMA Model

The difference of the series $\{X_t\}$ at lag s is a convenient way of eliminating a seasonal component of period 's'. When we fit an ARMA (p,q) model $\phi(B)Y_t = \theta(B)Z_t$ to the differenced series $Y_t = (1-B_s)X_t$, then the model for the original series is

$$\phi(B)(1-B_s)X_t = \theta(B)Z_t$$

This is a special case of the general seasonal ARIMA (SARIMA) model defined as follows:

If d and D are non-negative integers, then $\{X_t\}$ is a seasonal ARIMA (p, d, f) ×(P, D,Q)$_s$ process with period s if the differenced series $Y_t = (1-B)^d(1-B_s)^D X_t$ is ARMA process defined by

$$\phi(B)\Phi(B_s)Y_t = \theta(B)\Theta(B^s)Z_t,\quad (Z_t) \sim WN(0,\sigma^2)$$

Where $\phi(z) = 1-\phi_{1Z} - \ldots - \phi_{pZ^p}$, $\Phi(z) = 1-\Phi_{1Z} - \ldots - \Phi_{pZ^p}$, $\theta(z) = 1+\theta_{1Z} + \ldots + \theta_{pZ^p}$ and $\Theta(z) = 1+\Theta_{1Z} + \ldots + \Theta_{QZ^Q}$.

3.2. Mathematical formation of PAR Model

In a univariate time series y_t which is observed quarterly for N years; i.e., t=1, 2,…,n = 4N. A periodic autoregressive model of order p for y_t can be written as

$$y_t = \phi_{1s} y_{t-1} + ... + \phi_{ps} y_{t-p} + \varepsilon_t or \tilde{\phi}_{ps}(L) y_t = \varepsilon_t$$

Where L is the usual lag operator and where ϕ_{1s} through ϕ_{ps} are autoregressive parameters, which may take different values across the seasons s = 1, 2, 3, 4. The distribution with constant variance σ^2 or for different variances $\sigma_s{}^2$ in each season.

The periodic process described by model from above equation is nonstationary, as the variance and autocovariances are time varying within a year. A time invariant form of PAR (p) process is rewrite and as the PAR (p) model considers different AR(p) models for different seasons.

After analysis of our data by using seasonal and periodic autoregressive model we forecast our data. We check accuracy of forecasting use error measure gives us a summary of the skill and capability of the forecast model which made the forecasting. We used error measure which is RMSPE and forecast encompassing test.

$$RMSPE = \sum_{i=1}^{n} \sqrt{\frac{(\hat{y}_i - y_i)^2}{n} \times 100}$$

Where $\hat{y}$ is a vector of n forecast values, y is the vector of the true values , n is the size of the out of sample.

The forecast encompassing test for forecast accuracy is developed by Diebold and Mariano(1995) and they have calculated applied work in forecast comparison. The DM-test is based on loss differential between the models i.e. $d_t = e_{0,t}^2 - e_{k,t}^2$. To test the null hypothesis we assumed that both the forecast models are equal in nature is given as equal forecast accuracy $H_0 : E(d_t) = 0$. The DM statisitic is:

$$DM = P^{1/2} \frac{\bar{d}}{\hat{\sigma} DM}$$

Where $\bar{d}$ is the average loss differential, P is the out of sample size, the $\hat{\sigma} DM$ is the squared root of long run variance of d_t. The test statistics DM is asymptotically N(0, 1) distributed.

4. GRAPHIC ANALYSIS OF DATA

In this section we analyzed the graphic reperesentation of rainfall data. Firstly we plot the box-plot of data monthly as well as quarterly for identifying the outliers in data. Figure 1 below the box-plot for monthly and quarterly rainfall data for checking the presence of outliers in time series.

