

Skin-Friction and Rate of Heat And Mass Transfer Effects on MHD Flow Past A Parabolic Flow Past An Infinite Isothermal Vertical Plate In The Presence of Thermal Radiation And Chemical Reaction

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Abstract: The Rate Heat and Mass transfer effects on a parabolic flow past an infinite isothermal vertical plate with chemical reaction is investigated. The dimensionless governing partial differential equations are solved by Laplace transform method. Exact analytical solutions satisfying the governing coupled partial differential equations and boundary conditions are obtained. Numerical evaluation of the analytical results is performed for the velocity, temperature and concentration profiles within the boundary layer and tabulated results for the skin friction coefficient, Nusselt Number and the Sherwood Number are presented and discussed.

Keywords: Parabolic, Isothermal, Vertical plate, skin-friction, Nusselt Number, Sherwood Number.

INTRODUCTION

Radiative heat and mass transfer effects plays an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines, various propulsion device for aircraft, combustion, furnace design, material processing, energy utilization, temperature measurements, remote sensing for astronomy, space exploration, food processing, as well as numerous agricultural health and military applications. The interaction of convection and radiation in absorbing- emitting media occurs in many practical cases, such as atmospheric phenomena, shock problems, racket nozzles, and industrial furnaces. If the temperature of the surrounding fluid is rather high, radiation effects plays an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal and mass diffusion.

The Study of heat and mass transfer with chemical reaction is of considerable importance in chemical process industries such as wire drawing, food processing and polymer production. Hydromagnetic convection plays an important role in petroleum industries, geophysics and in astrophysics. It also finds applications in many engineering problems such as magneto hydrodynamic (MHD) generator, plasma studies, in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in the movement of earth's core.

The effect of a chemical reaction depends on whether the reaction is homogeneous or heterogeneous. This in turn depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to concentration. In many chemical engineering processes, there is the chemical reaction between a foreign mass and the fluid. These processes take place in numerous industrial applications such as manufacturing of ceramics, food processing and polymer production.

Chambre and Young [1] have analyzed the first order chemical reaction in the neighbourhood of a horizontal plate. Das *et al.*, [2] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, a mass transfer effect on moving isothermal vertical plate in the presence of chemical reaction was studied by Das *et al.*, [3]. The dimensionless governing equations were solved by the usual Laplace- transform technique and the solutions are valid only at lower time level.

England and Emery [4] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effects on mixed convection along a isothermal vertical plate were studied by

Hossain and Takhar [5]. The governing equations were solved analytically. Das *et al.*, [6] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Gupta *et al.*, [7] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [8] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [9]. Basant Kumar Jha *et al.*, [10] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Muthucumaraswamy and Ganesan [11] studied the effects of a first order homogeneous chemical reaction on the flow past an impulsively started semi-infinite vertical plate with uniform heat flux and mass diffusion. The governing equations were solved numerically. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [12].

Hence it is proposed to study the effect of MHD and radiative heat transfer effects on unsteady flow past a parabolic isothermal infinite vertical plate in the presence of chemical reaction and thermal radiation. Two problems are discussed here. The first problem deals with the chemical reaction effects on parabolic isothermal infinite vertical plate in the presence of chemical reaction and thermal radiation. The second problem discusses the MHD effects on vertical plate with uniform mass diffusion in the presence of thermal radiation and chemical reaction. Here the solutions are obtained by Laplace transform technique and the solutions are in terms of exponential and complementary error functions. The solutions are implemented MATLAB and the variations with respect to the different physical aspects of the problem are discussed.

FLOW ANALYSIS OF THE PROBLEM

The homogeneous first order chemical reaction between the fluid and species concentration to study radiation effects on unsteady parabolic MHD free convection flow of a viscous incompressible, electrically, conducting, radiating fluid past an infinite vertical plate with uniform mass diffusion in the presence of transverse applied magnetic field. The X-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_{∞} and concentration C'_{∞} . At time t' > 0, the plate is subjected to a velocity $u = u_0 t'^2$ in its own plane against gravitational field. The temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w . A chemically reactive species which transforms according to a simple reaction involving the concentration is emitted from the plate and diffuses into the fluid. The plate is also subjected to a uniform magnetic field of strength B_0 which is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small. Then under the above assumptions, the governing boundary layer equations of momentum with magnetic field, energy and mass diffusion with chemical reaction for free convective flow with usual Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_{\infty}) + g\beta^{*}(C' - C'_{\infty}) + v\frac{\partial^{2}u}{\partial y^{2}} - \frac{\sigma B_{0}^{2}}{\rho}u \dots (2.1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \qquad (2.2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - k_1 (C' - C'_{\infty}) \dots (2.3)$$

