Influence of Cesaro summation and the Fejer Kernel onto the climate change model

Rajendra Bapurao Vhatkar
Research Scholar, JJT University,
Jhunjhunu, Churela, Rajasthan, India

Dr. Vishwajeet S. Goswami
Department of Mathematics, Dilla University,
Ethiopia

ABSTRACT

Throughout past centuries our planet has experienced a vast climate change periodically along with the other geographical variations. The ecological diversities that occur over a period of time vary with the effect on its subsidiaries and thereby cause an overall change in its surroundings. The main focus of the work is to relate climate change model considered as in the form of a mathematical expression and also study its effects onto the elements of nature that are eventually affected during a particular event. These parameters are developed in the form of a function which further manifests according to the types of events that are resulting out of it. The current paper works on to facilitate an attempt to form a mathematical model to relate such climatic changes and the parameters affecting it and also relate it to the previously existing similar model. There are further influences of the cesaro summation and later convolutes with the Fejer kernel that links the Fourier condition to manifest the efficiency of this model. Major factors that we focus are the heat change, atmospheric pressure and the humidity change that form the major components of the Fourier series when treated with the Fejer kernel once they are declared under the Cesaro conditions of summation.

Keywords: Climate change, Cesaro Sum, Fejer Kernel, Model

1. INTRODUCTION

1.1 Climatology & Climate change:

This is one such element of the nature that exists way before mankind has stepped onto the planet. Ever since then there was a continuous change in the climatic conditions which results into a new form periodically. Ages later the introduction of mankind also did see many such changes in nature and climate. These changes did affect mankind at a large and thus resulted in evolution to the present conditions. There are various approaches and methodologies towards studying the climate change in India and throughout the world. Every moment there is a model formed approach developed to understand the behavior of climate change and its effect on the particular surroundings. The outcome of these studies deals with the aspect of not improving any such climatic conditions as it is not in the hands of mankind to control nature, however we may try to take necessary measures that can lead us to avoid certain losses due to climate change.

As known climate change globally drastically is responsible for a climate change in a particular region. In aspect of India the climatic conditions are very diversified and volatile. This changes are due to the surrounding conditions that result the final climatic conditions over the Indian subcontinent.
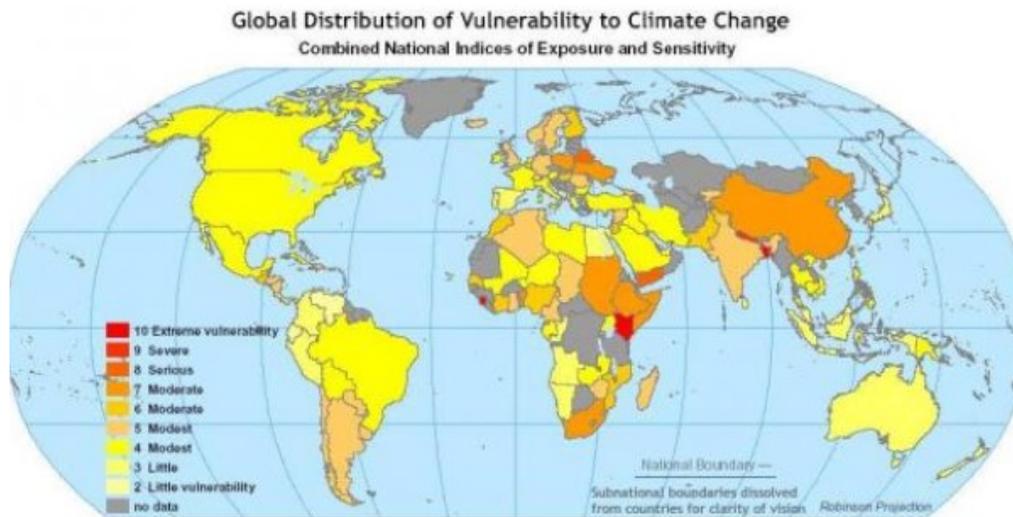
Global Distribution of Vulnerability to Climate Change
Combined National Indices of Exposure and Sensitivity