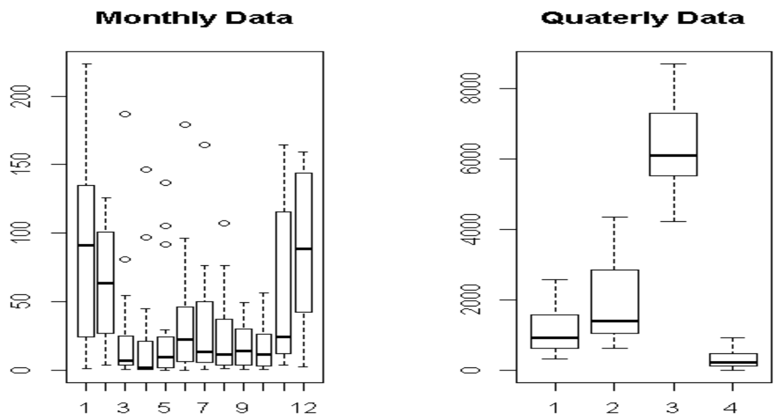

Figure 1: Graph of box-plot

Above graph of monthly data show few observation that can be identified as outliers. But when we take quarterly data there is no shows any observation outside the box-plot therefore we are taking quarterly data for analyzing. In above section 2 we discuss information about the source of data. Following figure 2 shows the graphical representation of data:

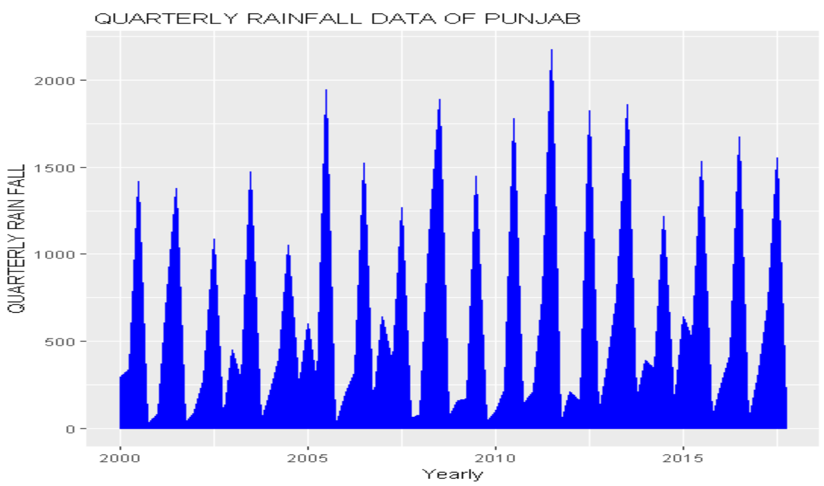

Figure 2: Graph of Quarterly Rainfall Data

5. AUTOCORRELATION FUNCTION FOR SEASONAL AND PERIODIC AUTOREGRESSIVE MODELS

The Autocorrelation functions plots are widely used in time series analysis and forecasting. These plots are graphically explanation of the strength of a relationship between observation of time series. Here we discuss the autocorrelation functions for both models.

5.1. Seasonal ARIMA (SARIMA)

The ACF plot of the quarterly rainfall data is given below:

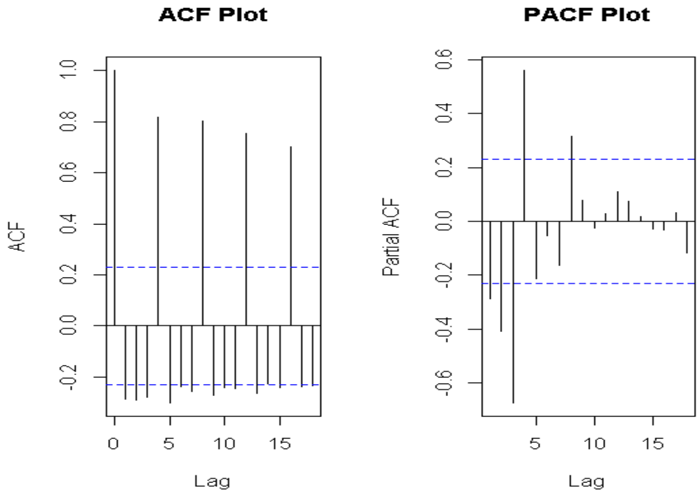

Figure 3: Graph of Autocorrelation Functions

In fig.3, the ACF plot describe the autocorrelation between observation and also include direct and indirect dependence information between them and show seasonality in data. The ACF and PACF plots shows the order of AR and MA terms. PACF plot shows the direct relationship between observation and its lags.

