JOURNAL OF OPTIMIZATION, DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS (JODEA)
Volume **26**, Issue **1**, June **2018**, pp. 78–88, DOI 10.15421/141806

ISSN (print) 2617–0108
ISSN (on-line) xxxx–xxxx

ON SOME TYPES OF INVERSE PROBLEMS FOR DIFFERENTIAL EQUATIONS

Yu. L. Menshikov*

Abstract. The inverse problems for differential equations are investigated, the solutions of which do not use information about the exact characteristics of the physical process. Such inverse problems have not yet become widespread, but they are of great practical importance. Some approaches to solving inverse problems of this type are suggested.

Key words: differential equations,inverse problems, classification, solution algorithms.

2010 Mathematics Subject Classification: 34A55, 45D05.

Communicated by Prof. V. Ye. Belozyorov

1. Introduction

Consider some types of inverse problems for differential equations [9], [10].

Let the physical process be characterized in the general case by a certain number of variables $x_1, x_2, ..., x_n$(state variables). The choice of the physical process characteristics is determined by the ultimate research goals. Let us assume that the variables $x_1, x_2, ..., x_n$ satisfy a linear system of ordinary differential equations with constant coefficients:

$$\dot{x} = Ex + Fz, \tag{1.1}$$

where $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T$ is a vector function of state variables, $z(t) = (z_1(t), z_2(t), \ldots, z_m(t))^T$ is a vector function of external loads (some of components are unknown), $(.)^T$ is transpose sign; $E = \{e_{ik}\}, 1 \leq i, k \leq n, F = \{f_{jl}\}, 1 \leq j \leq n, 1 \leq l \leq m$ are matrices with constant coefficients of the corresponding dimension, $t \in [0, T], [0, T]$ is interval of time where the solution of inverse problem is investigated. By mathematical description of the physical process we consider the set of the system of equations (1.1) with symbols of external loads, the vector function of the external loads as $z(t) = (z_1(t), z_2(t), \ldots, z_m(t))^T$ in the special form for individual problem and the initial condition $x(0) = x^0$. Thus, the mathematical description is a collection of mathematical model, vector function of external loads and initial conditions.

Briefly concerning the justification of such the definition. In inverse problems for the system (1.1) there are two main classes of inverse problems: - the determination of the coefficients of the matrices E, F using the some experimentally determined components of the vector function x(t), the given initial conditions and the

*Department of Differential Equations, Oles Honchar Dnipro National University, 72, Gagarin av., Dnipro, 49010, Ukraine, `Men0605Ude@gmail.com`

vector function $z(t)$; - the unknown components of the vector function $z(t)$ are determined under the some experimentally determined of the components of the vector function $x(t)$, the given initial conditions and the coefficients of the matrices E, F [7]. For the purpose of separating these two basic problems it is necessary to introduce the mathematical description of the physical process in the form of a collection of mathematical model (differential equations (1.1)) with symbols of external loads (general form of external loads) and separately the vector function of external loads $z(t)$ in the special form for individual problem [9].

The combination of these two inverse problems into one inverse problem is undesirable since it leads to a high degree of uncertainty. This is confirmed by the results of a review of the literature. Among the mathematical descriptions, one can single out descriptions that give the results of mathematical simulation which coincide with the given experimental measurements of the components of the vector function $x(t)$ with the experiment accuracy. Such descriptions are called adequate mathematical descriptions (AMD) [11].

2. Statement of problem

For system (1.1) it is possible to obtain several types of inverse problems in frame of second class of inverse problems for ordinary differential equations [9], [10], [11]. Let us investigate only two inverse problems from them.

Inverse problems of type I: it is necessary to define all unknown components $z_1(t), z_2(t), \ldots, z_m(t)$ of vector function of external loads $z(t)$ on segment $[0, T]$ using given from experiment functions of state variables $x_1(t), x_2(t), \ldots, x_n(t)$ on segment $[0, T]$, matrixes E, F and initial conditions aimed at constructing of AMD. Such inverse problems have not yet become widespread, but they have important mean for problems of mathematical modeling [11], as well as for problems of physical processes prediction [16], [15] and control problems [8], [4], [2].

Inverse problems of type II:it is necessary to define all unknown components of vector function of external loads $z_1(t), \ldots, z_m(t)$ on segment $[0, T]$ using some components of vector function of state variables $x_1(t), \ldots, x_n(t)$ on segment $[0, T]$ (which are given from experiment) and initial condition $x(0) = x^0$ aimed at obtaining the useful information about exact characteristics of real unknown external loads on system (1.1).

