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μ-β-generalized α-closed sets in generalized topological spaces

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ABSTRACT

In this paper, we have introduced a new class of sets in generalized topological spaces called μ-β-generalized α-closed sets. Also we have investigated some of their basic properties.

Keywords: *Generalized topology, generalized topological spaces, μ-α-closed sets, μ-β-generalized α-closed sets.*

1. Introduction

The concept of generalized topological spaces is introduced by A. Csaszar [1] in 2002. He also introduced many μ-closed sets like μ-semi closed sets, μ-pre closed sets, μ-β-closed sets, μ-α-closed sets, μ-regular closed sets etc., in generalized topological spaces. In 2016 Srija. S and Jayanthi. D [4] introduced g_u-semi closed set in generalized topological spaces. In this paper, we have introduced a new class of sets in generalized topological spaces called μ-β-generalized α-closed sets. Also we have investigated some of their basic properties and produced many interesting theorems.

2. Preliminaries

Definition 2.1: [1] Let X be a nonempty set. A collection μ of subsets of X is a generalized topology (or briefly GT) on X if it satisfies the following:

i. Ø, X ∈ μ and
ii. If $\{M_i: i \in I\} \subseteq \mu$, then $\cup_{i \in I} M_{i} \in \mu$

If μ is a GT on X, then (X, μ) is called a generalized topological space (or briefly GTS), and the elements of μ are called μ-open sets and their complement are called μ-closed sets.

Definition 2.2: [1] Let (X, μ) be a GTS and let A ⊆ X. Then μ-closure of A, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A.

Definition 2.3: [1] Let (X, μ) be a GTS and let A ⊆ X. Then μ-interior of A, denoted by $i_\mu(A)$, is the union of all μ-open sets contained in A.

Definition 2.4: [1] Let (X, μ) be a GTS. A subset A of X is said to be

i. μ-semi-closed if $i_\mu(c_\mu(A)) \subseteq A$
ii. μ-pre-closed if $c_\mu(i_\mu(A)) \subseteq A$
iii. μ-α-closed if $c_\mu(i_\mu(c_\mu(A))) \subseteq A$
iv. μ-β-closed if $i_\mu(c_\mu(i_\mu(A))) \subseteq A$
v. μ-regular closed if $A = c_\mu(i_\mu(A))$

Definition 2.5: [3] Let (X, μ) be a GTS. A subset A of X is said to be

i. μ-regular generalized closed if $c_\mu(A) \subseteq U$ whenever A ⊆ U, where U is μ-regular open in X,
ii. μ-generalized closed set if $c_\mu(A) \subseteq U$ whenever A ⊆ U, and U is μ-open in X,
iii. μ-generalized α-closed set if $\alpha c_\mu(A) \subseteq U$ whenever A ⊆ U, and U is μ-open in X.

The complement of μ-semi-closed (respectively μ-pre-closed, μ-α-closed, μ-β-closed, μ-regular closed, etc.,) is called μ-semi-open (respectively μ-pre-open, μ-α-open, μ-β-open, μ-regular open, etc.,) in X.

3. μ-β-generalized α-closed sets in generalized topological spaces

In this section we have introduced μ-β-generalized α-closed sets in generalized topological spaces and studied some of their basic properties.

Definition 3.1: A subset A of a GTS (X, μ) is called a μ-β-generalized α-closed set (briefly μ-βGαCS) if $\alpha c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-β-open in X.

Example 3.2: Let X= {a, b, c} and let μ= {Ø, {a}, {b}, {a, b}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Then A = {c} is a μ-βGαCS in (X, μ).

Theorem 3.3: Every μ-closed set in (X, μ) is a μ-βGαCS in (X, μ) but not conversely.

Proof: Let A be a μ-closed set in (X, μ) then $c_\mu(A) = A$. Now let $A \subseteq U$ where U is μ-β-open in (X, μ). Then $\alpha c_\mu(A) \subseteq c_\mu(A) = A \subseteq U$, by hypothesis. Therefore $\alpha c_\mu(A) \subseteq U$. This implies, A is a μ-βGαCS in (X, μ).

Example 3.4: Let X= {a, b, c, d} and let μ = {Ø, {a}, {c}, {a, c}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {c}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, c, d}, {a, b, d}, X}. Then A = {d} is a μ-βGαCS in (X, μ), but not a μ-closed set as $c_\mu(A) = c_\mu(\{d\}) = \{b, d\} \neq A$.

Theorem 3.5: Every μ-α-closed set in (X, μ) is a μ-βGαCS in (X, μ).

