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NONLINEAR ASYMMETRIC KELVIN-HELMHOLTZ INSTABILITY OF CYLINDRICAL FLOW WITH MASS AND HEAT TRANSFER AND THE VISCOUS LINEAR ANALYSIS

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Abstract

The nonlinear asymmetric Kelvin-Helmholtz stability of the cylindrical interface between the vapor and liquid phases of a fluid is studied when the phases are enclosed between two cylindrical surfaces coaxial with the interface, and when there is mass and heat transfer across the interface. The method of multiple time expansion is used for the investigation. The evolution of amplitude is shown to be governed by a nonlinear first order differential equation. The stability criterion is discussed, and the region of stability is displayed graphically. Also investigated in this paper is the viscous linear potential flow.

Keywords Kelvin-Helmholtz stability, Mass and heat Transfer, Cylindrical flow.[0]

1. Introduction

In dealing with flow of two fluids divided by an interface, the problem of interfacial stability is usually studied with the neglect of heat and mass transfer across the interface. However, there are situations when the effect of mass and heat transfer across the interface should be taken into account in stability discussions. For instance, the phenomenon of boiling accompanies high heat and mass transfer rates which are significant in determining the flow field and the stability of the system.

Hsieh [1] presented a simplified formulation of interfacial flow problem with mass and heat transfer, and studied the problems of Rayleigh-Taylor and Kelvin-Helmholtz stability in plane geometry.

The mechanism of heat and mass transfer across an interface is important in various industrial applications such as design of many types of contacting equipment, e.g., boilers, condensers, pipelines, chemical reactors, and nuclear reactors, etc.

In the nuclear reactor cooling of fuel rods by liquid coolants, the geometry of the system in many cases is cylindrical. We have, therefore, considered the interfacial stability problem of a cylindrical flow with mass and heat transfer. Nayak and Chakraborty[2] studied the Kelvin-Helmholtz stability of the cylindrical interface between the vapor and liquid phases of a fluid, when there is a mass and heat transfer across the interface, while Elhefnawy[3] studied the effect of a periodic radial magnetic field on the Kelvin-Helmholtz stability of the cylindrical interface between two magnetic fluids when there is mass and heat transfer across the interface. The analysis of these studies was confined within the frame work of linear theory. They both found that the dispersion relations are independent of the rate of interfacial mass and heat transfer. Hsieh[4] found that from the linearized analysis, when the vapor region is hotter than the liquid region, as is usually so, the effect of mass and heat transfer tends to inhibit the growth of the instability. Thus for the problem of film boiling, the instability would be reduced yet would persist according to linear analysis.

It is clear that such a uniform model based on the linear theory is inadequate to answer the question of whether and how the effect of heat and mass transfer would stabilize the system, but the nonlinear analysis is needed to answer the question.

The purpose of this paper is to investigate the Kelvin-Helmholtz asymmetric nonlinear stability of cylindrical interface between the vapor and liquid phases of a fluid when there is a mass and heat transfer across the interface.

The nonlinear problem of Rayleigh-Taylor instability of a system in a cylindrical geometry is, however, studied by the present author in (Lee[5-6]).

The multiple time scale method is used to obtain a first order nonlinear differential equation, from which conditions for the stability and instability are determined.

In more recent years,Awashi, Asthana and Zuddin[7] considered a problem in which a viscous potential flow theory is used to study the nonlinear Kelvin-Helmholtz instability of the interface between two viscous ,incompressible and thermally conducting fluids.

The basic equations with the accompanying boundary conditions are given in Sec.2. The first order theory and the linear dispersion relation are obtained in Sec.3. In Sec .4 we have derived second order solutions. In Sec.5 a first order nonlinear differential equation is obtained, and the situations of the stability and instability are summarized. In Sec.6 we investigate linear viscous potential flow. In Sec.7 some numerical examples are presented.

2. Formulation of the problem and basic equations[0]

We shall use a cylindrical system of coordinates (r, θ, z) so that in the equilibrium state $z-$axis is the axis of asymmetry of the system. The central solid core has a radius a. In the equilibrium state the fluid phase "1", of density $\rho^{(1)}$, occupies the region $a < r < R$, and, the fluid phase "2", of density $\rho^{(2)}$, occupies the region $R < r < b$. The inner and outer fluids are streaming along the z axis with uniform velocities U_1 and U_2, respectively. The temperatures at $r = a, r = R$, and $r = b$ are taken as T_1, T_0, and T_2 respectively. The bounding surfaces $r = a$, and $r = b$ are taken as rigid. The interface, after a disturbance, is given by the equation

$$F(r, z, t) = r - R - \eta(\theta, z, t) = 0, \qquad (2.1)$$

where η is the perturbation in radius of the interface from its equilibrium value R, and for which the outward normal vector is written as

$$\mathbf{n} = \frac{\nabla F}{|\nabla F|} = \left\{1 + \left(\frac{1}{r}\frac{\partial \eta}{\partial \theta}\right)^2 + \left(\frac{\partial \eta}{\partial z}\right)^2\right\}^{-1/2} \times \left(\mathbf{e}_r - \frac{1}{r}\frac{\partial \eta}{\partial \theta}\mathbf{e}_\theta - \frac{\partial \eta}{\partial z}\mathbf{e}_z\right), \qquad (2.2)$$

we assume that fluid velocity is irrotational in the region so that velocity potentials are $\phi^{(1)}$ and $\phi^{(2)}$ for fluid phases 1 and 2. In each fluid phase

$$\nabla^2 \phi^{(j)} = 0. \quad (j = 1, 2) \qquad (2.3)$$

The solutions for $\phi^{(j)} (j = 1, 2)$ have to satisfy the boundary conditions. The relevant boundary conditions for our configuration are

(i) On the rigid boundaries $r = a$ and $r = b$:

The normal field velocities vanish on both central solid core and the outer bounding surface.

$$\frac{\partial \phi^{(1)}}{\partial r} = 0 \qquad \text{on} \quad r = a, \tag{2.4}$$

$$\frac{\partial \phi^{(2)}}{\partial r} = 0 \qquad \text{on} \quad r = b, \tag{2.5}$$

(ii) On the interface $r = R + \eta(\theta, z, t)$:

(1) The conservation of mass across the interface:

$$\left[\!\left[\rho \left(\frac{\partial F}{\partial t} + \nabla \phi \cdot \nabla F \right) \right]\!\right] = 0,$$

$$\text{or} \quad \left[\!\left[\rho \left(\frac{\partial \phi}{\partial r} - \frac{\partial \eta}{\partial t} - \frac{1}{r}\frac{\partial \eta}{\partial \theta}\frac{\partial \phi}{\partial \theta} - \frac{\partial \eta}{\partial z}\frac{\partial \phi}{\partial z} \right) \right]\!\right] = 0, \tag{2.6}$$

where $[\![h]\!]$ represents the difference in a quantity as we cross the interface,*i.e.,* $[\![h]\!] = h^{(2)} - h^{(1)}$, where superscripts refer to upper and lower fluids, respectively.

