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Study of Silica Fume Blended Concrete using Statistical Mixture Experiment and Optimizing of Compressive Strength of Cement Concrete

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ABSTRACT

The prime objective of this study is to design, analyze and optimize the silica fume blended concrete using Statistical Mixture Experiment and to explore the practicability of Statistical mixture experiment in mix design of concrete containing the several components. The traditional mix design methods have drawbacks. Traditional method of mix design of concrete fails in designing the complex concrete mixture containing the cementitious materials, pozzolanic materials and several admixtures. Traditional methods are based on trial and error method in which effect of only one component can be determined. Trial and error does not provide the whole picture of concrete mixture.

In this study, the statistical mixture design method is employed to analyze the effects of silica fume on 28 days compressive strength of concrete and to design and optimize the silica fume blended concrete. The components of concrete mixture are Cement, silica fume, water and aggregate. Total of 15 runs or design points are generated which are combinations of proportions and laboratory experiment is carried out on these design points and 28 days compressive strength is measured. Mathematical relationship is established between 28 days compressive strength and proportions of components using least square technique and then mathematical model is studied to analyze the effects of silica fume in concrete.

Keywords: *Silica fume, Compressive strength, Slump Value, Total sum of square, Model sum of square*

1. INTRODUCTION

Concrete is a mixture of many components such as cement, water, aggregates, and supplementary cementitious materials such as silica fume etc. For given set of materials, the proportions of components directly manipulate the properties of concrete mixture, both fresh and hardened. It is essential to optimize this complex mixture to meet the certain performance criteria such as compressive strength, workability etc.

Combined with numerous performance criteria, the number of trial batches required to find optimal proportions using traditional methods could become prohibitive.

The common approach to concrete mixture proportioning can be described by the following steps:

1. Identifying a preliminary set of mixture proportions.
2. Performing many trial batches, first with the mixture identified in step 1 above, and adjusting the proportions in following trial batches until all criteria are fulfilled.

1.1 Experimental Investigation

- To use statistical mixture experiment as a systematic approach in design and optimization of mix design of silica fume blended concrete.
- To statistically investigate the effects of silica fume in concrete using statistical mixture experiment.

2. BACKGROUND ON STATISTICAL MIXTURE EXPRIMENT

2.1 Mixture Experiments

The experiments which involve blending of several parts are known as mixture experiments. In these experiments the quality or response of the end product depends upon relative proportion of the parts in a mixture.

In mixture experiments, the treatments are either:

1. A grouping of two or more components whose sum is a fixed amount implying that their levels are not self-determining (Montgomery 2005)
2. Quantity of inputs functional at various experimental steps such that sum total of the quantities functional at different experimental steps is fixed.

2.2 Statistical Mixture Experiment

Statistical mixture experiment is used to conduct experiments on mixture of two or more components. In mixture experiment, the input variables or factors are proportion of components of mixture and the sum of the proportion of components is unity i.e. the quantity of mixture is constant. The proportions of components are not independent i.e. if there is n number of components in mixture; only n−1 can be set independently. Responses of mixture are various

Properties resulting from blending of components of mixture and depend upon the proportion of components. Let us consider a mixture in which there are n components. If X1, X2,….,Xn are the proportions of n components respectively, then relation between proportions is given by

$$0 \leq X_i \leq 1, \; i = 1,2,3, \dots \dots \dots . n \qquad \textbf{(2.1)}$$

$$X_1 + X_2 + X_3 + \cdots \dots \dots . + X_n = 1 \qquad \textbf{(2.2)}$$

Equation (2.1) and (2.2) describe the constraints on proportions. Thus $n - 1$ components vary between 0 and 1 and remaining component is dependent on settings of $n - 1$ components. (Murty & Das, 1968).

In the case of concrete mixture, components are cement, pozzolanic materials, water, coarse aggregate and fine aggregate and response or output variables are compressive strength, tensile strength, flexural strength, workability etc.