Proof: Let A be a μ-α-closed in (X, μ), then $c_\mu(i_\mu(c_\mu(A))) \subseteq A$. Now let $A \subseteq U$ where U isμ-β-open in (X, μ). Then $\alpha c_\mu(A) = A \cup c_\mu(i_\mu(c_\mu(A))) = A \cup A = A \subseteq U$, by hypothesis. Therefore $\alpha c_\mu(A) \subseteq U$ and hence A is a μ-βGαCS in (X, μ).

Theorem 3.6: Every μ-regular closed set in (X, μ) is a μ-βGαCS in (X, μ) but not conversely.

Proof: Let A be a μ-regular closed set in (X, μ), then $c_\mu(i_\mu(A)) = A$. Now let $A \subseteq U$ where U is a μ-β-open set in (X, μ). Then $\alpha c_\mu(A) = A \cup c_\mu(i_\mu(c_\mu(A))) = A \cup c_\mu(i_\mu(c_\mu(c_\mu(i_\mu(A))))) = A \cup c_\mu(i_\mu(c_\mu(i_\mu(A)))) = A \cup c_\mu(i_\mu(A)) = A \subseteq U$, by hypothesis. Therefore $\alpha c_\mu(A) \subseteq U$. This implies, A is a μ-βGαCS in (X, μ).

Example 3.7: Let X= {a, b, c} and μ = {Ø, {a}, {b}, {a, b}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Then A = {c} is a μ-βGαCS in (X, μ), but not a μ-regular closed set as $c_\mu(i_\mu(A)) = c_\mu(i_\mu(\{c\}) = Ø \neq A$.

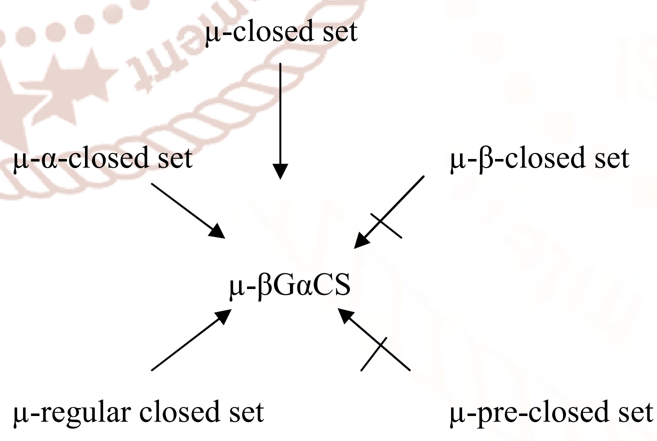
Remark 3.8: A μ-pre-closed is not a μ- β-GαCS in (X, μ) in general.

Example 3.9: Let X = {a, b, c} and μ= {Ø, {a, b}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Then A= {a} is a μ-pre-closed in (X, μ) as $c_\mu(i_\mu(A)) = c_\mu(i_\mu(\{a\})) = Ø \subseteq A$, but not a μ-βGαCS in (X, μ) as $\alpha c_\mu(A) = X \not\subseteq \{a, b\} = U$.

Remark 3.10: A μ-β-closed is not a μ-βGαCS in (X, μ) in general.

Example 3.11: Let X = {a, b, c} and μ = {Ø, {a, b}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Then A = {a} is a μ-β-closed in (X, μ) as $i_\mu(c_\mu(i_\mu(A)) = i_\mu(c_\mu(i_\mu(\{a\})) = Ø \subseteq A$, but not a μ-βGαCS as in Example 3.9.

In the following diagram, we have provided relations between various types of closed sets.



Theorem 3.12: Let A and B be μ-βGαCSs in (X, μ) such that $c_\mu(A) = \alpha c_\mu(A)$ and $c_\mu(B) = \alpha c_\mu(B)$, then $A \cup B$ is a μ-βGαCS in (X, μ).

Proof: Let $A \cup B \subseteq U$, where U is a μ-β-open set in X. Then $A \subseteq U$ and $B \subseteq U$. since A and B are μ-βGαCS, $\alpha c_\mu(A) \subseteq U$ and $\alpha c_\mu(B) \subseteq U$. Now, $\alpha c_\mu(A\cup B) \subseteq c_\mu(A\cup B) = c_\mu(A) \cup c_\mu(B) = \alpha c_\mu(A) \cup \alpha c_\mu(B) \subseteq U \cup U = U$. Hence, A∪B is a μ-βGαCS in (X, μ).