(2) The interfacial condition for energy is

$$L\rho^{(1)} \left(\frac{\partial F}{\partial t} + \nabla \phi^{(1)} \cdot \nabla F \right) = S(\eta), \tag{2.7}$$

where L is the latent heat released when the fluid is transformed from phase 1 to phase 2. Physically, the left-hand side of (2.7) represents the latent heat released during the phase transformation, while $S(\eta)$ on the right-hand side of (2.7) represents the net heat flux, so that the energy will be conserved.

In the equilibrium state, the heat fluxes in the direction of r increasing in the fluid phase 1 and 2 are $-K_1(T_1 - T_0)/R\log(a/R)$ and $-K_2(T_0 - T_2)/R\log(R/b)$, where K_1 and K_2 are the heat conductivities of the two fluids. As in Hsieh(1978), we denote

$$S(\eta) = \frac{K_2(T_0 - T_2)}{(R+\eta)(\log b - \log(R+\eta))} - \frac{K_1(T_1 - T_0)}{(R+\eta)(\log(R+\eta) - \log a)}, \tag{2.8}$$

and we expand it about $r = R$ by Taylor's expansion, such as

$$S(\eta) = S(0) + \eta S'(0) + \frac{1}{2}\eta^2 S''(0) + \cdots, \tag{2.9}$$

and we take $S(0) = 0$, so that

$$\frac{K_2(T_0 - T_2)}{R\log(b/R)} = \frac{K_1(T_1 - T_0)}{R\log(R/a)} = G(\text{say}), \tag{2.10}$$

indicating that in equilibrium state the heat fluxes are equal across the interface in the two fluids.

From (2.1), (2.7), and (2.9), we have

$$\rho^{(1)} \left(\frac{\partial \phi^{(1)}}{\partial r} - \frac{\partial \eta}{\partial t} - \frac{1}{r}\frac{\partial \eta}{\partial \theta}\frac{\partial \phi^{(1)}}{\partial \theta} - \frac{\partial \eta}{\partial z}\frac{\partial \phi^{(1)}}{\partial z} \right) = \alpha(\eta + \alpha_2\eta^2 + \alpha_3\eta^3), \tag{2.11}$$

where

$$\alpha = \frac{G\log(b/a)}{LR\log(b/R)\log(R/a)},$$

$$\alpha_2 = \frac{1}{R}\left(-\frac{3}{2} + \frac{1}{\log(b/R)} - \frac{1}{\log(R/a)} \right),$$

$$\alpha_3 = \frac{1}{R^2}\left[\frac{11}{6} - \frac{2\log(R^2/ab)}{\log(b/R)\log(R/a)} + \frac{\log^3(b/R) + \log^3(R/a)}{\{\log(b/R)\log(R/a)\}^2\log(b/a)} \right].$$

(3) The conservation of momentum balance, by taking into account the mass transfer across the interface, is

$$\rho^{(1)}(\nabla\phi^{(1)} \cdot \nabla F)\left(\frac{\partial F}{\partial t} + \nabla \phi^{(1)} \cdot \nabla F \right) = \rho^{(2)}(\nabla\phi^{(2)} \cdot \nabla F)\left(\frac{\partial F}{\partial t} + \nabla \phi^{(2)} \cdot \nabla F \right) + (p_2 - p_1 + \sigma\nabla\cdot\mathbf{n})|\nabla F|^2, \tag{2.12}$$

where p is the pressure and σ is the surface tension coefficient, respectively.[0] By eliminating the pressure by Bernoulli's equation we can rewrite the above condition (2.12) as

$$\left[\!\!\left[\rho\left\{\frac{\partial\phi}{\partial t}+\frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2+\frac{1}{2}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)^2+\frac{1}{2}\left(\frac{\partial\phi}{\partial z}\right)^2-\left\{1+\left(\frac{1}{r}\frac{\partial\eta}{\partial\theta}\right)^2+\left(\frac{\partial\eta}{\partial z}\right)^2\right\}^{-1}\right.\right.\right.$$
$$\left.\left.\left.\times\left(\frac{\partial\phi}{\partial z}\frac{\partial\eta}{\partial z}+\frac{1}{r^2}\frac{\partial\phi}{\partial\theta}\frac{\partial\eta}{\partial\theta}-\frac{\partial\phi}{\partial r}\right)\left(\frac{\partial\eta}{\partial t}+\frac{\partial\phi}{\partial z}\frac{\partial\eta}{\partial z}+\frac{1}{r^2}\frac{\partial\phi}{\partial\theta}\frac{\partial\eta}{\partial\theta}-\frac{\partial\phi}{\partial r}\right)\right\}\right]\!\!\right]$$
$$=\frac{\sigma}{(R+\eta)|\nabla F|}\left\{1+\left(\frac{1}{r}\frac{\partial\eta}{\partial\theta}\right)^2\frac{2}{|\nabla F|^2}\right\}$$
$$-\frac{\sigma}{|\nabla F|^3}\left[\frac{\partial^2\eta}{\partial z^2}\left\{1+\left(\frac{1}{r}\frac{\partial\eta}{\partial\theta}\right)^2\right\}-\frac{2}{r^2}\frac{\partial\eta}{\partial\theta}\frac{\partial^2\eta}{\partial\theta\partial z}\frac{\partial\eta}{\partial z}+\frac{1}{r^2}\frac{\partial^2\eta}{\partial\theta^2}\left\{1+\left(\frac{\partial\eta}{\partial z}\right)^2\right\}\right]. \tag{2.13}$$