5.2. Periodic Autoregressive Model (PAR)

In PAR model we check the pattern of PeACF and PePACF for PAR model from following plots:

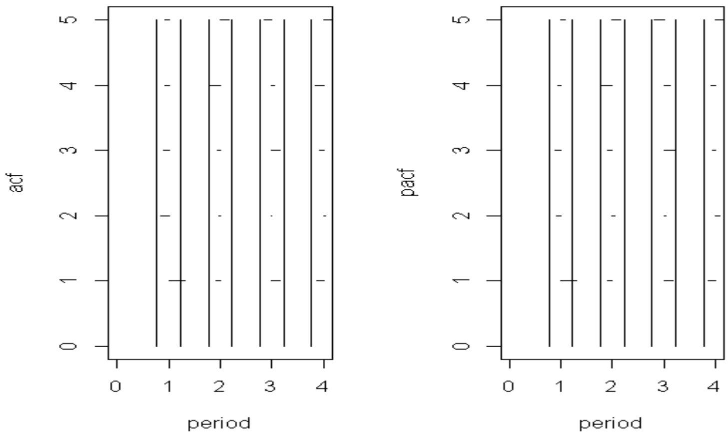

Figure 4: Graph of Periodic autocorrelation function

In fig.4 we plot the PeACF and PePACF plots of periodic autoregressive models for pure PAR process autocorrelation function becomes zero for lags beyond p_s and the order of the process can be decided according to the sample of PePACF. Once the presence of periodic correlation has been detected, a suitable PAR model can be selected either by examining plots of PeACF and PePACF or by using information criterion methods i.e. AIC or BIC.

6. STATIONARITY TEST

In most widely used stationary test in time series analysis is Augmented Dicky Fuller Unit Root test. In ADF test has alternative hypothesis can be taken as the series is stationary. When we apply the test on our data we obtain p-value is 0.01 .

7. MODEL BUILDING PROCEDURE OF SARIMA

Analysis of time series dealt with stationary time series data and fitting of seasonal autoregressive integrated moving average model for performing the steps of modeling. In this section we discuss the following steps for model building.

Decomposition of time series – Time series decomposition used for check the observation, seasonal effect or variation occurred in series and following figure shows the decomposition of our time series data.

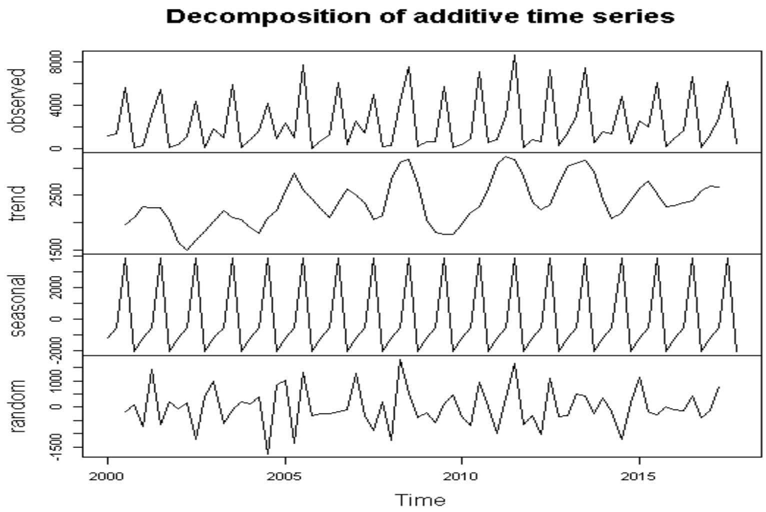

Figure 5: Decomposed time series plot of rainfall data

From above fig. 5 we observed existence of seasonal variation in the series which is constant over time, similarly for the observation of decomposed time series, the random effect change with respect to time.

- **Model Identification -** The model development process begins by examine the original plot of ACF, PACF and test of the raw data to be sure that it is stationary.
- **Model Estimation –** The model estimation procedure for choossing the model with the minimum AIC, AICc and BIC and find the best model for our purpose.

Table 1. Parameter estimation of the best fit model SARIMA(3,0,0)(1,0,1)[4]

	Ar1	Ar2	Ar3	Sar1	Sar2
Coefficients	-	-	-	-	-
	1.2885	0.7966	0.3384	0.5150	0.8558
S. E.	0.1131	0.1662	0.1450	0.2291	0.1366

In above table1 we summarised the best SARIMA (3,0,0)(1,0,1)[4] model with mean zero on the basis of minimum information ciretion.

