Rp-99: Formulation of Standard Cubic Congruence of Composite Modulus- A Multiple of the Power of the Modulus

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ABSTRACT

In this paper, two special standard cubic congruence of composite modulus of higher degree are considered for study to establish a formulation of the solutions of the congruence. The formulation of the solutions is established. The author's rigorous labour works efficiently and succeed to formulate the solutions of the congruence under consideration. This paper presents the author's successful efforts. The discovered formula is tested and verified true by illustrating different suitable examples. No effective method is found in the literature of mathematics for its solutions. Formulation is the merit of the paper. Formulation is proved time-saving, easy and simple. The study of standard cubic congruence of composite modulus of higher degree is now an interesting subject of mathematics.

KEYWORDS: *Cubic Expansion, Cubic Congruence, Composite Modulus, Formulation*

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INTRODUCTION

A congruence of the type: $x^3 \equiv a^3\ (mod\ m), m$ being a composite positive integer is called a standard cubic congruence of composite modulus. Its solutions are the values of x that satisfies the congruence. Such types of congruence have different numbers of solutions. There are standard cubic congruence having unique solutions; some has three solutions; some has four solutions and some others have $p\ or\ p^2$ solutions; p being a positive prime integer.

LITERATURE REVIEW

The author has gone through many books of Number Theory but did not find any discussion or formulation on standard cubic congruence. Earlier mathematicians did not show any interest to study the congruence. Hence no formulation or any discussion is found.

The author intentionally started his research on standard cubic congruence and successfully discovered formulations of standard cubic congruence of prime and composite modulus [1], [2], [3], [4], [5].The author already discovered a formulation of $x^3 \equiv a^3\ (mod\ a^m), m \geq 4$.
And $x^3 \equiv 0\ (mod\ a^m); m \leq 3$ [6].

Now he wishes to formulate the congruence of the type:
1. $x^3 \equiv a^3\ (mod\ a^m.b); m \geq 4; b \neq ka;\ and\ also$
2. $x^3 \equiv 0\ (mod\ a^m.b)$.

NEED OF RESEARCH

The author found no discussion or any formulation for the said congruence in the literature of mathematics. He tried his best to formulate the congruence. His efforts of establishment of the formulation is presented in this paper. This is the need of the research.

PROBLEM- STATEMENT

"To formulate a very special standard cubic congruence of composite modulus of the types
$(\mathbf{1})\boldsymbol{x^3} \equiv \boldsymbol{a^3}\ (\boldsymbol{mod}\ \boldsymbol{a^m.b}); \boldsymbol{m} \geq \mathbf{4}\ \&\ a, b\ any\ positive\ integers."$
$(\mathbf{2})\ \boldsymbol{x^3} \equiv \mathbf{0}\ (\boldsymbol{mod}\ \boldsymbol{a^m.b}); \boldsymbol{m} \leq \mathbf{3}\ \&\ a, b\ any\ positive\ integers."$

ANALYSIS & DISCUSSION

Case-I: If $x \equiv a^{m-2}.bk + a$, then

$x^3 \equiv (a^{m-2}.bk + a)^3$
$\equiv (a^{m-2}bk\ )^3 + 3.(a^{m-2}bk)^2.a + 3.a^{m-2}bk.a^2 + a^3\ (mod\ a^m.b)$
$\equiv\ a^{m-2}bk\{\ (a^{m-2}bk)^2 + 3a.a^{m-2}bk +$ *3a2+a3 (mod am.b)*
$\equiv\ a^{m-2}bk\{\ a^2t\} +\ a^3\ \ (mod\ a^m.b),$
$\equiv a^m bkt + a^3\ (mod\ a^m.b)$
$\equiv a^3\ (mod\ a^m.b).$

Thus, it can be said that $x \equiv a^{m-2}.bk + a\ (mod\ a^m.b)$ is a solution of it for some

$k = 0, 1, 2, \dots \dots . a^2 - 1, a^2, a^2 + 1, \dots \dots \dots$

It can also be seen that if $k = a^2, a^2 + 1, \dots \dots$, $x \equiv a^{m-2}. a^2 + a = a^m + a$

$$\equiv a \ (mod\ a^m. b)$$

It is the same solution as for $k = 0,1, \dots \dots \dots, a^2 - 1$.

Thus, the said congruence has a^2 incongruent solutions.

Case-II

Consider the congruence $x^3 \equiv 0 \ (mod\ a^m. b)$.

It is seen that for $x \equiv abt \ (Mod\ a^m. b); m \leq 3$,

$x^3 \equiv (abt)^3 \equiv a^3 b^3 t^3 \equiv 0 \ (mod\ a^3. b)$

Thus, $x \equiv abt \ (mod\ a^3. b) for\ t = 1, 2, 3, \dots \dots \dots$

are the solutions of the congruence under consideration.

ILLUSTRATIONS

ILLUSTRATIONS OF CASE-I

Consider the congruence $x^3 \equiv 5^3 \ (mod\ 5^4. 3)$.

It is of the type $x^3 \equiv a^3 \ (mod\ a^m. b)\ with\ a = 5, m = 4, b = 3$. It has $5^2 = 25$ solutions.