When the interface is perturbed from the equilibrium $\eta=0$ to $\eta=A\exp[i(kz+m\theta-\omega t)]$, the dispersion relation for the linearized problem is

$$D(\omega,k,m)=a_0\omega^2+(a_1+ib_1)\omega+a_2+ib_2=0, \tag{2.14}$$

where

$$\begin{aligned}
a_0 &= \rho^{(1)}E_m^{(1)}-\rho^{(2)}E_m^{(2)},\\
a_1 &= 2k\{\rho^{(2)}E_m^{(2)}U_2-\rho^{(1)}E_m^{(1)}U_1\},\\
b_1 &= \alpha\{E_m^{(1)}-E_m^{(2)}\},\\
a_2 &= k^2\{\rho^{(1)}E_m^{(1)}U_1^2-\rho^{(2)}E_m^{(2)}U_2^2\}\\
&\quad -\frac{\sigma}{R^2}(R^2k^2+m^2-1),\\
b_2 &= \alpha k\{E_m^{(2)}U_2-E_m^{(1)}U_1\},
\end{aligned}$$

where for the simplicity of notation, we used

$$E_m^{(j)}=E_m^{(j)}(k,R),\qquad (j=1,2)$$

where $E_m^{(j)}(k,R),(j=1,2)$ are explained by (3.4)-(3.5). (i) When $\alpha=0$, (2.14) reduces to

$$a_0\omega^2+a_1\omega+a_2=0. \tag{2.15}$$

Therefore the system is stable if

$$a_1^2-4a_0a_2>0, \tag{2.16}$$

or

$$\frac{\sigma}{R^2}(R^2k^2+m^2-1)$$
$$+k^2\frac{\rho^{(1)}\rho^{(2)}E_m^{(1)}E_m^{(2)}(U_2-U_1)^2}{\rho^{(1)}E_m^{(1)}-\rho^{(2)}E_m^{(2)}}>0. \tag{2.17}$$

It is clear from the above inequality that the streaming has a destabilizing effect on the stability of a cylindrical interface, because $E_m^{(2)}$ is always negative from the properties of Bessel functions. (ii) when $\alpha\neq 0$, we find that necessary and sufficient stability conditions for (2.14) are [3]

$$b_1>0, \tag{2.18}$$

and

$$a_0b_2^2-a_1b_1b_2+a_2b_1^2<0, \tag{2.19}$$

since a_0 is always positive.[0]

Putting the values of a_0,a_1,a_2,b_1 and b_2 from(2.14) into(2.18) and(2.19) we notice that the condition (2.18) is trivially satisfied since α is always positive, and from properties of Bessel functions $E_m^{(2)}$ is always negative. From (2.19), it can be shown that the condition for the stability of the system is

$$\frac{\sigma}{R^2}(R^2k^2+m^2-1)+k^2\frac{\rho^{(1)}\rho^{(2)}E_m^{(1)}E_m^{(2)}(U_2-U_1)^2}{\rho^{(1)}E_m^{(1)}-\rho^{(2)}E_m^{(2)}}$$
$$\times\left[1-\frac{E_m^{(1)}E_m^{(2)}(\rho^{(1)}-\rho^{(2)})^2}{(E_m^{(1)}-E_m^{(2)})^2\rho^{(1)}\rho^{(2)}}\right]>0. \tag{2.20}$$

The stability condition (2.20) differs from (2.17) by the additional last term: $E_m^{(1)}E_m^{(2)}(\rho^{(1)}-\rho^{(2)})^2/[\rho^{(1)}\rho^{(2)}(E_m^{(1)}-E_m^{(2)})^2]$. Thus the condition (2.20) is valid for infinitesimal α and when $\alpha=0$ the last term is absent.

We now employ multiscale expansion near the critical wave number. The critical wave number is attained when $a_2 = b_2 = 0$. The corresponding critical frequency, ω_c is zero for this case.

Introducing ϵ as a small parameter, we assume the following expansion of the variables:

$$\eta = \sum_{n=1}^{3} \epsilon^n \eta_n(\theta, z, t_0, t_1, t_2) + O(\epsilon^4), \qquad (2.21)$$

$$\phi^{(j)} = \sum_{n=0}^{3} \epsilon^n \phi_n^{(j)}(r, \theta, z, t_0, t_1, t_2) + O(\epsilon^4), (j = 1, 2) \qquad (2.22)$$

where $t_n = \epsilon^n t (n = 0, 1, 2)$.[0] The quantities appearing in the field equations (2.3) and the boundary conditions (2.6), (2.11), and (2.13) can now be expressed in Maclaurin series expansion around $r = R$. Then, we use (2.21), and (2.22) and equate the coefficients of equal power series in ϵ to obtain the linear and the successive nonlinear partial differential equations of various orders.

To solve these equations in the neighborhood of the linear critical wave number k_c, because of the nonlinear effect, we assume that the critical wave number is shifted to

$$k = k_c + \epsilon^2 \mu.$$

3. First Order Solutions.

We take

$$\phi_0^{(j)} = U_j z. \qquad (j = 1, 2)$$

The first order solutions will reproduce the linear wave solutions for the critical case and the solutions of (2.3) subject to boundary conditions yield

$$\eta_1 = A(t_1, t_2) e^{i\vartheta} + \bar{A}(t_1, t_2) e^{-i\vartheta}, \qquad (3.1)$$

$$\phi_1^{(1)} = \left(\frac{\alpha}{\rho^{(1)}} + ikU_1 \right) A(t_1, t_2) E_m^{(1)}(k, r) e^{i\vartheta} + c.c., \qquad (3.2)$$

$$\phi_1^{(2)} = \left(\frac{\alpha}{\rho^{(2)}} + ikU_2 \right) A(t_1, t_2) E_m^{(2)}(k, r) e^{i\vartheta} + c.c., \qquad (3.3)$$

where

$$E_m^{(1)}(k, r) = \frac{I_m(kr) K'_m(ka) - I'_m(ka) K_m(kr)}{I'_m(kR) K'_m(ka) - I'_m(ka) K'_m(kR)}, \qquad (3.4)$$

$$E_m^{(2)}(k, r) = \frac{I_m(kr) K'_m(kb) - I'_m(kb) K_m(kr)}{I'_m(kR) K'_m(kb) - I'_m(kb) K'_m(kR)}, \qquad (3.5)$$

$$\vartheta = kz + m\theta, \quad I'_m(ka) = \frac{\partial}{\partial r} I_m(kr)\big|_{r=a}, \text{etc.}$$

with I_m and K_m are the modified Bessel functions of the first and second kinds, respectively.

4. Second order solutions.

With the use of the first order solutions , we obtained the equations for the second order problem

$$\nabla^2 \phi_2^{(j)} = 0, \qquad (j = 1, 2) \qquad (4.1)$$

and the boundary conditions at $r = R$.