These inverse problems have applications for problems of diagnostics [3], for problems where it is necessary to obtain a new knowledge about the world around us [12].

Suppose that system (1.1) has only one unknown component $z_j(t), 1 < j \leq m$ of vector function $z(t)$ and only one given component $x_1(t)$ of vector function $x(t)$.

Using the linearity of the system (1.1), Volterra's integral equation of the first kind with respect to the unknown function $z_1(t)$ can be obtained with use the additional condition (see later)

$$\int_0^t K(t, \tau) z_j(\tau)\, d\tau = u_{\delta,j}(t), or A_{p,j} z_j = u_{\delta,j}, t \in [0, T], \tag{2.1}$$

where $z_j(t) \in Z, u_{\delta,j} \in U; U, Z$ are some normed functional spaces.

The function $u_{\delta,j}(t)$ is defined in terms of the initial conditions $x(0) = x^0$, the given function $x_1(t)$ and the known components $z_2(t), \dots, z_m(t)$ of the vector function of the external loads $z(t)$ with a predetermined error

$$\| u_{\delta,j} - u^{ex}_{\delta,j} \|_U \leq \delta_j, \tag{2.2}$$

where $u^{ex}_{\delta,j}$ are exact right parts in (2.1).

Notice that inverse problems of type I, II have the same equations for unknown functions $z_j(t)$ [9], [11].

The properties of operators $A_{p,j}$ are saved for inverse problems of both types: operators $A_{p,j}$ are compact operators for typical cases of functional spaces choice. Let us consider the set of possible solutions $Q_{\delta,p,j}$ of equation (2.1):

$$Q_{\delta,p,j} = \{z \in Z : \| A_{p,j} z_j - u_{\delta,j} \|_U \leq \delta_j\}. \tag{2.3}$$

The sets $Q_{\delta,p,j}$ are not bounded at any δ_j while operators $A_{p,j}$ are compact operators. So inverse problems of type I belongs to class of incorrect problems [9], [17] and special methods have to be used for their solution [9], [17].

In inverse problem of type I it is enough to obtain any function from the set $Q_{\delta,p,j}$. The inverse problems with such ultimate goals will be called as the inverse problems of synthesis [9], [11]. Let us consider exact equation z^{ex}_j of exact equation (2.1)

$$A^{ex}_{p,j} z^{ex}_j = u^{ex}_j, 1 < j \leq m, \tag{2.4}$$

where $A^{ex}_{p,j}$ is the exact operator in (2.1), u^{ex}_j is exact right part (initial data).

Besides the exact solution z^{ex}_j of the equation (2.4) may not belong to set of possible solutions $Q_{\delta,p,j}$ since the operator $A_{p,j}$ is being described inexactly of the real physical process.

In such inverse problems there is no need to require the convergence of the approximate solution of the problem to an exact solution z^{ex}_j of equation (2.4) by $\delta_j \to 0$ [10]. Therefore, the approximate solution cannot have properties of regularization [10], [11].

In addition these inverse problems have the following features:

- the approximate solution can considerably differ from the exact solution z^{ex}_j as $z^{ex}_j \notin Q_{\delta,p,j}$;
- the size of an error of the approximate solution in relation to the exact solution z^{ex}_j has no importance for further use of the approximate solution;
- the exact solution z^{ex}_j of an inverse problem in aggregate with initial mathematical model can give worse results of mathematical modeling than the approximate solution as $z^{ex}_j \notin Q_{\delta,p,j}$;

- the error of the operator $A_{p,j}$ to the exact operator $A^{ex}_{p,j}$ is possible not to take into account, as the initial inexact mathematical model of physical process will be used at mathematical modeling further.

The solution of such problems can interpret only as a good model for purposes of mathematical modeling.

In [9], [17] it is assumed that exact solution of equation (2.1) z^{ex}_j belongs to the set of possible solutions $Q_{\delta,p,j}$. This property was used by construction of regularized algorithms. So in case of inverse problems of type I ($z^{ex}_j \notin Q_{\delta,p,j}$) it is necessary to choose another approaches.

Now the conditions will be obtained when the set of possible solution is not empty.