These solutions are given by

$x \equiv a^{m-2}. bk + a \ (mod\ a^m. b)$

$\equiv 5^{4-2}. 3k + 5 \ (mod\ 5^4. 3)$

$\equiv 5^2. 3k + 5 \ (mod\ 5^4. 3)$

$\equiv 75k + 5 \ (mod\ 1875)$

$\equiv$

5, 80, 155, 230, 305, 380, 455, 530, 605, 680, 755, 830,905, 980, 1055, 1130, 1205, 1280, 1355, 1430, 1505, 1580, 1655, 1730, 1805 (mod 1875).

These solutions are also verified true.

Consider another congruence $x^3 \equiv 5^3 \ (mod\ 5^4. 16)$

It is of the type $x^3 \equiv a^3 \ (mod\ a^m. b)\ with\ a = 5, m = 4, b = 16$. It has $5^2 = 25$ solutions.

These solutions are given by

$x \equiv a^{m-2}. bk + a \ (mod\ a^m. b)$

$\equiv 5^{4-2}. 16k + 5 \ (mod\ 5^4. 16)$

$\equiv 5^2. 16k + 5 \ (mod\ 5^4. 16)$

$\equiv 400k + 5 \ (mod\ 10{,}000)$

$\equiv$ 5, 405, 805, 1205,1605, 2005, 2405, 2805, 3205, 3605, 4005, 4405, 4805, 5205, 5605, 6005, 6405, 6805, 7205, 7605, 8005,8405,8805, 9205,9605 (mod 10,000).

These solutions are also verified true.

ILLUSTRATIONS of case-II

Consider the congruence $x^3 \equiv 0 \ (mod\ 7^3. 4)$. It is of the type $x^3 \equiv 0 \ (mod\ a^m. b)$

$with\ m = 3, a = 7, b = 4.$

Its solutions are given by

$x \equiv abt \ (mod\ a^m. b) for\ any\ integer\ t.$

$i.e.x \equiv 7.4t = 28t \ (mod\ 7^3. 4) for\ t = 1, 2, 3, \dots \dots \dots$

$i.e.x \equiv 28, 56, 84, \dots \dots \dots \dots \dots . (mod\ 7^3. 4).$

Consider the congruence $x^3 \equiv 0 \ (mod\ 7^2. 4)$. It is of the type $x^3 \equiv 0 \ (mod\ a^m. b)$

$with\ m = 2, a = 7, b = 4.$

Its solutions are given by

$x \equiv abt \ (mod\ a^m. b) for\ any\ integer\ t.$

$i.e.x \equiv 7.4t = 28t \ (mod\ 7^2. 4) for\ t = 1, 2, 3, \dots \dots \dots$

$i.e.x \equiv 28, 56, 84, \dots \dots \dots \dots \dots . (mod\ 7^2. 4).$

Consider the congruence $x^3 \equiv 0 \ (mod\ 7.4)$. It is of the type $x^3 \equiv 0 \ (mod\ a^m. b)$

$with\ m = 1, a = 7, b = 4.$

Its solutions are given by

$x \equiv abt \ (mod\ a^m. b) for\ any\ integer\ t.$

$i.e.x \equiv 7.4t = 28t \ (mod\ 7^1. 4) for\ t = 1, 2, 3, \dots \dots \dots$

$i.e.x \equiv 28, 56, 84, \dots \dots \dots \dots \dots . (mod\ 7^1. 4).$

Thus all such congruence have the same solutions.

These solutions are also verified true.

CONCLUSION

Thus, it can be concluded that the congruence $x^3 \equiv a^3 \ (mod\ a^m. b); m \geq 4$, a, b positive integers has a^2 incongruent solutions given by

$x \equiv a^{m-2}bk + a \ (mod\ a^m. b); k = 0, 1, 2, \dots \dots, a^2 - 1.$

Similarly it can also be proved that $x = abt\ are\ the\ solutions\ of\ x^3 \equiv 0 \ (mod\ a^m. b)$ for $m = 1,2,3; t = 1, 2, 3, \dots \dots \dots$

REFERENCE

[1] Roy, B M, *Formulation of Two Special Classes of Standard Cubic Congruence of Composite Modulus- a Power of Three*, International Journal of Scientific Research and Engineering Development (IJSRED), ISSN: 2581-7175, Vol-02, Issue-03, May-June 2019, Page-288-291.

[2] Roy, B M, *Formulation of a class of solvable standard cubic congruence of even composite modulus*, International Journal of Advanced Research, Ideas & Innovations in Technology (IJARIIT), ISSN: 2454-132X, Vol-05, Issue-01, Jan-Feb 2019.

[3] Roy, B M, *Formulation of a class of standard cubic congruence of even composite modulus-a power of an odd positive integer multiple of a power of three*, International Journal of Research Trends and Innovations (IJRTI), ISSN: 2456-3315, Vol-04, issue-03, March-2019.

[4] Roy, B M, *Formulation of Solutions of a Special Standard Cubic Congruence of Prime-power Modulus*, International Journal of Science & Engineering Development Research (IJSDR), ISSN: 2455-2631, Vol-04, Issue-05, May-2019.

[5] Roy, B M, *Formulation of Solutions of a Special Standard Cubic Congruence of Composite Modulus--an Integer Multiple of Power of Prime*, International Journal of Advanced Research, Ideas & Innovations in Technology (IJARIIT), ISSN: 2454-132X, Vol-05, Issue-03, May-Jun-2019 .

[6] Roy, B M, *Formulation of Solutions of a Special Standard Cubic Congruence of Composite Modulus*, International Journal of Research Trends and Innovations (IJRTI), ISSN: 2456-3315, Vol-04, Issue-06, Jun-2019.