$$\rho^{(j)} \left\{ \frac{\partial \phi_2^{(j)}}{\partial r} - \frac{\partial \eta_2}{\partial z} U_j \right\} - \alpha \eta_2 = \left[\rho^{(j)} \left\{ \frac{\alpha}{\rho^{(j)}} + iU_j \right\} \left\{ \frac{1}{R} - 2\left(k^2 + \frac{m^2}{R^2} \right) E_m^{(j)} \right\} + \alpha \alpha_2 \right]$$
$$\times A^2 e^{2i\vartheta} + \rho^{(j)} \frac{\partial A}{\partial t_1} e^{i\vartheta} + c.c. + 2\alpha \left(\frac{1}{R} + \alpha_2 \right) |A|^2, \qquad (j = 1, 2) \qquad (4.2-4.3)$$

$$\rho^{(2)} U_2 \frac{\partial \phi_2^{(2)}}{\partial z} - \rho^{(1)} U_1 \frac{\partial \phi_2^{(1)}}{\partial z} + \sigma \left(\frac{\partial^2 \eta_2}{\partial z^2} + \frac{1}{R^2} \frac{\partial^2 \eta_2}{\partial \theta^2} + \frac{\eta_2}{R^2} \right)$$
$$= -\frac{1}{2} \left\{ \left[\!\left[\rho \left(\frac{\alpha}{\rho} + ikU \right)^2 \left\{ -1 - \left(\frac{m^2}{R^2} + k^2 \right) E_m^2 \right\} + 3\alpha U k i - 2\rho U^2 k^2 \right]\!\right] \right.$$

$$+\frac{\sigma}{R^3}(R^2k^2+2-7m^2)\Big\}A^2e^{2i\vartheta}+\left[\!\!\left[\frac{\rho}{k}\left(\frac{\alpha}{\rho}+ikU\right)E_m\right]\!\!\right]\frac{\partial A}{\partial t_1}e^{i\vartheta}+c.c.$$

$$-\left\{\left[\!\!\left[\rho\left(\frac{\alpha^2}{\rho^2}+k^2U^2\right)\left\{-1+E_m^2\left(\frac{m^2}{R^2}+k^2\right)\right\}\right]\!\!\right]+\frac{\sigma}{R^3}(R^2k^2+m^2-2)\right\}|A|^2. \tag{4.4}$$

The non secularity condition for the existence of the uniformly valid solution is[0]

$$\frac{\partial A}{\partial t_1}=0. \tag{4.5}$$

Equations (4.1) to (4.4) furnish the second order solutions:

$$\eta_2=-2\left(\frac{1}{R}+\alpha_2\right)|A|^2+A_2A^2e^{2i\vartheta}+\bar{A}_2\bar{A}^2e^{-2i\vartheta}, \tag{4.6}$$

$$\phi_2^{(j)}=B_2^{(j)}A^2e^{2i\vartheta}E_{2m}^{(j)}(2k,r)+c.c.+b^{(j)}(t_0,t_1,t_2), \qquad (j=1,2) \tag{4.7}$$

where

$$A_2=\frac{1}{D(0,2k,2m)}\left\{\left[\!\!\left[-\rho i2kUE_{2m}\beta+\frac{\rho}{2}\left\{E_m^2\left(\frac{m^2}{R^2}+k^2\right)+1\right\}\left(\frac{\alpha}{\rho}+ikU\right)^2\right.\right.\right.$$

$$\left.\left.\left.+2\rho(kU)^2-i3\alpha kU\right]\!\!\right]+\frac{\sigma}{2R^3}(2+R^2k^2-7m^2)\right\}, \tag{4.8}$$

$$B_2^{(j)}=\beta^{(j)}+\left\{\frac{\alpha}{\rho^{(j)}}+2ikU_j\right\}A_2, \tag{4.9}$$

$$\beta^{(j)}=\left\{\frac{\alpha}{\rho^{(j)}}+ikU_j\right\}\left\{\frac{1}{R}-2E_m^{(j)}\left(\frac{m^2}{R^2}+k^2\right)\right\}+\frac{\alpha\alpha_2}{\rho^{(j)}}, \tag{4.10}$$

$$\rho^{(2)}\frac{\partial b^{(2)}}{\partial t_0}-\rho^{(1)}\frac{\partial b^{(1)}}{\partial t_0}=\left\{\left[\!\!\left[\rho\left(\frac{\alpha^2}{\rho^2}+k^2U^2\right)\left\{1-E_m^2(k,R)\left(\frac{m^2}{R^2}+k^2\right)\right\}\right]\!\!\right]\right.$$

$$\left.-\frac{\sigma}{R^3}\left(k^2R^2+m^2-4-2R\alpha_2\right)\right\}|A|^2, \tag{4.11}$$

where$E_{2m}^{(j)}=E_{2m}^{(j)}(2k,R)$.

5. Third order solutions

We examine now the third order problem:

$$\nabla_0^2\phi_3^{(i)}=0. \qquad (i=1,2) \tag{5.1}$$

On substituting the values of η_1, $\phi_1^{(i)}$ from (3.1)-(3.3) and η_2, $\phi_2^{(i)}$ from (4.6)-(4.7) into (A.7), we obtain

$$\phi_3^{(j)}=C_3^{(j)}E_{2m}^{(j)}(k,r)A^2\bar{A}e^{i\vartheta}+E^{(j)}(k,r)\frac{\partial A}{\partial t_2}e^{i\vartheta}+c.c., \tag{5.2}$$

where

$$C_3^{(j)}=-\left[\left\{E_{2m}^{(j)}2\left(\frac{m^2}{R^2}+k^2\right)-\frac{1}{R}\right\}B_2^{(j)}-2\left\{E_m^{(j)}\left(\frac{m^2}{R^2}+k^2\right)-\frac{1}{R}\right\}\left(\frac{\alpha}{\rho^{(j)}}+ikU_j\right)\right.$$

$$\times\left(\frac{1}{R}+\alpha_2\right)+\frac{1}{2}\left\{k^2+\frac{2+m^2}{R^2}-\frac{E_m^{(j)}}{R}\left(\frac{3m^2}{R^2}+k^2\right)\right\}\left(\frac{3\alpha}{\rho^{(j)}}+ikU_j\right)$$

$$+\left(\frac{\alpha}{\rho^{(j)}}-ikU_j\right)\left(\frac{2m^2}{R^3}E_m^{(j)}-\frac{m^2}{R^2}-k^2\right)+\frac{\alpha}{\rho^{(j)}}\left\{4\alpha_2\left(\frac{1}{R}+\alpha_2\right)-3\alpha_3\right\}$$
$$-\left\{\left(\frac{\alpha}{\rho^{(j)}}-ikU_j\right)\left\{E_m^{(j)}\left(\frac{m^2}{R^2}+k^2\right)+\frac{1}{R}\right\}+\frac{2\alpha\alpha_2}{\rho^{(j)}}\right\}A_2\Bigg]. \qquad (j=1,2) \tag{5.3}$$