Let us assume that in system (1.1) there is only one state variable $\tilde{x}_1$, which is obtained by experiment and only one unknown component $z_j(t), 1 < j \le m$ of vector function $z_(t)$ of external loads. The matrix $F = \hat{F} = f_{k,i}, 1 \le k \le n, 1 \le i \le m$, has the special form: by fixed $l, j\,(1 \le j \le m, 2 \le l \le m)$ element $\hat{f}_{lj} \ne 0, \hat{f}_{ki} = 0, k \ne l, i \ne j$.

Then first equation of system (1.1) with matrices $E, \hat{F}$ is differentiates $(n-1)$-times on t:

$$\left.\begin{aligned} &\frac{d^2\tilde{x}_1}{dt^2} = B_1E^2\tilde{x}(t) + \hat{f}_{lj}z_j(t),\\ &\frac{d^3\tilde{x}_1}{dt^3} = B_1E^3\tilde{x}(t) + \hat{f}_{lj}\dot{z}_j(t),\\ &\dots\\ &\frac{d^{(n-1)}\tilde{x}_1}{dt^{(n-1)}} = B_1E^{(n-1)}\tilde{x}(t) + \hat{f}_{lj}z_j^{(n-3)}(t), \end{aligned}\right\} \tag{2.5}$$

where matrix-string B_1 is defined as $B_1 = \{b^1_i\}, 1 \le i \le n, b^1_1 = 1, b^1_k = 0, k \ne 1, 1 \le k \le n$.

System (2.5) and first equation of (1.1) are solved relatively state variables $\tilde{x}_2(t), ..., \tilde{x}_n(t)$ through variables $\tilde{x}_1(t), \dot{\tilde{x}}_1(t), ..., \tilde{x}_1^{(n-1)}(t)$ and is substituted in following equation

$$\frac{d^n\tilde{x}_1}{dt^n} = B_1E^n\tilde{x}(t) + \hat{f}_{lj}z_j^{(n-2)}(t)$$

.

Consequently the next equation was obtained

$$\frac{d^n\tilde{x}_1}{dt^n} = \Phi(t, \tilde{x}_1, \dot{\tilde{x}}_1, \dots, \tilde{x}_1^{(n-1)}(t), \hat{f}_{lj}z_j, \dots, \hat{f}_{lj}z_j^{(n-2)}). \tag{2.6}$$

For solving system from equations (2.5) and first equation of (1.1) relatively state variables $\tilde{x}_2(t), \dots, \tilde{x}_n(t)$ it is sufficient that the Jacobian of such a transformation is nonzero.

Hence this condition has the form

$$J = \frac{D(B_1 E\tilde{x}, B_1 E^2\tilde{x}, \ldots, B_1 E^{n-1}\tilde{x})}{D(\tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n)} =$$

$$= \det \begin{pmatrix} \frac{\partial B_1 E\tilde{x}}{\partial \tilde{x}_2} & \frac{\partial B_1 E\tilde{x}}{\partial \tilde{x}_3} & \ldots & \frac{\partial B_1 E\tilde{x}}{\partial \tilde{x}_n} \\ \frac{\partial B_1 E^2\tilde{x}}{\partial \tilde{x}_2} & \frac{\partial B_1 E^2\tilde{x}}{\partial \tilde{x}_3} & \ldots & \frac{\partial B_1 E^2\tilde{x}}{\partial \tilde{x}_n} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial B_1 E^{n-1}\tilde{x}}{\partial \tilde{x}_2} & \ldots & \ldots & \frac{\partial B_1 E^{n-1}\tilde{x}}{\partial \tilde{x}_n} \end{pmatrix} \neq 0. \quad (2.7)$$

The differential equation of n -th order (2.6) is obtained under condition (2.7).

$$L_n[\tilde{x}_1(t)] = L_{n-2}[z_j(t)], \quad (2.8)$$

where $L_n[\tilde{x}_1(t)], L_{n-2}[z_j(t)]$ are linear differential operators of n -order and $(n-2)$ -order respectively.

Thus, the sufficient condition for the reduction of the inverse problem for system (1.1) to the inverse problem for one high-order equation (2.8) is condition (2.7).

In inverse problem for equation (2.8) it is necessary to define the function of external load $z_j(t)$, using function of state variable $\tilde{x}_1(t)$ that is obtained from experiment data with error.