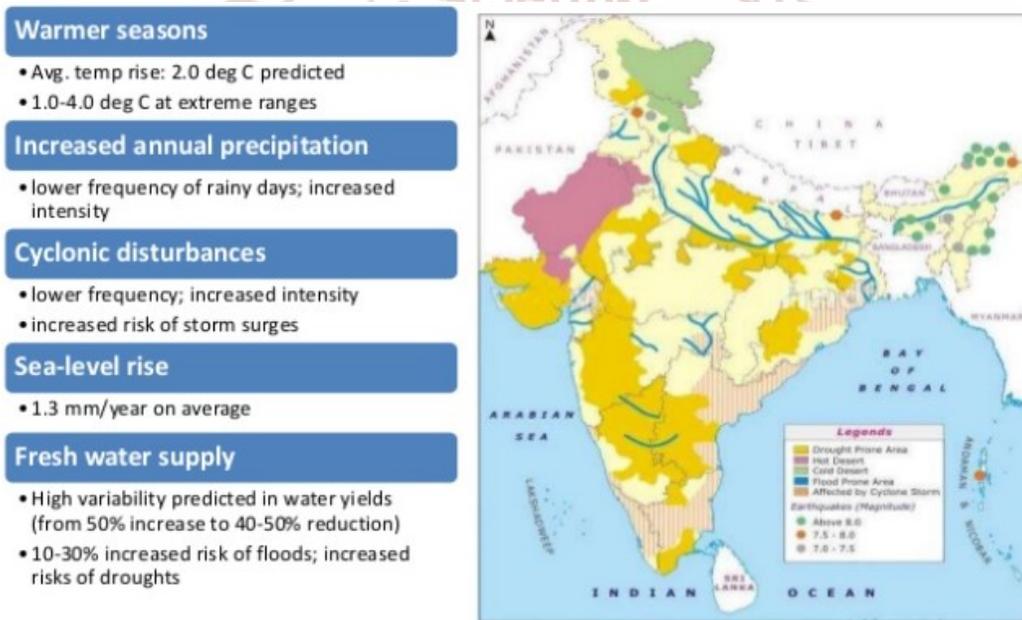**Figure 1.1 Spatial view of Climate change globally (Courtesy:NOAA)**

Figure 1.2 Climate change and its impact in India(Courtesy: INCCA)

1.2 Impact of Climate change:

As we see climate change is a continuous process that deals with a lot of other factors involving the ones that are either a part of the process or discrete. On the basis of their impact and effectiveness, climate change may be termed as long term and short term. However, a major change over a large demography is caused due to long term climatic change and the small changes that affect the demography of a place in less. We shall focus on one such mathematical model to understand the effect on population and allied parameters due to climate change
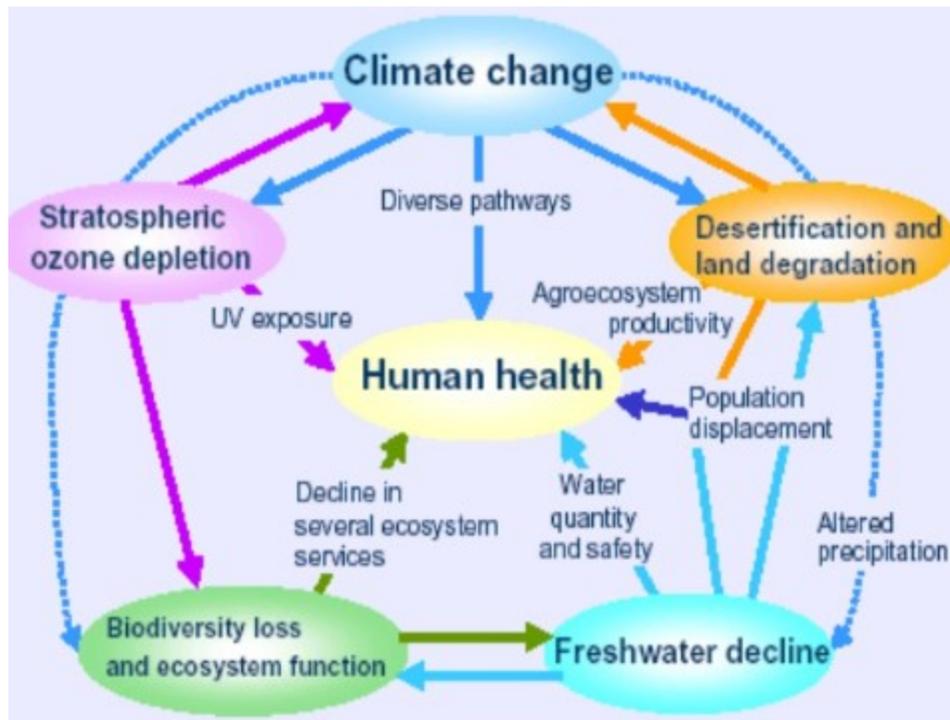
Figure 1.3 Effect of Climate change on various elements (Courtesy:Google images)