We substitute the first- and second-order solutions into the third order equation. In order to avoid nonuniformity of the expansion, we again impose the condition that secular terms vanish. Then from (A.8), we find[0]

$$i\frac{\partial D(0,k,m)}{\partial\omega}\frac{\partial A}{\partial t_2}+\left\{2\sigma k_c\mu+\left[\!\!\left[\rho U\left(\frac{\alpha}{\rho}+ikU\right)E_m\right]\!\!\right]k_c i\mu\right\}A+qA^2\bar{A}=0, \tag{5.4}$$

where

$$q=\Bigg[\!\!\Bigg[\rho\Bigg(ikUC_3E_m+A_2\left(i\frac{\alpha}{\rho}3kU-k^2U^2\right)$$
$$+B_2\left(\frac{\alpha}{\rho}-iUk\right)\left\{2E_mE_{2m}\left(\frac{m^2}{R^2}+k^2\right)-1\right\}-i\left(\frac{5}{2R}+2\alpha_2\right)kU\left(\frac{\alpha}{\rho}+ikU\right)$$
$$+\frac{3}{R}\frac{\alpha^2}{\rho^2}-E_m\left(\frac{m^2}{R^2}+k^2\right)\frac{kU}{2}i\left(7\frac{\alpha}{\rho}+5kUi\right)+\frac{m^2}{R^2}E_m^2\left(\frac{\alpha^2}{\rho^2}+3k^2U^2-i2\frac{\alpha}{\rho}kU\right)\Bigg]\!\!\Bigg]$$
$$-\frac{\sigma}{R^4}\{(2A_2R-4-4R\alpha_2)(1-m^2)-2A_2R(m^2+k^2R^2)-\frac{3}{2}(m^2+k^2R^2)^2+\frac{1}{2}(9m^2+k^2R^2-6)\}. \tag{5.5}$$

We rewrite (5.4) as

$$\frac{\partial A}{\partial t_2}+(\tilde{a}_1+\tilde{a}_2|A|^2)A=0, \tag{5.6}$$

which can be easily integrated as

$$|A(t_2)|^2=a_{1r}|A_0|^2\exp(-2a_{1r}t)$$
$$\times[a_{1r}+a_{2r}|A_0|^2-a_{2r}|A_0|^2\exp(-2a_{1r}t)]^{-1}, \tag{5.7}$$

where A_0 is the initial amplitude and $a_{jr}=\Re\tilde{a}_j, (j=1,2)$.

With a finite initial value $|A_0|$, $|A|$ may become infinite when the denominator in (5.7) vanishes. Otherwise, $|A|$ will be asymptotically bounded. The situation can be summarized as follows:

(1) $a_{2r}>0$; stable.

(i) $a_{1r}>0$; $|A|^2\to 0$, as $t_2\to\infty$

(ii) $a_{1r}<0$; $|A|^2\to -a_{1r}/a_{2r}$, as $t_2\to\infty$

(2) If $a_{2r}<0$,

(i) $a_{1r}<0$; unstable.

(ii) $a_{1r}>0$, and $|A_0|^2>-a_{1r}/a_{2r}$: unstable.

(iii) $a_{1r}>0$, and $|A_0|^2<-a_{1r}/a_{2r}$: stable and $|A|^2\to 0$ as $t_2\to\infty$. Thus, a sufficient condition for stability is $a_{2r}>0$, which is due to the finite amplitude effect. The cylindrical system is nonlinearly stabile if $a_{1r}>0$ and the initial amplitude is sufficiently small.

6.Viscous asymmetric linear cylindrical flow

In this section we consider the viscous potential flow. For the viscous fluid, (2.12) is now replaced by

$$\rho^{(1)}(\nabla\varphi^{(1)}\cdot\nabla F)\left(\frac{\partial F}{\partial t}+\nabla\varphi^{(1)}\cdot\nabla F\right)$$
$$=\rho^{(2)}(\nabla\varphi^{(2)}\cdot\nabla F)\left(\frac{\partial F}{\partial t}+\nabla\varphi^{(2)}\cdot\nabla F\right)$$
$$+(p_2-p_1-2\mu_2\mathbf{n}\cdot\nabla\otimes\nabla\varphi^{(2)}\cdot\mathbf{n}$$
$$+2\mu_1\mathbf{n}\cdot\nabla\otimes\nabla\varphi^{(1)}\cdot\mathbf{n}+\sigma\nabla\cdot\mathbf{n})|\nabla F|^2, \tag{6.1}$$

where μ_1, μ_2 are viscosities of fluid '1' and '2', respectively and we modify (2.13) accordingly. The nonlinear analysis for the viscous fluid is too onerous when the perturbation is asymmetric , we are content here with the linear analysis. Then linearizing (2.6), (2.11) and (6.1) we have

$$\left[\!\left[\rho\left(\frac{\partial\phi}{\partial r}-\frac{\partial\eta}{\partial t}\right)-\frac{\partial\eta_1}{\partial z}U\right)\right]\!\right]=0, \tag{6.2}$$

0

$$\rho^{(1)}\left(\frac{\partial\phi^{(1)}}{\partial r}-\frac{\partial\eta}{\partial t}-\frac{\partial\eta}{\partial z}U\right)=\alpha\eta, \tag{6.3}$$

$$\left[\!\left[\rho\left(\frac{\partial\phi}{\partial t}+\frac{\partial\phi}{\partial z}U\right)+2\mu\frac{\partial^2\phi}{\partial r^2}\right]\!\right]$$

$$=-\sigma\left(\frac{\partial^2\eta}{\partial z^2}+\frac{\eta}{R^2}+\frac{1}{r^2}\frac{\partial^2\eta}{\partial\theta^2}\right). \tag{6.4}$$

When the interface is perturbed to $\eta = A\exp[i(kz+m\theta-\omega t)$, we recover the first order solutions (3.1)-(3.3), and the dispersion relation for the viscous fluid is same as (2.14), however

$$\begin{aligned}
a_0 &= \rho^{(1)}E_m^{(1)}-\rho^{(2)}E_m^{(2)},\\
a_1 &= 2k\{\rho^{(2)}E_m^{(2)}U_2-\rho^{(1)}E_m^{(1)}U_1\},\\
b_1 &= \alpha\{E_m^{(1)}-E_m^{(2)}\}+2(\mu_1E_t^{(1)}-\mu_2E_t^{(2)}),\\
a_2 &= k^2\{\rho^{(1)}E_m^{(1)}U_1^2-\rho^{(2)}E_m^{(2)}U_2^2\}\\
&\quad -\frac{\sigma}{R^2}(R^2k^2+m^2-1)\\
&\quad -2\alpha\left(\frac{\mu_1}{\rho^{(1)}}E_t^{(1)}-\frac{\mu_2}{\rho^{(2)}}E_t^{(2)}\right),\\
b_2 &= \alpha k\{E_m^{(2)}U_2-E_m^{(1)}U_1\}\\
&\quad -2k(\mu_1U_1E_t^{(1)}-\mu_2U_2E_t^{(2)}),
\end{aligned}$$

with

$$E_t^{(i)}=E_m^{(i)}\left(k^2+\frac{m^2}{k^2}\right)-\frac{1}{R},$$

and necessary and sufficient stability conditions are

$$b_1>0, \tag{6.5}$$

and

$$a_0b_2^2-a_1b_1b_2+a_2b_1^2<0, \tag{6.6}$$

since a_0 is always positive.

