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The Differential Transform Method for (n+1) - dimensional Equal Width Wave equation with damping term

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ABSTRACT

We have employed the differential transform method to solve the (n + 1)-dimensional Equal Width wave equations with damping term as.

Keywords: *Differential Equation Method (DTM), DTM for Equal Width wave equations with damping term*

1. INTRODUCTION

Zhou[2] was the first one to use the differential transform method (DTM) in engineering applications. He employed DTM in solution of the initial boundary value problems in electric circuit analysis. In recent years concept of DTM has broad end to problems involving partial differential equations and systems of differential equations[3-5].Some researchers have lately applied DTM for analysis of uniform and non-uniform beams [6-10].In the few decades, the traditional integral transform methods such as Fourier and Laplace has equations into the algebraic equations which are easier to deal with

The well known Korteweg and de Vries (KdV) equation $u_t + uu_x + u_{xxx} = 0$ dimension. Morrison et al.[1] proposed the one dimensional PDE $u_t + uu_x\, u_{xxx} = 0$, as an equally valid and accurate method for the same wave phenomena simulated by the KdV equation. This PDE is called the equal width wave equation because the solutions for the solitary waves with a permanent form and speed, for given value of the parameter, are waves with an equal width or wavelength for all wave amplitudes. In this chapter, we have employed the differential transform method to solve the (n+1)- dimensional Equal Width wave equation with damping term as,

$$u_t = v_1 u_{x_1 x_1 t} + v_2 u_{x_2 x_2 t} + \ldots + v_n u_{x_n x_n t} + \gamma u u_{x_1} + \beta u \tag{1}$$

under the initial condition

$$u(x_1, x_2, \ldots, x_n, 0) = u_0(x_1, x_2, \ldots, x_n) \tag{2}$$

Where $v_i^{\prime}$s $i = 1, 2, \ldots, n$,γ and β are constants

2. Differential Transform method (DTM)

In this section, we give some basic definitions of the differential transformation. Let D denotes the di erential transform operator and D 1 the inverse differential transform operator.

2.1 Basic Definition of DTM

Definition: 1

If u (x_1; x_2; :::; x_n; t) is analytic in the domain then its (n+1)- dimensional differential transform is given by

$$U\left(k_1,k_2,...,k_n,k_{n+1}\right)=\left(\frac{1}{k_1!,k_2!,...,k_n!,k_{n+1}!}\right)\times \frac{\partial^{k_1+k_2+...+k_n+k_{n+1}}}{\partial_{x_1}^{k_1}\partial_{x_2}^{k_2}...\partial_{x_n}^{k_n}\partial_{x_t}^{k_{n+1}}}.u(x_1,x_2,...,x_n,t)|x_1=0,x_2=0,...,x_n=0,t=0 \tag{3}$$

where

$$u(x_1,x_{2,...,x_n,t})=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}...\sum_{k_n=0}^{\infty}\sum_{k_{n+1}=0}^{\infty}U\left(k_1,k_2,...,k_n,k_{n+1}\right).x_1^{k_1}x_2^{k_2}...x_n^{k_n}t^{k_{n+1}} = D^{-1}\left[U\left(k_1,k_2,...,k_n,k_{n+1}\right)\right] \tag{4}$$

Definition:2

$$\text{If } U\left(x_1,x_2,...,x_n,t\right)=D^{-1}[U(k_1,k_2,...,k_n,k_{n+1})],v(x_1,x_{2,...,x_n,t}) = D^{-1}[V(k_1,k_2,...,k_n,k_{n+1})]$$

and $\otimes$ denotes convolution , then the fundamental operations of the differential transform are expressed as follows:

$$(a).D\left[U\left(x_1,x_2,...,x_n,t\right)\right]v\left(x_1,x_2,...,x_n,t\right)=U\left(k_1,k_2,...,k_n,k_{n+1}\right)\otimes V\left(k_1,k_2,...,k_n,k_{n+1}\right) = \sum_{a_1=0}^{k_1}\sum_{a_2=0}^{k_2}...\sum_{a_n=0}^{k_n}\sum_{a_{n+1}=0}^{k_{n+1}}U\left(a_1,k_2-a_2,...,k_{n+1}-a_{n+1}\right) V\left(k_1-a_1,a_2,...,a_{n+1}\right) \tag{5}$$