1.2 Mathematical models to understand climate change and its impact on allied parameters:

a) Probability and climate change:
As known one of the most primitive approaches in understanding the climate change with respect to event occurrence is the theory of probability.
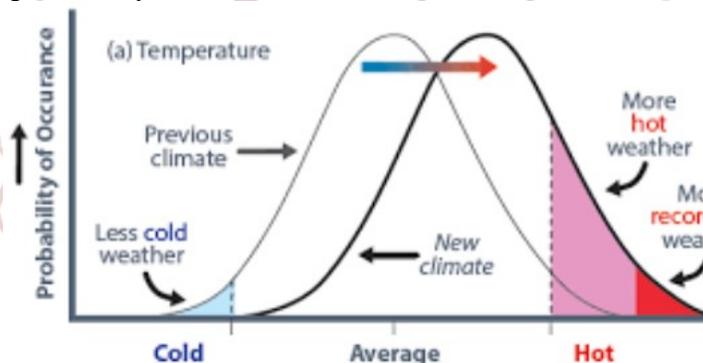

Figure 1.4 General example of probability to study atmospheric temperature.

As seen in the above diagram the data for a particular demography is taken under study and various conditions are convoluted with reference to conditional probability and the further effectiveness is seen ahead.

Let X and Y be two continuous random variables, with joint probability density function f(x,y) and marginal probability density functions g(x) and h(y). The conditional probability density function of the random variable Y, given that X = x, is

$$f(y \mid x) = \frac{f(x, y)}{g(x)}, \qquad g(x) > 0$$

Similarly, the conditional probability density function of the random variable X, given that Y = y, is

$$f(x \mid y) = \frac{f(x, y)}{h(y)}, \qquad h(y) > 0$$

Figure 1.5 General statement of conditional probability for various events

Based on these methods of probability the further estimation of climate change could be estimated up to an extent. The further diagram describes it well.
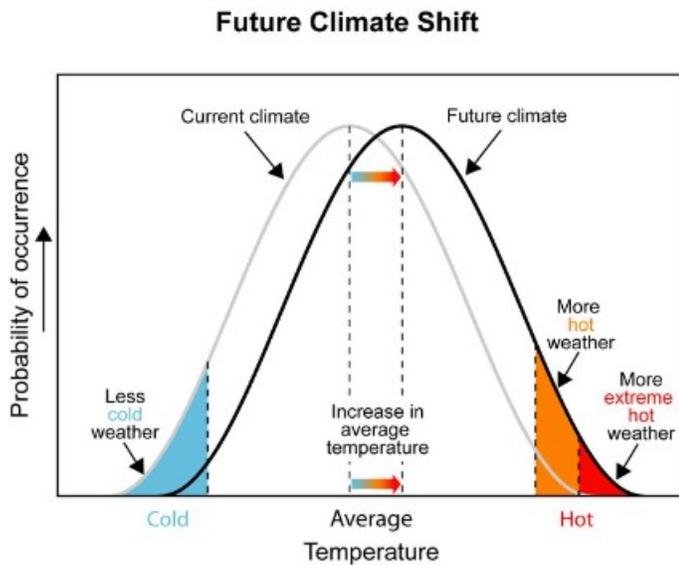

Figure 1.6 General estimation of climate change based on conditional probabilistic approach

b) Graphical approach in the form of spatial graphs to understand climate change:

The attempts further could not effective give an overview of the events happening due to climate change over a particular demography. There were alternate mathematical approaches that involved graph theory to study such changes which were more effective when combined with a particular probability theory.
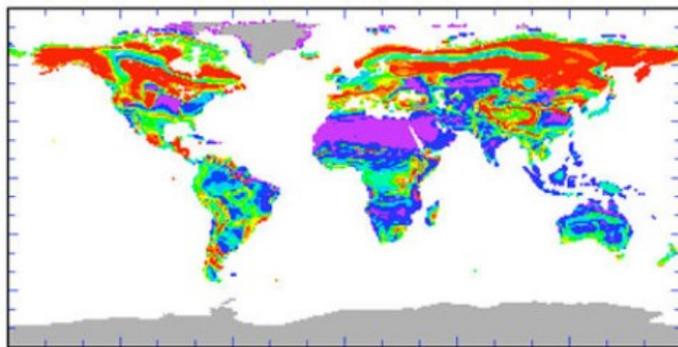