7. Numerical examples

In this section we do numerical works using the expressions presented in previous sections for the film boiling conditions. The vapor and liquid are identified with phase 1 and phase 2, respectively,

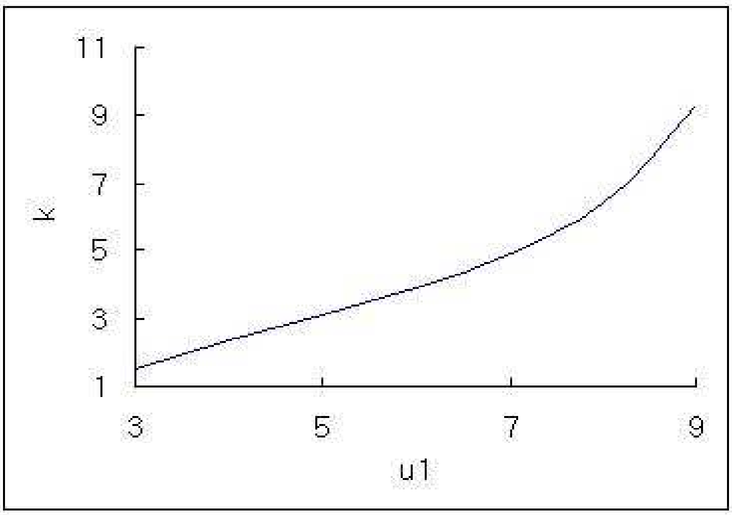

FIGURE 1. The critical wave number for m=1.

so that $T_1 > T_0 > T_2$.

In the film boiling, the liquid-vapor interface is of saturation condition and the temperature T_0 is set equal to the saturation temperature. The properties of both phases are determined from this condition. First, in figure 1 we display critical wave number k_c, i.e., the value for which $\omega = 0$ in (2.14) Here we chose $\rho_1 = 0.001gm/cm^3, \rho_2 = 1gm/cm^3, \sigma = 72.3dyne/cm, b = 2cm, a = 1cm, R = 1.2cm, \alpha = 0.1gm/cm^3s$ [0]

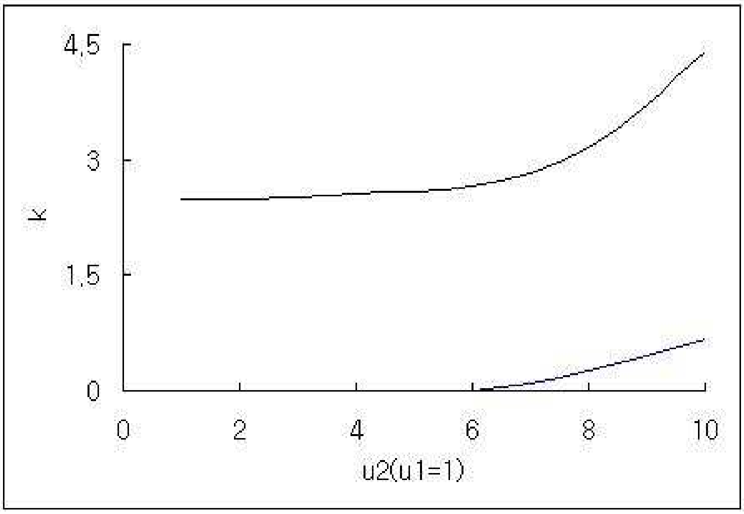

FIGURE 2. The stability diagram for the flow when m=1. The system is stable in the region between the two upper and lower curves.

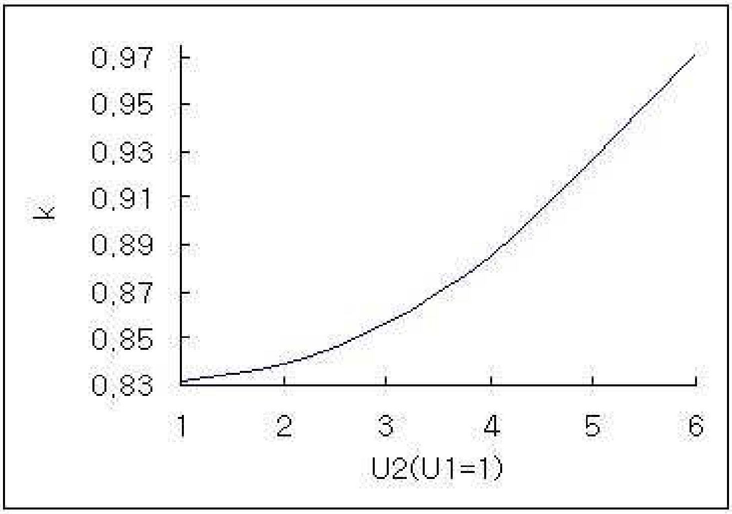

Fig.3.Viscous cylindrical flow for m=0.The region above the curve is stable region.

From this figure we can notice that critical wave number increases as the velocity of fluid increases, the increment rate of the inviscid fluid being sharper at higher fluid velocities. In figure 2 we display

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the region of stability of fluid in the nonlinear analysis as the velocity of one fluid increases while that of the other fluid remains unchanged. In these figures , u_1 remains constant as 1 cm/sec while u_2 varies from 1 cm/sec to 10cm/sec. The region between the two curves is the region of stability, while in the region above the upper curve, the fluid is unstable.[0]

In Fig.3 and Fig.4 we present the results for viscous cylindrical linear flow.Here we chose $\rho_1 = 0.0001 gm/cm^3, \rho_2 = 1 gm/cm^3, \sigma = 72.3 dyne/cm, b = 2cm, a = 1cm, R = 1.2cm, \alpha = .1 gm/cm^3 s, \mu_1 = 0.00001 poise, \mu_2 = 0.01 poise$

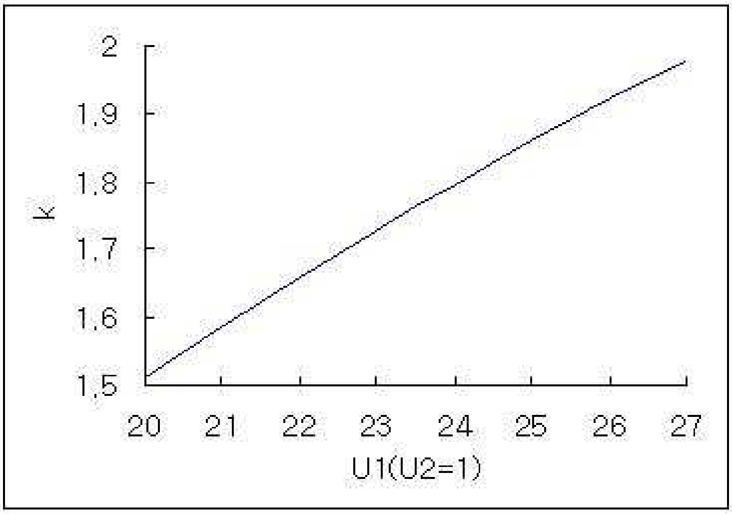

Fig.4.Viscous cylindrical flow for m=1.The region above the curve is stable region.