$$(b).D\left[\alpha u\left(x_1,x_2,...,x_n,t\right)\right]\pm\beta v\left(x_1,x_2,...,x_n,t\right)=\alpha U\left(k_1,k_2,...,k_n,k_{n+1}\right) \pm\beta V\left(k_1,k_2,...,k_n,k_{n+1}\right) \tag{6}$$

$$(c).D\{\frac{\partial^{r_1+r_2+...+r_n+r_{n+1}}}{\partial_{x_1}{}^{r_1}\partial_{x_2}{}^{r_2}...\partial_{x_n}{}^{r_n}\partial_{x_t}{}^{r_{n+1}}}.u\left(x_1,x_2,...,x_n,t\right)\} = (k_1+1)+(k_1+2)$$
$$...\left(k_1+r_1\right)\left(k_2+1\right)\left(k_2+2\right)...\left(k_1+r_2\right)$$
$$...\left(k_{n+1}+1\right)\left(k_{n+1}+2\right)$$
$$...\left(k_{n+1}+r_{n+1}\right).U\left(k_1+r_1,...,k_{n+1}+r_{n+1}\right). \tag{7}$$

3. Computational illustrations of (n+1)-dimensional Equal width wave equation with damping term

Here we describe the method explained in the previous section, by the following examples to validate the efficiency of the DTM.

Example:1

Consider the (n+1)-dimensional equal width wave equation with damping by assuming v_i's and = = 1 as,

$\gamma = \beta = 1$ as,

$$u_t = u_{x_1x_1t} + u_{x_2x_2t} + ... + u_{x_nx_nt} + uu_{x_1} + u \tag{8}$$

Subject to the initial condition

$$u\left(x_1,x_2,...,x_n,0\right) = u_0\left(x_1,x_2,...,x_n,t\right) = x_1+x_2+...+x_n \tag{9}$$

3.1. DTM for equal wave equation with damping term

Taking the differential transform of eq.(8)., we have

$$\begin{aligned}(k_{n+1}+1)U(k_1,k_2,...,k_n,k_{n+1}+1) &= (k_1+2)(k_1+1)(k_{n+1}+1)U(k_1+2,k_2,k_3,...,k_{n+1}+1)\\ &+ (k_2+2)(k_2+1)(k_{n+1}+1)U(k_1,k_2+2,k_3,...,k_{n+1}+1)+...\\ &+ (k_n+2)(k_n+1)(k_{n+1}+1)U(k_1,k_2+2,k_3,...,(k_{n+2})k_{n+1}+1)\\ &+ \sum_{a_1=0}^{k_1}\sum_{a_2=0}^{k_2}...\sum_{a_n=0}^{k_n}\sum_{a_{n+1}=0}^{k_{n+1}}(k_1+1-a_1)(k_{n+1}-a_{n+1}).\\ & U\left(k_1+1-a_1,a_2,a_3,...,a_n,k_{n+1}-a_{n+1}\right)\\ & \times U\left(a_1k_2-a_2,k_3-a_3,...,k_n-a_n,a_{n+1}\right)\end{aligned} \tag{10}$$

From the initial condition eq.(9).,it can be seen that

$$u\left(x_1,x_2,...,x_n,0\right) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}...\sum_{k_n=0}^{\infty}U\left(k_1,k_2,...,k_n,0\right).x_1^{k_1}.x_2^{k_2}...x_n^{k_n} \tag{11}$$

$$= x_1 + x_2 + ... + x_n$$

$$U(k_1, k_2, ..., k_n, 0) = \begin{cases} 1 & \text{if } k_i = 1, k_j = 0,\ i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

using eq.,(12) into eq.(11) one can obtain the values of $U\left(k_1, k_2, ..., k_n, k_{n+1}\right)$ as

$$U(k_1, k_2, ..., k_n, 1) = \begin{cases} 1 & \text{if } k_i = 1, k_j = 0,\ i \neq j,\ i, j = 1, 2, ..., n \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

$$U(k_1, k_2, ..., k_n, 2) = \begin{cases} 1 & \text{if } k_i = 1, k_j = 0,\ i \neq j\ ,\ i, j = 1, 2, ..., n \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

$$U(k_1, k_2, ..., k_n, 3) = \begin{cases} 1 & \text{if } k_i = 1, k_j = 0, i \neq j\ ,\ i, j = 1, 2, ..., n \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

$$U(k_1, k_2, ..., k_n, n) = \begin{cases} 1 & \text{if } k_i = 1, k_j = 0, i \neq j, i, j = 1, 2, ..., n \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