With the following initial and boundary conditions:

$$u = 0,$$
 $T = T_{\infty},$ $C' = C'_{\infty}$ for all $y, t' \le 0$

$$t' > 0: u = u_0 t'^2, \quad T = T_w, \quad C' = C'_w \quad \text{at} \quad y = 0 \dots (2.4)$$

 $u \to 0 \qquad T \to T_w, \quad C' \to C'_w \quad \text{as} \quad y \to \infty$

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On introducing the following non-dimensional quantities:

The equations (2.1) to (2.3) reduces to the following dimensionless form

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \dots (2.6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr} \theta \dots (2.7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC \dots (2.8)$$

The corresponding initial and boundary conditions in dimensionless form are as follows: $U = 0, \ \theta = 0, \ C = 0$ for all $Y, t \le 0$

$$t > 0$$
: $U = t^2$, $\theta = 1$, $C = 1$ at $Y = 0$(2.9)

$$U \to 0, \ \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty$$

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SOLUTION PROCEDURE ERROR FUNCTIONS

During the process of finding inverse Laplace transform, we frequently encountered error functions (also known as Gauss error function) and complementary error functions of real arguments and it is defined as follows

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-u^2} du$$
 and $erfc(x) = 1 - erf(x)$

The dimensionless governing equations (2.6) to (2.8) and the corresponding initial and boundary conditions (2.9) are tackled using the Laplace transform technique and the solutions are derived as follows:

$$\begin{split} & U = 2 \Biggl[\frac{(\eta^2 + Mt)t}{4M} \Biggl[\exp(2\eta\sqrt{Mt}) \operatorname{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta\sqrt{Mt}) \operatorname{erfc}(\eta - \sqrt{Mt}) \Biggr] \\ & + \frac{\eta\sqrt{t}(1 - 4Mt)}{8M^{\frac{3}{2}}} \Biggl[\exp(-2\eta\sqrt{Mt}) \operatorname{erfc}(\eta - \sqrt{Mt}) - \exp(2\eta\sqrt{Mt}) \operatorname{erfc}(\eta + \sqrt{Mt}) \Biggr] \\ & - \frac{\eta t}{2M\sqrt{\pi}} \exp(-(\eta^2 + Mt)) + d \Biggl[\frac{1}{2} \Biggl[\exp(2\eta\sqrt{Mt}) \operatorname{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta\sqrt{Mt}) \operatorname{erfc}(\eta - \sqrt{Mt}) \Biggr] \\ & - \frac{\exp(bt)}{2} \Biggl[\exp(2\eta\sqrt{(M+b)t}) \operatorname{erfc}(\eta + \sqrt{(M+b)t}) + \exp(-2\eta\sqrt{Mt}) \operatorname{erfc}(\eta - \sqrt{(M+b)t}) \Biggr] \\ & - \frac{1}{2} \Biggl[\exp(2\eta\sqrt{Pr(at)}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Pr(at)}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{at}) \Biggr] \\ & - \frac{1}{2} \Biggl[\exp(2\eta\sqrt{Pr(a+b)t}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{(a+b)t}) + \exp(-2\eta\sqrt{Pr(a+b)t}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{(a+b)t}) \Biggr] \Biggr] \\ & + \exp(-2\eta\sqrt{Pr(a+b)t}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{(a+b)t}) \Biggr] \\ & + \exp\left(-2\eta\sqrt{Pr(a+b)t}\right) \operatorname{erfc}(\eta + \sqrt{Mt}) + \exp\left(-2\eta\sqrt{Mt}\right) \operatorname{erfc}(\eta - \sqrt{Mt}) \Biggr] \\ & - \frac{\exp(ct)}{2} \Biggl[\Biggl[\exp\left(2\eta\sqrt{Mt}\right) \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp\left(-2\eta\sqrt{Sc}Kt\right) \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) \Biggr] \\ & + \exp\left(-2\eta\sqrt{Sc}(K+c)t\right) \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{(K+c)t}) \Biggr] \Biggr]$$

Where,

$$a = \frac{R}{\Pr}$$
, $b = \frac{R-M}{1-\Pr}$, $c = \frac{ScK-M}{1-Sc}$, $d = \frac{Gr}{b(1-\Pr)}$, $e = \frac{Gc}{c(1-Sc)}$ and $\eta = \frac{y}{2\sqrt{t}}$

DISCUSSION OF RESULTS

In order to get the physical insight of the problem, the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number are computed for various values of Prandtl number, Schmidt number, thermal Grashof number and mass Grashof number. The values of Prandtl number is chosen in such a way that, it represents air (Pr = 0.73), water (Pr = 7).For this purpose the values of Schmidt number Sc are chosen to represent various species at low concentration in air. Schmidt numbers under consideration are Hydrogen (Sc = 0.16), water vapor (Sc = 0.60).