8. Conclusions.

The stability of liquids in a cylindrical flow when there is mass and heat transfer across the interface which depicts the film boiling is studied. Using the method of multiple time scales, a first order nonlinear differential equation describing the evolution of nonlinear waves is obtained.With the linear theory the region of stability is the whole plane above a curve like in Fig.3,4, however with the nonlinear theory it is in the form of a band as shown in Fig.2. Unlike linear theory, with nonlinear theory, it is evident that the mass and heat transfer plays an important role in the stability of fluid, in a situation like film boiling.

Appendix

The interfacial conditions are given on $r = R$ as

Order $O(\epsilon)$

$$\left[\!\left[\rho\left(\frac{\partial\phi_1}{\partial r} - \frac{\partial\eta_1}{\partial T_0}\right) - \frac{\partial\eta_1}{\partial z}\frac{\partial\phi_0}{\partial z}\right)\right]\!\right] = 0, \tag{A.1}$$

$$\rho^{(1)}\left(\frac{\partial\phi_1^{(1)}}{\partial r} - \frac{\partial\eta_1}{\partial T_0} - \frac{\partial\eta_1}{\partial z}\frac{\partial\phi_0}{\partial z}\right) = \alpha\eta_1, \tag{A.2}$$

$$\left[\!\left[\rho\left(\frac{\partial\phi_1}{\partial T_0}+\frac{\partial\phi_1}{\partial z}\frac{\partial\phi_0}{\partial z}\right)\right]\!\right]=-\sigma\left(\frac{\partial^2\eta_1}{\partial z^2}+\frac{\eta_1}{R^2}+\frac{1}{r^2}\frac{\partial^2\eta_1}{\partial\theta^2}\right). \tag{A.3}$$

Order $O(\epsilon^2)^0$

$$\left[\!\left[\rho\left(\frac{\partial\phi_2}{\partial r}+\frac{\partial^2\phi_1}{\partial r^2}\eta_1-\frac{\partial\eta_2}{\partial T_0}-\frac{\partial\eta_1}{\partial T_1}-\frac{\partial\eta_1}{\partial z}\frac{\partial\phi_1}{\partial z}-\frac{1}{r^2}\frac{\partial\eta_1}{\partial\theta}\frac{\partial\phi_1}{\partial\theta}-\frac{\partial\eta_2}{\partial z}\frac{\partial\phi_0}{\partial z}\right)\right]\!\right]=0, \tag{A.4}$$

$$\rho^{(1)}\left(\frac{\partial\phi_2^{(1)}}{\partial r}+\frac{\partial^2\phi_1^{(1)}}{\partial r^2}\eta_1-\frac{\partial\eta_2}{\partial T_0}-\frac{\partial\eta_1}{\partial T_1}-\frac{\partial\eta_1}{\partial z}\frac{\partial\phi_1^{(1)}}{\partial z}-\frac{1}{r^2}\frac{\partial\eta_1}{\partial\theta}\frac{\partial\phi_1}{\partial\theta}-\frac{\partial\eta_2}{\partial z}\frac{\partial\phi_0}{\partial z}\right)=\alpha(\eta_2+\alpha_2\eta_1^2), \tag{A.5}$$

$$\begin{aligned}&\left[\!\left[\rho\left\{\frac{\partial\phi_2}{\partial T_0}+\frac{\partial\phi_1}{\partial T_1}+\frac{\partial^2\phi_1}{\partial T_0\partial r}\eta_1+\frac{1}{2}\left[\left(\frac{\partial\phi_1}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial\phi_1}{\partial\theta}\right)^2+\left(\frac{\partial\phi_1}{\partial z}\right)^2\right]+\frac{\partial\phi_2}{\partial z}\frac{\partial\phi_0}{\partial z}+\frac{\partial\phi_1}{\partial r}\left(\frac{\partial\eta_1}{\partial T_0}-\frac{\partial\phi_1}{\partial r}\right)\right.\right.\right.\\&\qquad\left.\left.\left.+\frac{\partial\phi_0}{\partial z}\left(-\frac{\partial\eta_1}{\partial z}\frac{\partial\eta_1}{\partial t_0}+2\frac{\partial\eta_1}{\partial z}\frac{\partial\phi_1}{\partial r}-\left(\frac{\partial\eta_1}{\partial z}\right)^2\frac{\partial\phi_0}{\partial z}+\frac{\partial^2\phi_1}{\partial z\partial r}\eta_1\right)\right\}\right]\!\right]\\&=-\sigma\left\{\frac{\partial^2\eta_2}{\partial z^2}+\frac{1}{2R}\left(\frac{\partial\eta_1}{\partial z}\right)^2+\frac{\eta_2}{R^2}-\frac{\eta_1^2}{R^3}-\frac{3}{2}\frac{1}{R^3}\left(\frac{\partial\eta_1}{\partial\theta}\right)^2+\frac{1}{r^2}\frac{\partial^2\eta_2}{\partial\theta^2}-\frac{2}{r^3}\eta_1\frac{\partial^2\eta_1}{\partial\theta^2}\right\}.\end{aligned} \tag{A.6}$$

Order $O(\epsilon^3)$

$$\begin{aligned}&\rho^{(j)}\left\{\frac{\partial\phi_3^{(j)}}{\partial r}+\frac{\partial^2\phi_2^{(j)}}{\partial r^2}\eta_1+\frac{\partial^2\phi_1^{(j)}}{\partial r^2}\eta_2+\frac{1}{2}\frac{\partial^3\phi_1^{(j)}}{\partial r^3}\eta_1^2-\frac{\partial\eta_3}{\partial T_0}-\frac{\partial\eta_2}{\partial T_1}-\frac{\partial\eta_1}{\partial T_2}\right.\\&-\frac{\partial\eta_1}{\partial z}\left(\frac{\partial\phi_2^{(j)}}{\partial z}+\frac{\partial^2\phi_1^{(j)}}{\partial z\partial r}\eta_1\right)-\frac{\partial\eta_2}{\partial z}\frac{\partial\phi_1^{(j)}}{\partial z}-\frac{\partial\eta_3}{\partial z}\frac{\partial\phi_0^{(j)}}{\partial z}-\frac{1}{R^2}\frac{\partial\eta_2}{\partial\theta}\frac{\partial\phi_1^{(j)}}{\partial\theta}-\frac{1}{R^2}\frac{\partial\eta_1}{\partial\theta}\frac{\partial\phi_2^{(j)}}{\partial\theta}\\&\left.-\frac{1}{R^2}\eta_1\frac{\partial\eta_1}{\partial\theta}\frac{\partial^2\phi_1^{(j)}}{\partial\theta\partial r}+\frac{2}{R^3}\eta_1\frac{\partial\eta_1}{\partial\theta}\frac{\partial\phi_1^{(j)}}{\partial\theta}\right\}=\alpha(\eta_3+2\alpha_2\eta_1\eta_2+\alpha_3\eta_1^3),\quad(j=1,2),\end{aligned} \tag{A.7}$$