Then from eqn.(4) we have

$$U(x_1, x_2, ..., x_n, t) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} ... \sum_{k_n=0}^{\infty} \sum_{k_{n+1}=0}^{\infty} U(k_1, k_2, ..., k_n, k_{n+1})(x_1^{k_1}, x_2^{k_2} ... x_n^{k_n} t^{k_{n+1}}) \tag{17}$$

$$= (x_1 + x_2 + ... + x_n)(1 + 2t + 3t^2 + ...).$$

Thus,the exact solution is given by

$$u\left(x_1, x_2, ..., x_n, t\right) = \frac{x_1 + x_2 + ... + x_n}{(1-t)^2}, \text{provided that } 0 \leq t < 1 \tag{18}$$

Example:2

Consider the (3+1)-dimensional equal width wave equation with damping as,

$$u_t = u_{xxt} + u_{yyt} + u_{zzt} + uu_x + u \tag{19}$$

Subject to the initial condition

$$u(x,y,z,0)=u_0(x,y,z)=x+y+z \tag{20}$$

3.2. DTM for Equal Width Wave Equation with damping term

Talking the differential transform of eq.(19).,we have

$$\begin{aligned}(k_4+1)U(k_1,k_2,k_3,k_{4+1}) &= (k_1+2)(k_1+1)(k_{4+1}+1)U(k_1+2,k_2,k_3,k_{4+1})\\ &+(k_2+2)(k_2+1)(k_{4+1})U(k_1,k_2+2,k_3,k_{4+1})\\ &+(k_3+2)(k_3+1)(k_{4+1})U(k_1,k_2,k_{3+2},(k_{4+1})\\ &+\sum_{a_1=0}^{k_1}\sum_{a_2=0}^{k_2}\sum_{a_3=0}^{k_3}\sum_{a_4=0}^{k_4}(k_1+1-a_1)(k_4-a_4).\\ &U(k_1+1-a_1,a_2,a_3,a_n,k_4-a_4)\times U(a_1,k_2-a_2,k_3-a_3,a_4).\end{aligned} \tag{21}$$

From the initial condition eq.(20),it can be seen that

$$\begin{aligned}u(x,y,z,t,0) &= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}U(k_1,k_2,k_3,0).x^{k_1}.y^{k_2}z^{k_3}\\ &= x+y+z\end{aligned} \tag{22}$$

where

$$U(k_1,k_2,k_3,0)=\begin{cases}1 & \text{if } k_i=1,k_j=0,i\neq j,i,j=1,2,3\\ 0 & \text{otherwise}\end{cases} \tag{23}$$

Using eq.(32)into eq.(21)one can obtain the values of $U(k_1,k_2,k_3,k_4)$ as

$$U(k_1,k_2,k_3,1)=\begin{cases}1 & \text{if } k_i=1,k_j=0,i\neq j,i,j=1,2,3\\ 0 & \text{otherwise}\end{cases} \tag{24}$$

$$U(k_1,k_2,k_3,2)=\begin{cases}1 & \text{if } k_i=1,k_j=0,i\neq j,i,j=1,2,3\\ 0 & \text{otherwise}\end{cases} \tag{25}$$

$$U(k_1,k_2,k_3,3)=\begin{cases}1 & \text{if } k_i=1, k_j=0, i\neq j, i,j=1,2,3\\ 0 & \text{otherwise}\end{cases} \tag{26}$$

Then from eqn.(4) we have

$$\begin{aligned} u(x,y,z,t) &= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty} U(k_1,k_2,k_3,k_4)\, x^{k_1}.y^{k_2}z^{k_3}t^{k_4} \\ &= (x+y+z)\left(1+2t+3t^2+...\right) \end{aligned} \tag{27}$$