Let the inaccuracy of function $\tilde{x}_1(t)$ relatively to exact function $x_1^{ex}(t)$ in metric $C[0,T]$ is given as follow

$$\| \tilde{x}_1(t) - x_1^{ex} \|_{C[0,T]} \leq \delta_1. \quad (2.9)$$

Theorem 2.1. *The set of possible solutions $Q_{\delta,p,j}$ of inverse problem for system* (1.1) *is non-empty if condition* (2.7) *is valid.*

Proof. The inverse problem for system (1.1)(1.1) can be reduced to inverse problem for equation (2.8) if the condition (2.7) is satisfied.

It is well-known that any continuous function $\tilde{x}_1(t)$ can be approximate in metric $C[0,T]$ with any accuracy δ by polynomial of q -order $x_{1,q}(t)$ [1]:

$$\| \tilde{x}_1(t) - x_{1,q}(t) \|_{C[0,T]} \leq \delta, q = q(\delta). \quad (2.10)$$

Initial conditions for function $x_{1,q}(t)$ are values

$$x_{1,q}(0) = x_{1,q}^{0,0}, \dot{x}_{1,q}(0) = x_{1,q}^{1,0}, \ddot{x}_{1,q}(0) = x_{1,q}^{2,0}, ..., x_{1,q}^{(n-1)}(0) = x_{1,q}^{(n-1),0}. \quad (2.11)$$

Let us consider the function $\gamma_1(t)$

$$\gamma_1(t) = L_n[x_{1,q}(t)]. \quad (2.12)$$

It is evident that $\gamma_1(t) \in C[0, T]$.

The solutions $x_{1,q}(t)$ of nonhomogeneous differential equation

$$\gamma_1(t) = L_{n-2}[z_{j,q}(t)]. \tag{2.13}$$

with different initial conditions form the set $Z_{j,q}$, $z_{j,q}(t) \in Z_{j,q}$.

The solution $\hat{x}_{1,q}(t)$ of differential equation (2.8) with initial conditions (2.11) and any functions from set $Z_{j,q}$ will satisfy the inequality (2.10).

The right part $u_{\delta,j}$ of equation (2.1) is determined with the help of function $\tilde{x}_1(t)$ only through continuous operations. So the set of possible solutions $Q_{\delta,p,j}$ for equation (2.1) is non-empty for any δ and this set contains the functions continuously differentiable any number number of times. □

3. Possible approaches of solving synthesis inverse problems

Let us consider the possibility of constructing stable algorithms for solving inverse synthesis problems without assuming that the exact solution z_j^{ex} of the inverse problem belongs to the set of possible solutions $Q_{\delta,p,j}$.

If the functional spaces Z, U are Banach spaces and the operator $A_{p,j}$ is linear, then the set of functions $Q_{\delta,p,j}$ is convex, closed, and unbounded. Let's show it.

Let $z_1, z_2 \in Q_{\delta,p,j}$. Then for $z_\alpha = \alpha z_1 + (1-\alpha) z_2 \in Q_{\delta,p,j}$ inequality is realized

$$\|A_{p,j} z_\alpha - u_{\delta,j}\|_U \leq \| \alpha A_{p,j} z_1 + (1+\alpha) A_{p,j} z_2 - \alpha u_{\delta,j} - (1-\alpha) u_{\delta,j} \|_U \leq \delta_j.$$

Consequently, the sets $Q_{\delta,p,j}$ are convex.

Let the sequence $z_k \in Q_{\delta,p,j}$ strongly converges to the element z_0.

Then

$$\begin{aligned} \| A_{p,j} z_0 - u_{\delta,j} \|_U &\leq \| A_{p,j} z_k - A_{p,j} z_0 \|_U + \| A_{p,j} z_k - u_{\delta,j} \|_U \\ &\leq \| A_{p,j} \| \| z_k - z_0 \| + \delta j. \end{aligned}$$

Let us pass to the limit in the last inequality when $k \to \infty$. Then we have

$$\| A_{p,j} z_0 - u_{\delta,j} \|_U \leq \delta j.$$

Thus, the sets $Q_{\delta,p,j}$ are closed.