SKIN-FRICTION

Now we study skin-friction from velocity field. It is given in non-dimensional form as

From equations (2.12) and (2.13), we get skin-friction as follows

$$\tau = \left(t^{2} + 2A + 2B\right) \left[\sqrt{M} \operatorname{erf}\left(\sqrt{Mt}\right) + \frac{1}{\sqrt{\pi t}} e^{-Mt}\right]$$
$$-\left[\frac{1}{4M\sqrt{M}} - \frac{t}{\sqrt{M}}\right] \operatorname{erf}\left(\sqrt{Mt}\right) + \frac{\sqrt{t}}{2M\sqrt{\pi}} e^{-Mt}$$
$$-2Ae^{bt} \left[\left(\sqrt{M+b}\right)\operatorname{erf}\left(\sqrt{(M+b)t}\right) + \frac{1}{\sqrt{\pi t}} e^{-(M+b)t}\right]$$
$$+2Ae^{bt} \left[\left(\sqrt{\operatorname{Pr}(a+b)}\right)\operatorname{erf}\left(\sqrt{(a+b)t}\right) + \frac{\sqrt{\operatorname{Pr}}}{\sqrt{\pi t}} e^{-(a+b)t}\right]$$
$$-2Be^{ct} \left[\left(\sqrt{M+c}\right)\operatorname{erf}\left(\sqrt{(M+c)t}\right) + \frac{1}{\sqrt{\pi t}} e^{-(M+c)t}\right]$$
$$+2Be^{ct} \left[\left(\sqrt{Sc(K+c)}\right)\operatorname{erf}\left(\sqrt{(K+c)t}\right) + \frac{\sqrt{Sc}}{\sqrt{\pi t}} e^{-(K+c)t}\right]$$
$$-2A \left[\sqrt{\operatorname{Rerf}}\left(\sqrt{at}\right) + \frac{\sqrt{\operatorname{Pr}}}{\sqrt{\pi t}} e^{-at}\right] - 2B \left[\sqrt{\operatorname{ScK}\operatorname{erf}}\left(\sqrt{Kt}\right) + \frac{\sqrt{Sc}}{\sqrt{\pi t}} e^{-Kt}\right]$$

Where,

$$a = \frac{R}{\Pr}$$
, $b = \frac{R-M}{1-\Pr}$, $c = \frac{ScK-M}{1-Sc}$, $A = \frac{Gr}{2b(1-\Pr)}$, $B = \frac{Gc}{2c(1-Sc)}$,

From Table-1, it is observed that skin-friction value increases with increasing value of thermal Grashof number or mass Grashof number or radiation parameter or Schmidt number and time. It also observed that the skin-friction values increases with decreasing values of the magnetic field parameter and chemical reaction parameter.

М	Gr	Gc	K	R	Sc	t	τ
2	2	2	2	5	0.16	0.2	5.0252
2	2	2	2	5	0.3	0.2	5.0858
2	2	2	2	5	0.6	0.2	5.1593

Table-1: Values of skin friction for air (Pr = 0.71)

2	2	2	4	5	0.16	0.2	4.9372
2	2	2	6	5	0.16	0.2	4.7703
2	2	2	8	5	0.16	0.2	4.4215
2	5	5	2	5	0.16	0.2	11.6118
2	10	10	2	5	0.16	0.2	22.5894
0.8	2	2	2	5	0.16	0.2	7.5425
0.6	2	2	2	5	0.16	0.2	7.9136
0.2	2	2	2	5	0.16	0.2	11.1342
2	2	2	2	5	0.16	0.4	6.2352
2	2	2	2	10	0.16	0.2	41.3841

NUSSELT NUMBER

From temperature field, now we study Nusselt number (rate of change of heat transfer) which is given in nondimensional form as

From equations (2.10) and (2.14), the Nusselt number calculated as

$$Nu = \left\lfloor \sqrt{R} erf\left(\sqrt{at}\right) + \frac{\sqrt{\Pr}}{\sqrt{\pi t}} e^{-at} \right\rfloor$$

Table-2 shows that the variation in temperature profile with respect to y at y = 0 (Nusselt number). It is noticed that the variation in Nusselt number for different values of time and Prandtl number in the presence of radiation parameter. It is observed that for different values of time (t = 0.2, 0.4, 0.6) when time increases, Nusselt number also decreases. In the case of Prandtl number, for different values of Prandtl number (Pr = 0.71, 7) when Nusselt number increases at time (t = 0.2, 0.4, 0.6) and R = 2. It also explains that the radiation parameter enhances for the increasing values of nusselt number in the presence or air (Pr = 0.71)