Thus,the exact solution is given by

$$u(x,y,z,t)=\frac{(x+y+z)}{(1-t)^2}\text{provided that } 0\leq\ t<1 \tag{28}$$

Example:3

Consider the (2+1)-dimensional Equal width wave equation with damping as,

$$u_t=u_{xxt}+u_{yyt}+uu_x+u \tag{29}$$

Subject to the initial condition

$$u(x,y,0)=u_0(x,y)=x+y \tag{30}$$

3.3 DTM for Equal width Wave equation with damping term

Talking the differential transform of eq.(29).,we have

$$\begin{aligned} (k_3+1)U(k_1,k_2,k_{3+1}) &= (k_1+2)(k_1+1)(k_3+1)U(k_1+2,k_2,k_3+1) \\ &+(k_2+2)(k_2+1)(k_3+1)U(k_1,k_2+2,k_3+1) \\ &\sum_{a_1=0}^{k_1}\sum_{a_2=0}^{k_2}\sum_{a_3=0}^{k_3}(k_1+1-a_1)(k_3-a_3) \\ &.U(k_1+1-a_1,a_2,k_3-a_3)XU(a_1,k_2,a_3). \end{aligned} \tag{31}$$

From the initial condition eq.(30),it cna be seen that

$$u(x,y,0)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}U(k_1,k_2,0).x^{k_1}.y^{k_2}=x+y \tag{32}$$

Where

$$U(k_1,k_2,0)=\begin{cases}1 & \text{if } k_i=1,k_j=0,\ i\neq j\text{,i,j=1,2}\\ 0 & \text{otherwise}\end{cases} \tag{33}$$

Using eq.(33)into eq.(31)one can obtain the values of

$$U(k_1,k_2,k_3)U(k_1,k_2,1)=\begin{cases}1 & \text{if } k_i=1,k_j=0,\ i\neq j\text{,i,j=1,2}\\ 0 & \text{otherwise}\end{cases} \tag{34}$$

$$U(k_1,k_2,2)=\begin{cases}1 & \text{if } k_i=1,\ k_j=0, i\neq j,\ i,\ j=1,2\\ 0 & \text{otherwise}\end{cases} \tag{35}$$

Then from eqn.(4)we have

$$u(x,y,t)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}U(k_1,k_2,k_3)$$

$$x^{k_1}.y^{k_2}.t^{k_3}=(x+y)(1+2t+3t^2+...). \tag{36}$$

Hence,the exact solution is given by

$$u(x,y,t)=\frac{(x+y)}{(1-t)^2}, \text{provided that } 0\leq t<1 \tag{37}$$

4. CONCLUSION

1. The differential transform method have been successfully applied for solving the (n+1)-dimensional equal width wave equation with damping term.

2. The solutions obtained by this method is an in nite power series for the appropriate initial condition, which can, in turn be expressed in a closed form, the exact solution.

3. The results reveal that this method is very effective, convenient and quite accurate mathematical tools for solving the (n+1)-dimensional equal width wave equation with damping term.

4. This method can be used without any need to complex computations except the simple and elementary operations are also promising technique for solving the other nonlinear problems.

REFERENCES

1. Morrison.P.J., Meiss.J.D.,Carey.J.R.: cattering of RLW Solitary waves, Physica D 11.

2. Zhou.J.K.,1986, Differential transform and its application for electrical circuits. Huazhong University press, china.

3. Jang.M.J., c.L.Chen and Y.C.Lic, 2001, Two dimensional Differential transform for partial differential equations. Applied Mathematics and computation, 121: 261-270

4. Ayaz,Fatma,2003,on the-dimensional differential transform method. Applied Mathematics and Computation,143:361-374

5. Ayaz,Fatma,2004,solutions of the system of the differential equations using the differential transform method. Applied Mathematics and Computation, 147: 547-567

6. H.O.,S.H. and C.K.Chen,1998,Analysis of general elastically and restrained non-uniform beams using the differential transform ,Applied Mathematical, Modelling, 22: 219-234.

7. Ozdemir .O., and M.O.Kaya,kaya,2006,Flapwise bending vibration analysis of a rotating tapered contilever Bernoulli-Euler beam by differential transform method. Journal of sound and vibration, 289: 413-420.

8. Ozgumus.O. and M.O.Kaya, 2006, Flapwise bending vibration analysis of double tapered rotating Euler-Bernoulli beam by using the differential transform method. Meccanica, 41: 661-670.

9. Seval, Catal, 2008. Solution of free vibration equations of beam on elastic soil by using the differential transform method. Applied Mathematical modelling, 32: 1744-1757

10. Mei.C,2008. Application of differential transformation technique to free vibration analysis of a centrifugally stiffened beam. Computers and structures, 86: 1280-1284