In inverse synthesis problems the exact solution z_j^{ex} of the inverse problem does not belong to the set of possible solutions $Q_{\delta,p,j}$. Therefore, it is impossible to construct the regularizing algorithms which give solutions converging to an exact solution. However, in inverse problems of synthesis there are no need in such algorithms property, but it is sufficient to find any function from the set of possible solutions $Q_{\delta,p,j}$. This set is unbounded for arbitrary δ because of the compactness of the operator $A_{p,j}$. Therefore, it makes sense to choose from the set $Q_{\delta,p,j}$ a non-arbitrary element but an element with additional properties which are suitable for further research. For example, we can choose as the solution of inverse problem the most "convenient"element from the set $Q_{\delta,p,j}$ for the mathematical modeling purposes. It is possible to choose from $Q_{\delta,p,j}$ the simplest element [9], [11] that is the most stable to small changes of the initial data, the best element for prediction purposes [9], [11] and so on. Some algorithms of a choice from set $Q_{\delta,p,j}$ of an element with additional properties based on a variation principle can are proposed.

Consider now the following extreme problem

$$\Omega[z_{\delta_j,p}] = \inf_{z \in Q_{\delta,p,j}} \Omega[z], \tag{3.1}$$

where functional $\Omega[z]$ has been defined on set Z [17].

Theorem 3.1. *Assume that system* (1.1) *has only one state variable* $\tilde{x}_1(t)$*, which is obtained by experiment, and only one unknown component* $z_j(t), 1 \le j \le m$ *of vector function of external loads. The matrix* $F = \hat{F} = f_{k,i}, 1 \le k \le n, 1 \le i \le m$*, has the special form: by fixed* $l, j\,(1 \le j \le m, 2 \le l \le m)$ *element* $\hat{f}_{lj} \ne 0, \hat{f}_{ki} = 0, k \ne l, i \ne j$*. Suppose that* Z *is a reflexive Banach function space, that the functional* $\Omega[z]$ *is convex and lower semi continuous on* $Q_{\delta,p,j}$ *, that the Lebesgue set* $M(v)$ *bounded for a certain function from* $v \in Q_{\delta,p,j}$*:* $M(v) = \{z \in Q_{\delta,p,j} : \Omega[z] \le \Omega[v]\}$ *. Then the solution of the extreme problem (3.1) exists and belongs to* $Q_{\delta,p,j}$*.*

Proof. It is obvious that the exact lower bound of the functional $\Omega[z]$ on $Q_{\delta,p,j}$ can be achieved only from the points of the set $M(v)$ [19].

We show that the set $M(v)$ closed. Let the sequence $z_k \subset M(v)$ strongly converges to w . Since the set $Q_{\delta,p,j}$ is closed we have $w \in Q_{\delta,p,j}$. The following inequality holds since the functional $\Omega[z]$ is lower semi continuous on $Q_{\delta,p,j}$:

$$\Omega[w] \le \underline{\lim}_{k\to\infty} \Omega[z_k] \le C = \Omega[v].$$

Hence we have $w \in M(v)$. The closedness of the set $M(v)$ is proved.

Let us show that the set $M(v)$ is convex. Let $z_1, z_2 \in M(v)$. Since the functional $\Omega[z]$ is convex, we have

$$\Omega[z_\alpha] = \Omega[\alpha z_1 + (1-\alpha) z_2] \le \alpha \Omega[z_1] + (1-\alpha)\Omega[z_2] \le$$

$$\le \alpha \Omega[v] + (1-\alpha)\Omega[v] = \Omega[v].$$

Consequently the element $z_\alpha = \alpha z_1 + (1-\alpha) z_2 \in M(v)$. The convexity of set $M(v)$ is proved.

It is known that any bounded closed convex set from a reflexive Banach space Z is weakly compact. So the set $M(v)$ is weakly compact set.

We choose an arbitrary minimizing sequence $z_k \in M(v)$

$$\lim_{k\to\infty} \Omega[z_k] = \Omega^*.$$

Since the set $M(v)$ is weakly compact, it follows that there is at least one subsequence $z_{k_m} \in M(v)$ that weakly converges to some point z^*

$$z_{k_m} \longrightarrow z^* \text{ as } m \to \infty,\ z^* \in M(v).$$

The following inequality is valid

$$\Omega^* \leq \Omega[z^*] \leq \varliminf_{k\to\infty} \Omega[z_{k_m}] = \lim_{k\to\infty} \Omega[z_k] = \Omega^*, \Omega^* = \Omega[z^*].$$