Figure 1.7 Spatial analysis of heat waves globally
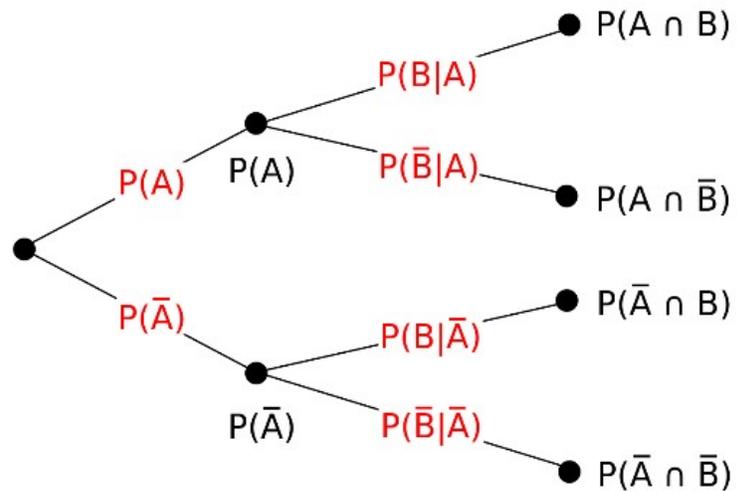

Figure 1.8 Relating conditional probability and graph theory to estimate further events.

The above views give an effective approach to minimize the losses ignored due to either graph theory or conditional probability. However there are clauses for this approach as well as it is studying a very vulnerable system and it is not a uniformly controlled body. We need a more descriptive method to relate such a system and also to predict its further outcome. We look at a similar concept described in one of the literature paper given by Burgess and Deschenes in one of their works on climate change which uses parameters like climate change associated to the components associated to its outcome. We shall carry out a mathematical analysis of these components in a parallel way so as to relate other components under our study to develop and understand the effectiveness.

2. Mathematical model relating the components associated to climate change and its outcome:

The process of climate change is primarily bifurcated into two aspects. The primary one is the long term climate change and the other one is the short term climate change. Both these factors cause changes according to their intensity over the environmental parameters including financial aspect and human population. We shall develop a differential model that changes with respect to time as a parameter and also associates with the other atmospheric components.

Let us consider the following abbreviations to form the fundamental assumptions of the model.

Cc: Climate change parameter.

$\partial Cc/\partial t$: Differential of climate change parameter with respect to time.

Δ_t: heat change parameter with respect to time.

ρ_t: atmospheric pressure parameter change with respect to time.

$\mathcal{H}_t$: humidity parameter change with respect to time.

The pre requisites of the measure of our climate change Cc and its differential with respect to time given by $\partial Cc/\partial t$ that further affects the other two parameters simultaneously with respect to time.

Where Cc may be consisting of various other factors we shall take into consideration the (Δ_t) heat change and atmospheric pressure (ρ_t) convoluted with the humidity parameter ($\mathcal{H}_t$) to cause a primary climate change over a particular demography.

Let us first know that these are not the only factors responsible for a climate change over a particular region but may be the prominent factors following which we are taking them into consideration at a primary level. To develop this model we may also assume various other factors in the further case of study. Initially we propose a simple expression Relating these parameters and further intend to analyze them mathematically.

$$C_c^{\,*} = \sum_{i=0}^{\infty} \rho_t(\Delta_t \cdot \mathcal{H}_t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.1$$

This above equation is our assumption of the parameters associated with each other that are in a positive correlation. A slight change in one of the parameters effects a change in the other components accordingly. The final output of climate change parameter C_c is a result of all these factors and shall be seen in the below expression.

Differentiating the equation 2.1 partially with respect to time.

$$(C_c^{\,*})' = \sum_{i=0}^{\infty} \frac{\partial \rho}{\partial t}\left(\frac{\partial \Delta}{\partial t} \cdot \frac{\partial \mathcal{H}}{\partial t}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.2$$

Further understanding the limits of each of these parameters we see,

$$\rho_t = \lim_{n\to\infty}(\rho_t)_n \; ;$$
$$\mathcal{H}_t = \lim_{n\to\infty}(\mathcal{H}_t)_n \; ;$$
$$\Delta_t = \lim_{n\to\infty}(\Delta_t)_n$$

The final limit of the climate change depends on the individual components. We introduce the concept of Cesaro summation and the fejer kernel to get an efficient quantity of the climate change parameter over these possibilities. Before we introduce the concept, we shall see their basic explanation over a particular sum.