Diagnositic Checking – In time series analysis after selecting the best model fit, we daigonised the residuals analysis. We assume that the best model residuals must follows a white noise process means that the residuals have zero mean, constant variance and also uncorrelated.

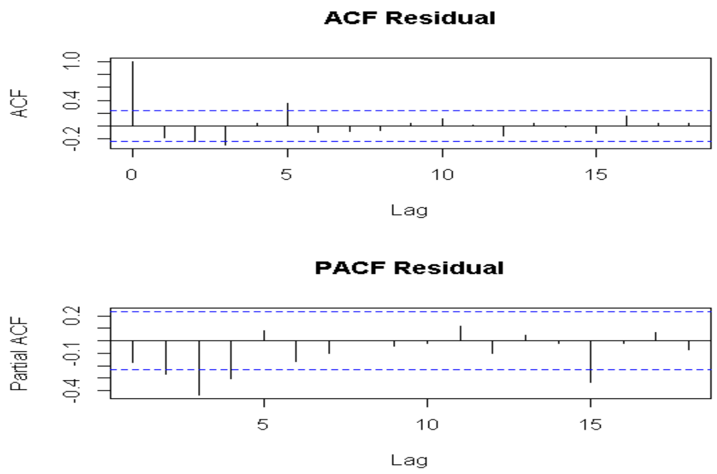

Figure 6: Graph of Residuals Checking

From above fig. 6, the ACF and PACF of the residuals of the model shows that autocorrelation of the residuals are almost zero and uncorrelated, we find very few spikes out side the boundary. We can say that our resulting model is good for forecasting.

8. MODEL BUILDING PROCEDURE OF PAR

The PAR models extended class of autoregressive(AR) models by allowing the autoregressive parameters to vary with the seasons. If the number of seasons should be one then PAR model become AR model. In this section we gives the steps for analysing data using PAR model.

Model Identification – We trace seasonal component and seasonal variability of our data in sec.4 of graphic analysis of data. To stabalized the series we transform the series by taking difference. The presence of periodic correlation in time series has detected by examing the plot of PePACF, In section V, autocorrelation function for seasonal and periodic autoregressive model is explained or it can be done by using information criterion.

Model Estimation and Order Selection - For this estimation and order selection we select the lowest value of AIC and BIC criteria. On the basis of this method order of model is one. Results of order selection is based on table 2 as given below:

Table 2: Periodic Autoregressive order selection

Criterion	Periodic Autoregressive Order			
	1	**2**	**3**	**4**
AIC	1134.293	1149.518	1163.444	1171.681
BIC	1180.903	1187.498	1192.675	1192.046
$F(\phi_{p+1,s}=0)$	0.19	1.26	1.41	1.32
P- value	0.94	0.29	0.24	0.27

We are also test the periodicity in the autoregressive parameters of the model, we applied F-test for the null hypothesis of non- periodicity, $\phi_{is}=\phi_i$ for s=1,2,…,4 and i=1,2,….,p. The results of the test rejected the null hypothesis at 5% level of significance that means periodic model fits better to the data rather than an AR model, which is constrained to seasonally constant parameters.

Test for a Unit Root – The multivariate representation of data obtained on the basis of eigen values which provide the information on the prospective unit roots. From the results it is clear that only one positive value is less than one hence it seems that there is no unit root. To check whether a unit root exist or not we apply the Likelihood Ratio test for a single unit root in PAR model.We apply the test and check positive and negative seasonal unit root which shows the rejection for existence of unit root.

Model Diagnostic Checking – For residual checking we apply F-statistic test. According to this test we check whether seasonal heterokedasticity exist in the residuals of the fitted model. Our F-test result show that seasonal heterokedasticity is rejected. The ACF of the original time series shows long – memory behaviour and it can also observe that the residual of the first differenced series better than the first and seasonal differenced of the original series.