□

Theorem 3.2. *Let us assumed that system* (1.1) *has only one state variable $\tilde{x}_1(t)$, which is obtained by experiment, and there is only one unknown component $z_j(t), 1 \leq j \leq m$ of vector function of external loads. The matrix $F = \hat{F} = f_{k,i}, 1 \leq k \leq n, 1 \leq i \leq m$, has the special form: by fixed $l, j\,(1 \leq j \leq m, 2 \leq l \leq m)$ element $\hat{f}_{lj} \neq 0, \hat{f}_{ki} = 0, k \neq l, i \neq j$. Suppose that Z is a Gilbert functional space, that the functional $\Omega[z]$ is strongly convex and lower semi continuous on $Q_{\delta,p,j}$, that the Lebesgue set $M(v)$ is bounded for a certain function from $v \in Q_{\delta,p,j}$: $M(v) = \{z \in Q_{\delta,p,j} : \Omega[z] \leq \Omega[v]\}$. Then the solution of the extreme problem* (3.1) *exists, unique and belongs to $Q_{\delta,p,j}$.*

Proof. According to Theorem 3.1, the set $Z^* = z \in Q_{\delta,p,j} : \Omega[z] = \Omega^*$ is not empty. Since the functional $\Omega[z]$ is strictly convex the set Z^* consists of a single point.

□

The choice of the function v is determined from physical considerations.

Thus, there is a principal possibility of constructing solutions of inverse synthesis problems on the basis of the variation principle. Satisfaction of additional conditions is determined by a special choice of the functional $\Omega[z]$. These solutions should be interpreted only as functions necessary for the subsequent mathematical modeling of physical processes in order to predict the behavior of physical processes, optimize the characteristics of these processes, etc.

In inverse problems of type II it is necessary to find such function from the set $Q_{\delta,p,j}$ which gives the useful information about exact solution of equation (2.1). However, such information cannot be obtained if the exact solution z_j^{ex} does not belong to the set $Q_{\delta,p,j}$.

Thus, it is necessary to take into account the error of operator $A_{p,j}$ in relation to the exact operator A_j^{ex}.

Let the characteristic of the deviation of the exact operator A_j^{ex} from the approximate operator $A_{p,j}$ be given [17] for linear operators $A_{p,j}$, A_j^{ex} in case U is normed functional space:

$$\| A_{p,j} - A_j^{ex} \|_{Z\to U} = \sup_{\|z\|\leq 1} \| A_{p,j}z - A_j^{ex}z \|_U \leq h_j. \tag{3.2}$$

A similar error characteristic can be determined in another way

$$\| A_{p,j} - A_j^{ex} \|_\Omega = \sup_{z \in Z_1} \frac{\rho_U(A_{p,j}z, A_j^{ex}z)}{\{\Omega[z]\}^{0.5}} \leq h_j \tag{3.3}$$

for fixed positive functional $\Omega[z]$ defined on $Z_1 \subset Z, Z_1$ everywhere dense in Z.

In this case the set of possible solutions of (2.1) $Q_{\delta,h,p,j}$ should be expanded so that the exact solution z_j^{ex} belongs to it with guarantee $z_j^{ex} \in Q_{\delta,h,p,j}$ [17]:

$$Q_{\delta,h,p,j} = \{z : \| A_{p,j}z - u_{\delta,j} \|_U \leq \delta_j + h_j\|z\|_Z\}, \tag{3.4}$$

where h_j is the error characteristic of the operator $A_{p,j}$.

The sets $Q_{\delta,h,p,j}$ are not bounded at any δ and any h_j while operators $A_{p,j}$ are compact operators. So inverse problems of type II belong to class of incorrect problems [9], [17] and special methods are used for their solutions [9], [10], [11], [17], [6].

To this type of inverse problems should be attributed the problems of diagnosis in various areas of activity [5], definition of real properties of physical objects [12] and etc. The inverse problems with such ultimate goals will be called as the inverse problems of measurement (or interpretation) [9], [10].

Consider now the following extreme problem:

$$\Omega[z_{h_j,\delta_j,p}] = \inf_{z \in Q_{\delta,h,p,j} \cap Z_1} \Omega[z], \tag{3.5}$$

where functional $\Omega[z]$ has been defined on set $Z_1 \subset Z, Z_1$ everywhere dense in Z [17].

During solving the practical inverse problems there are a big difficulties of definition of the value h_j since the structure and parameters of the exact operator A_j^{ex} cannot be determined in principle. Consequently, the exact solution z_j^{ex} of the inverse problem does not belong in an expanded set of possible solutions $Q_{\delta,h,p,j}$ with a guarantee as a rule.