Table-2 Nusselt number for different t, Pr and R

t	Pr	R	Nu
0.2	0.71	2	1.6114
0.4	0.71	2	1.4693
0.6	0.71	2	1.4341
0.2	7	2	3.5267
0.4	7	2	2.6249
0.6	7	2	2.2483
0.2	0.71	5	2.2874
0.2	0.71	10	3.1701

SHERWOOD NUMBER

From concentration field, now we study Sherwood number (rate of change of mass transfer) which is given in non-dimensional form as

$$Sh = -\left(\frac{\partial C(y,t)}{\partial y}\right)_{y=0}$$
(2.15)

From equations (2.11) and (2.15), we get Sherwood number as follows

$$Sh = \left\lfloor \sqrt{ScK} \, erf\left(\sqrt{Kt}\right) + \frac{\sqrt{Sc}}{\sqrt{\pi t}} \, e^{-Kt} \, \right\rfloor$$

The numerical values of Sherwood number is presented in Table-3. From this table, we conclude that the Sherwood number increases with an increase in Schmidt number (Sc) and Chemical reaction parameter (K). It is also found that that Sherwood number decreases with an increase in t.

t	Sc	K	Sh	
0.2	0.16	0.2	0.5247	
0.2	0.3	0.2	0.7184	
0.2	0.6	0.2	1.0160	
0.2	0.16	2	0.6940	
0.2	0.16	4	0.8620	
0.2	0.16	8	1.1499	
0.4	0.16	0.2	0.6095	
0.6	0.16	0.2	0.5848	
0.8	0.16	0.2	0.5750	

Table-3: Sherwood number for different t, Sc and K

CONCLUSION

The Theoretical solution of flow past a parabolic infinite vertical plate with uniform mass diffusion, in the presence of (i) chemical reaction and (ii) chemical reaction and magnetic field are studied. The dimensionless governing equations are solved using Laplace transform technique. The effects of the velocity, the concentration, the temperature, skin-friction, heat transfer coefficient and mass transfer coefficient for different parameters like Gr, Gc, Sc, Pr, R, K, M and t are studied. The conclusions of the study are as follows:

- Skin-friction value increases with increasing value of thermal Grashof number or mass Grashof number or radiation parameter or Schmidt number and time. It also observed that the skin-friction values increase with decreasing values of the magnetic field parameter and chemical reaction parameter.
- The Sherwood number increases with an increase in Schmidt number (Sc) and Chemical reaction parameter (K).

LIST OF NOMENCLATURE

- A =Constants
- B_0 =external magnetic field
- C' =species concentration in the fluid $kg m^{-3}$
- C'_{w} = concentration of the fluid near the plate
- C'_{∞} =concentration in the fluid far away from the plate

- *C* =dimensionless concentration
- C_p =specific heat at constant pressure $J.kg^{-1}.k$
- D =mass diffusion coefficient $m^2 . s^{-1}$
- *Gc* =mass Grashof number
- *Gr* =thermal Grashof number
- g =acceleration due to gravity $m.s^{-2}$
- k = thermal conductivity $W.m^{-1}.K^{-1}$
- K_l =chemical reaction parameter
- K =dimensionless chemical reaction parameter
- M =magnetic field parameter
- R =thermal radiation parameter
- Pr =Prandtl number
- Sc =Schmidt number
- T =temperature of the fluid near the plate K
- Sh =Sherwood number
- Nu =Nusselt number
- T_w =temperature of the fluid near the plate
- T_{∞} =temperature of the fluid far away from the plate
- t' =time s
- u =velocity of the fluid in the x'-direction $m.s^{-1}$
- u_0 =velocity of the plate $m.s^{-1}$
- U =dimensionless velocity
- x =spatial coordinate along the plate
- y =coordinate axis normal to the plate m
- Y =dimensionless coordinate axis normal to the plate

Greek symbols

- α =thermal diffusivity
- β =volumetric coefficient of thermal expansion K^{-1}
- β^* =volumetric coefficient of expansion with concentration K^{-1}
- μ =coefficient of viscosity *Ra.s*
- V =kinematic viscosity $m^2 . s^{-1}$
- ρ =density of the fluid kg.m⁻³
- τ =dimensionless skin-friction $kg.m^{-1}.s^2$
- θ =dimensionless temperature
- η =similarity parameter
- *erfc* =complementary error function

Subscripts

- *W* =conditions at the wall
- ∞ =free stream conditions

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