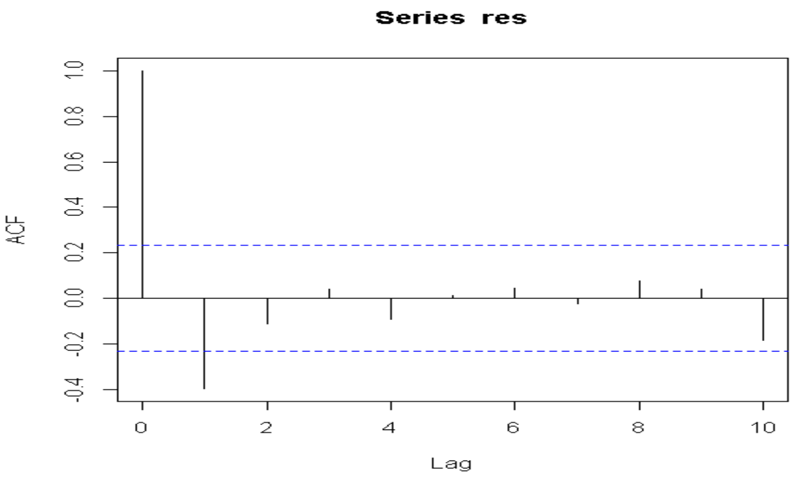

Figure 7: Residual plot of ACF

In above fig 7: acf residual graph shows dotted line for the level of siginificance. It has no residuals present in plot, all spikes are under the boundary line therefore fitted model is best model for forecasting.

9. MODEL COMPARISION ON THE BASIS OF FORECAST ENCOMPASSING TEST AND ERROR MEASURE

For comparision between both models, we have taken RMSPE and forecast encompassing test. For analysing our data set based on SARIMA and PAR model the RMSPE and use forecast encompassing test values are obtained.

Table 3: Forecast errors of SARIMA (3) and PAR(1) model

Forecast Error Models	RMSPE
SARIMA	13.08
PAR	10.07

From the table3, it is clear that PAR model is minimum error value as compare to SARIMA. So we conclude that PAR model gives more accurate value as compare to SARIMA. In this paper we apply Diebold-Mariano test for forecast encompassing testing. Diebold-Mariano test is applied for the null hypothesis for equal forecast accuracy between SARIMA and PAR model. Since the DM statistics converge to a normal distribution, we can reject the null hypothesis at the 5% level if $|DM| > 1.96$. Otherwise, if $|DM| \leq 1.96$, we accept the null hypothesis H_0. In our result, test value is greater than 1.96 which implies that null hypothesis should be rejected and alternative hypothesis accepected means that PAR model has more forecast accuracy than SARIMA model.

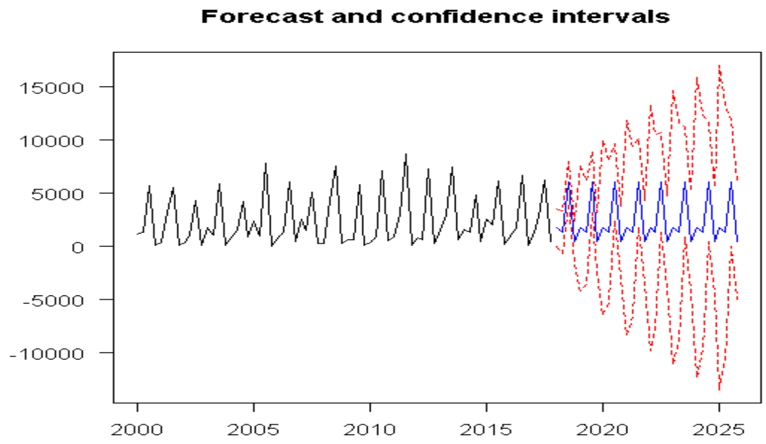

Figure 8: Forecast graph of Rainfall data using PAR model

In fig.8 we plot the forecast graph of rainfall data and show the 32 quarters ahead forecasts, the upper and lower has 95% confidence bound, as well as the input and the number of predictions. The minimum predict rainfall value is 472.33mm and the maximum predict rainfall value is 6032.39 mm.

10. CONCLUSIONS

In this paper we have discussed and forecast the analysis of quarterly rainfall data of Punjab, we have also evaluated their forecasting performance based on error measures and encompassing test. Periodic Autoregressive models are most widely used for modelling seasonal hydrological time series. In our meterological data, we applied seasonal and periodic autoregressive model and compare their performance measures and it is obtained that PAR model is best among the two.

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