Theorem 3.13: If a subset A of X is a μ-βGαCS and $A \subseteq B \subseteq \alpha c_\mu(A)$, then B is a μ-βGαCS in X.

Proof: Let A be a μ-βGαCS such that $A \subseteq B \subseteq \alpha c_\mu(A)$. Suppose U is a μ-β-open set of X such that $B \subseteq U$, by assumption, we have $\alpha c_\mu(A) \subseteq U$ as $A \subseteq B \subseteq U$. Now $\alpha c_\mu(B) \subseteq \alpha c_\mu(\alpha c_\mu(A)) = \alpha c_\mu(A) \subseteq U$. That is $\alpha c_\mu(B) \subseteq U$. Therefore, B is a μ-βGαCS in X.

Theorem 3.14: In a GTS X, for each x ∊ X, {x} is μ-β-closed or its complement X – {x} is a μ-βGαCS in (X, μ).

Proof: Suppose that {x} is not a μ-β-closed set in (X, μ). Then X – {x} is not a μ-β-open set in (X, μ). The only μ-β-open set containing X – {x} is X. Thus $X – \{x\} \subseteq X$ and so $\alpha c_\mu(X – \{x\}) \subseteq \alpha c_\mu(X) = X$. Therefore $\alpha c_\mu(X – \{x\}) \subseteq X$ and so X– {x} is a μ-βGαCS in (X, μ).

Theorem 3.15: If A is both a μ-β-open set and μ-βGαCS in (X, μ), then A is a μ-α-closed set.

Proof: Let A be a μ-β-open set and μ-βGαCS in (X, μ). Then, $\alpha c_\mu(A) \subseteq A$ as $A \subseteq A$. we have $A \subseteq \alpha c_\mu(A)$. Therefore, $A = \alpha c_\mu(A)$. Hence A is a μ-α-closed set in (X, μ).

Theorem 3.16: Every subset of X is a μ-βGαCS in X iff every μ-β-open set in X is μ-α-closed in X.

Proof: Necessity: Let A be a μ-β-open set in X and by hypothesis, A is a μ-βGαCS in X. Hence by Theorem 3.15, A is a μ-α-closed set in X.

Sufficiency: Let A be a subset of X and U be a μ-β-open set such that $A \subseteq U$, then by hypothesis, U is μ-α-closed. This implies that $\alpha c_\mu(U) = U$ and $\alpha c_\mu(A) \subseteq \alpha c_\mu(U) = U$. Hence $\alpha c_\mu(A) \subseteq U$. Thus A is a μ-βGαCS set in (X, μ).

Theorem 3.17: A subset A of X is a μ-βGαCS if and only if $\alpha c_\mu(A) – A$ contains no non-empty μ-β-closed set in X.

Proof: Let A be a μ-βGαCS in X. Suppose F is a non-empty μ-β-closed set such that $F \subseteq \alpha c_\mu(A) – A$. Then $F \subseteq \alpha c_\mu(A) \cap A^c$. Therefore $F \subseteq \alpha c_\mu(A)$ and $F \subseteq A^c$. This implies $A \subseteq F^c$. Since F^c is a μ-β-open set, by definition of μ-βGαCS, $\alpha c_\mu(A) \subseteq F^c$. That is $F \subseteq (\alpha c_\mu(A))^c$. Hence $F \subseteq \alpha c_\mu(A) \cap (\alpha c_\mu(A))^c = \emptyset$. That is F= Ø, which is a contradiction. Thus $\alpha c_\mu(A) – A$ contains no non-empty μ-β-closed set in X.

Conversely, assume that $\alpha c_\mu(A) – A$ contains no non-empty μ-β-closed set in X. Let $A \subseteq U$, where U is a μ-β-open set in X. Suppose that $\alpha c_\mu(A) \not\subseteq U$, then $\alpha c_\mu(A) \cap U^c$ is a non-empty closed subset of $\alpha c_\mu(A) – A$, which is a contradiction. Therefore $\alpha c_\mu(A) \subseteq U$ and hence A is a μ-βGαCS in X.

Theorem 3.18: If $A \subseteq Y \subseteq X$ and A is a μ-βGαCS in X, then A is a μ-βGαCS relative to Y.

Proof: Given that A ⊆ Y X and A is a μ-βGαCS in X. Let A Y ∩ U, where U is μ-β-open set in X. since A is a μ-βGαCS, A U implies $\alpha c_\mu(A)$ U. This implies Y ∩ $\alpha c_\mu(A)$ Y ∩ U and $\alpha c_\mu(A)$ Y ∩ U. That is A is a μ-βGαCS relative to Y.

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