$$\begin{aligned}&\left[\!\left[\rho\left\{\frac{\partial\phi_3}{\partial T_0}+\frac{\partial\phi_2}{\partial T_1}+\frac{\partial\phi_1}{\partial T_2}+\frac{\partial^2\phi_1}{\partial T_0\partial r}\eta_2+\left(\frac{\partial^2\phi_1}{\partial T_1\partial r}+\frac{\partial^2\phi_2}{\partial T_0\partial r}\right)\eta_1\right.\right.\right.\\&+\frac{1}{2}\frac{\partial^3\phi_1}{\partial T_0\partial r^2}\eta_1^2+\frac{\partial\phi_1}{\partial r}\left(\frac{\partial\phi_2}{\partial r}+\frac{\partial^2\phi_1}{\partial r^2}\eta_1\right)+\frac{\partial\phi_1}{\partial z}\left(\frac{\partial\phi_2}{\partial z}+\frac{\partial^2\phi_1}{\partial r\partial z}\eta_1\right)\\&+\frac{1}{R^2}\frac{\partial\phi_1}{\partial\theta}\left(\frac{\partial^2\phi_1}{\partial r\partial\theta}-\frac{1}{R}\frac{\partial\phi_1}{\partial\theta}\right)\eta_1+\frac{1}{R^2}\frac{\partial\phi_1}{\partial\theta}\frac{\partial\phi_2}{\partial\theta}\\&-\left(\frac{\partial\phi_1}{\partial r}-\frac{\partial\eta_1}{\partial t_0}\right)\left(\frac{\partial\phi_2}{\partial r}-\frac{\partial\phi_1}{\partial z}\frac{\partial\eta_1}{\partial z}-\frac{1}{R^2}\frac{\partial\phi_1}{\partial\theta}\frac{\partial\eta_1}{\partial\theta}\right)+\frac{\partial\phi_1}{\partial r}\left(\frac{\partial\eta_2}{\partial t_0}+\frac{\partial\eta_1}{\partial t_1}-\frac{\partial\phi_2}{\partial r}+\frac{\partial\phi_1}{\partial z}\frac{\partial\eta_1}{\partial z}+\frac{1}{R^2}\frac{\partial\phi_1}{\partial\theta}\frac{\partial\eta_1}{\partial\theta}\right)\\&+\eta_1\frac{\partial^2\phi_1}{\partial r^2}\left(\frac{\partial\eta_1}{\partial T_0}-2\frac{\partial\phi_1}{\partial r}\right)+\frac{\partial\phi_0}{\partial z}\left(\frac{\partial\phi_3}{\partial z}+\frac{\partial^2\phi_1}{\partial r\partial z}\eta_2+\frac{\partial^2\phi_2}{\partial r\partial z}\eta_1+\frac{\partial^3\phi_1}{\partial r^2\partial z}\frac{\eta_1^2}{2}\right.\\&+2\frac{\partial\phi_2}{\partial r}\frac{\partial\eta_1}{\partial z}+2\frac{\partial\phi_1}{\partial r}\frac{\partial\eta_2}{\partial z}+2\frac{\partial^2\phi_1}{\partial r^2}\frac{\partial\eta_1}{\partial z}\eta_1-\frac{\partial\eta_2}{\partial t_0}\frac{\partial\eta_1}{\partial z}-\frac{\partial\eta_1}{\partial t_0}\frac{\partial\eta_2}{\partial z}\\&\left.\left.\left.-2\frac{\partial\eta_1}{\partial z}\frac{\partial\eta_2}{\partial z}\frac{\partial\phi_0}{\partial z}-2\left(\frac{\partial\eta_1}{\partial z}\right)^2\frac{\partial\phi_1}{\partial z}-\frac{2}{R^2}\frac{\partial\eta_1}{\partial z}\frac{\partial\eta_1}{\partial\theta}\frac{\partial\phi_1}{\partial\theta}\right)\right\}\right]\!\right]=-\sigma\left[\frac{\partial^2\eta_3}{\partial z^2}-\frac{3}{2}\frac{\partial^2\eta_1}{\partial z^2}\left(\frac{\partial\eta_1}{\partial z}\right)^2\right.\end{aligned}$$

$$-\frac{1}{2}\frac{\eta_1}{R^2}\left(\frac{\partial\eta_1}{\partial z}\right)^2+\frac{1}{R}\frac{\partial\eta_1}{\partial z}\frac{\partial\eta_2}{\partial z}+\frac{\eta_3}{R^2}-\frac{2\eta_1\eta_2}{R^3}+\frac{\eta_1^3}{R^4}-\frac{3}{R^3}\frac{\partial\eta_1}{\partial\theta}\frac{\partial\eta_2}{\partial\theta}-\frac{1}{2R^2}\frac{\partial^2\eta_1}{\partial z^2}\left(\frac{\partial\eta_1}{\partial\theta}\right)^2$$

$$+\frac{9}{2R^4}\eta_1\left(\frac{\partial\eta_1}{\partial\theta}\right)^2+\frac{1}{R^2}\frac{\partial^2\eta_3}{\partial\theta^2}+\frac{1}{R^2}\frac{\partial^2\eta_1}{\partial\theta^2}\left\{-\frac{1}{2}\left(\frac{\partial\eta_1}{\partial z}\right)^2-\frac{2\eta_2}{R}+\frac{3\eta_1^2}{R^2}-\frac{3}{2R^2}\left(\frac{\partial\eta_1}{\partial\theta}\right)^2\right\}$$

$$-\frac{2}{R^3}\eta_1\frac{\partial^2\eta_2}{\partial\theta^2}-\frac{2}{R^2}\frac{\partial\eta_1}{\partial\theta}\frac{\partial^2\eta_1}{\partial\theta\partial z}\frac{\partial\eta_1}{\partial z}\Bigg]. \qquad (A.8)$$

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