2.1 Cesaro Summation

This concept is seen in most of the real analysis literature where a single entity is expressed as a collective order of discrete values. However these values are associated to each other over the sum and thereby cannot be counted separately over a domain. The values need to be in a real plain and the ideology developed in the studies of measure theory accordingly.

Consider a sequence of n tuples $(x_n)_{n=0}^{\infty}$ over a domain of a measurable space where components are discrete.

Then the nth partial sums given by

$$a_n = x_1 + x_2 + \cdots + x_n$$

To further check the convergence of a_n is to see the convergence of the below terms.

$$k_n = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

If $a_n \to a$ then $k_n \to a$ accordingly. We need not ignore that at times k_n converges even if a_n does not converge. But still it may be assumed that $a_n \to a$ with respect to the conditions prescribed in the cesaro forms.

2.2 Fejer Kernel

Whenever we consider a transformation of a particular element from one domain to another in the form of a function, we see a particular parameterization of these components. The scenario when such a similar situation happens thus gives thrust to the involvement of Fourier transformation as one of the forms.

The function $k_n f(x)$ forms partial Fourier sums towards a transformation seen as $f : T \to \mathbb{R}$

$$k_n f(p) = \frac{x_0 f(p) + x_1 f(p) + \cdots + x_{n-1} f(p)}{n}$$

Finally before defining the Fejer kernel we intend to express $k_n f(p)$ which means to convolute f with a particular kernel to get the below result.

$$k_n f(p) = \frac{x_0 f(p) + x_1 f(p) + \cdots + x_{n-1} f(p)}{n}$$

$$= \frac{1}{2\pi}\frac{1}{n}[\int_{-\pi}^{\pi} f(y)\,\Gamma_0(p-y)dy + \cdots$$

$$+ \int_{-\pi}^{\pi} f(y)\,\Gamma_{n-1}(p-y)dy]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(y)[\frac{1}{n}\sum_{v=0}^{n-1}\Gamma_v\ (p-y)]dy$$

Therefore the Fejer kernel for the above condition may be given as follows.

$$\mathcal{F}_n(t) = \frac{1}{n}\sum_{v=0}^{n-1}\Gamma_v(t)$$

Thereby combining the results with the above expressions we get;

$$k_n f(p) = \frac{1}{2\pi}\,(\mathcal{F}_n * f\,)(x)$$

This not only accumulates the three components of the climate change parameters associating together but also links them in the form of Fourier conditions. This being the main pursuit of the paper thereby declares the components of climate change given above to be expressed in the form of Fourier components.

3. Summary and conclusion:

In order to summarize the entire work in a particular form the motivation of the topic wherein the need for an efficient climate change model infuses the treatment of mathematical theories to link various factors together.

The theory of climate change began with the association of various factors that are primarily responsible to carry out a particular change in the outcome of climate change. The further involvement of Cesaro summation and the Fejer kernel simply relates the components of the climate change model as

per the Fourier forms. These give scope of treating the components of climate change known as the het change, humidity and the air pressure to be giving a different scope of analysis under this criterion.

REFERENCES:

1. Sathaye, J., Shukla, P. R., &Ravindranath, N. H. (2006). Climate change, sustainable development and India: Global and national concerns. Current Science,90(3),314-325.

2. Retrieved from http://www.ias.ac.in/currsci/feb102006/314.pdf.

3. Robin Burgess& Olivier Deschenes (2017). Weather, Climate Change and Death in India, Correspondence: ddonald@stanford.edu. Affiliations: Burgess: LSE and IGC; Deschenes: UCSB and NBER

4. Knox, J.; Hess, T.; Daccache, A.; Wheeler, T. Climate change impacts on crop productivity in Africa and South Asia. Environ. Res. Lett. 2012, 7, 034032. [CrossRef]

5. Cairns, J.E.; Sonder, K.; Zaidi, P.H.; Verhulst, N.; Mahuku, G.; Babu, R.; Nair, S.K.; Das, B.; Govaerts, B.; Vinayan, M.T.; et al. Maize production in a changing climate: Impacts, adaptation, and mitigation strategies. Adv. Agron. 2012, 114, 1–57. [CrossRef]