In addition, the approximate solution obtained in this way after substituted into equation (2.1), gives a big value of deviation from the experimental data which excludes an objective evaluation of the results of solving the inverse problem.

A certain different approach for solving of inverse measurement problems is proposed in the works [13], [18]. To obtain the useful information on the exact solution z_j^{ex}, a hypothesis is proposed [14]. Such an approach makes it possible to obtain objective estimates from below of the exact solution z_j^{ex} in the sense of a priori given functional $\Omega[z]$.

When solving the well known inverse problem of astrodynamics (the measurement problem), Adams and Leverier did not take into account the error of the operator $A_{p,j}$ [12]. Nevertheless, the solution was obtained which turned out to be quite accurate. This contradiction gives a strong impetus for the search of new methods for solving measurement inverse problems.

4. Conclusion

It is shown that the algorithms for finding approximate solutions of inverse synthesis problems and inverse measurement problems for differential equations can be created without assuming that an exact solution of inverse problem belongs to the set of possible solutions.

References

1. N. I. Akhieser, *Lectures on Approximation Theory*, Moscow, Nauka, 1965.
2. L. Beilina, M. V. Klibanov, *Synthesis of global convergence and adaptivity for a hyperbolic coefficient inverse problem in 3D*, Journal Inverse Ill-Posed Problems, (18) (2010),85–132.
3. V. Canndola, *Anomaly detection: A survey*, ACM Computing Surveys (CSUR), **41** (2008), 1-–15.
4. G. Dreyfus, *Neural networks: methodology and applications*, Springer Science and Business Media, USA, 2005.
5. Yu. Fan, Yu. Wejun, *Multi-fault diagnosis for industrial processes based on hybrid dynamic bayesian network*, Proceedings of the IASTED International Conference Modelling, Identification and Control (MIC 2013), 2013, February 11–13, Innsbruck, Austria, (2013), 501–506.
6. A. Goncharski, A. Leonov, A. Yagola, *On a regularizing algorithm for ill-posed problems with an approximately given operator*, Journal of Computational Mathematics and Mathematical Physics, **12** (6) (1972), 1592–1594.
7. C. I. Kabanichin, *Inverse and ill-posed problems*, Novosibirsk, Siberian Scientific Publishing House, 2009.
8. A. B. Kurzhansky, *Control and Observation under Conditions of Uncertainties*, Moscow, Nauka, 1977.
9. Yu. L. Menshikov, *Inverse problems for dynamic systems: classification and solution methods*, Journal Advances in Pure Mathematics, **3** 4 (2013), 390-–399.
10. Yu. L. Menshikov, *Inverse Problems of Synthesis: Basic Features*, Proc. of 8th International Conference on Inverse Problems, Kracow, Poland, 12–15 May, 2014, 107–108.
11. Yu. L. Menshikov, *Synthesis of Adequate Mathematical Description as Solution of Special Inverse Problems*, European Journal of Mathematical Science, **2** (3) (2013), 256–271.
12. Yu. L. Menshikov, *Inverse Problem of Astrodynamics*, World Journal Mechanics, **5** (2015), 249–256.
13. Yu. L. Menshikov, *Identification of external loads on mechanical systems by the regularization method*, The thesis abstract, Kharkov, Ukraine, 1978.
14. Yu. L. Menshikov, *One approach to solutions of measurement's inverse problems*, Journal Mathematical Inverse Problems, **1** (2) (2014), 71–85.
15. P. Nedyalko, I. Jordanov, *Gap analysis and optimization of Hydroprocessing prediction model*, Proceedings of the IASTED International Conference and 201 Applications Artificial Intelligence (AIA 3), February 11–13, Innsbruck, Austria, (2013), 7.
16. Fu. Shual, J. Yong, Xu. Shiqi, Zh. Kai, J. Yi, *Prediction of air pollution in Changchun based on OSR method*, World Journal of Modeling and Simulation, **13** (1) (2017), 12–18.

17. A. N. Tikhonov, V. Y. Arsenin, *Methods of incorrect problems solution*, Nauka, Moscow, 1979.
18. A. N. Tikhonov, *On the approximate functional equations of the first kind* , *Proc. of Mathematical Institute AS USSR*, **158** (1981), 197–202.
19. F. P. Vasilev, *Methods of Extreme Problems Solution*, Nauka, Moscow, 1981.

Received 06.02.2018