2.3 Experimental Design Detail

In the statistical mixture methodology, the measured responses are assumed to depend on the proportion of materials present in the mixture rather than on the amount of mixture. In general, in a mixture with n-components where xi represents the proportion of the i^{th} component in the mixture, the relation between variables is as in equation [2.3]

Scheffe's linear model is

$$Y = \sum_{i=1}^{n} \beta_i X_i \qquad \textbf{(2.3)}$$

Where,

Y = response of mixture

n = number of components of mixture

X_i = proportion of i^{th} component

β_i = a coefficient of regression. (Cornell, 1981)

For example, the linear model for the mixture of three components is

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \qquad \textbf{(2.4)}$$

The mathematical relationship is established between proportions i.e. input variables and output variables. In concrete mixture the input variables are proportions of cement, water, pozzolanic materials, coarse aggregate and fine aggregate and the output variable or response is compressive strength and slump value. The mathematical relationship between proportions of component and response or output variables is called as mathematical model of mixture.

3. METHODOLOGY

The whole procedure of statistical mixture experiment is as follows:

1) Selection of design points.
2) Selection of mathematical model.
3) Experiment on design points.
4) Mathematical Model fitting.
5) Mathematical Model validation.
6) Prediction and Optimization of response variables according to performance criteria.

3.1 Selection of Design Points

There are various methods available to determine design points i.e. the points or proportions on which experiment is carried out. The methods are simplex lattice design, and d-optimal design. In this study using d-optimal design for selection of design point.

3.1.2 D-Optimal Design

In d-optimal design, first mathematical model is chosen and design points are generated by computer algorithm.

Let the response of mixture is related to its proportions of components by equation (3.1).

$$Y = X\beta + e \qquad \textbf{(3.1)}$$

Where, Y is vector of observation, X is vector of proportions, β is vector of least square estimator and e is the vector of errors. These vectors are given by

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, \qquad X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1k} \\ X_{21} & X_{21} & \cdots & X_{2k} \\ \vdots & \vdots & & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nk} \end{bmatrix},$$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}, \qquad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix},$$

n is the number of observations and k is the number of components in mixture.

The least square estimator β is given by

$$\beta = (X^T X)^{-1} X^T Y \qquad \textbf{(3.2)}$$

And variance of β is given by

$$var(\beta) = \sigma^2 [X'X]^{-1} \qquad \textbf{(3.3)}$$

Where, σ^2 is error variance. The variance of β is directly proportional to $[X'X]^{-1}$ and variance of β should be minimum for best fit. In d-optimal design, the vector of mixture proportions are so chosen that $[X'X]^{-1}$ becomes minimum. (Montgomery, et al., 2002)

3.2 Selection of Mathematical Model

Appropriate mathematical model is chosen after selection of design points. There is close relation between selection of design points and model selection i.e. the selection of model is depend upon number of design points of experiment. The polynomial model is recommended however other models can also be chosen. The special Scheffe's canonical polynomial equations are used for modelling of mixture system which are as follows:

$$Y = \sum_{i=1}^{n} \beta_i X_i \qquad (3.4)$$

Where, Y is the response of mixture, n is the number of components of mixture, X_i is the proportion of i^{th} component and β_i is the coefficient of regression.

For example, the linear model for the mixture of three components is

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

Scheffe's Quadratic model can be represented as

$$Y = \sum_{i=1}^{n} \beta_i X_i + \sum \sum_{i<j}^{n} \beta_{ij} X_i X_j \qquad (3.5)$$

where, Y is the response of mixture, n is the number of components of mixture, X_i, X_j are the proportions of i^{th}, j^{th} component, β_i is a coefficient of linear portion of equation and β_{ij} is the coefficient of quadratic portion of equation. (Cornell, 1981)

For example, the quadratic model for the mixture of three components is

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3 \qquad (3.6)$$

3.3. Experiments on Design Points

The experiment is carried out on generated design points of mixture and response variables or properties of mixture are calculated for all design points. The design point is generated by design expert software based on d-optimal design.

3.4. Mathematical Model Fitting

The selected Model is fitted to data obtained in experiment using principle of least squares and Analysis of variance (ANOVA) and mathematical relationship is established between input variables i.e. proportions of components of mixture and responses of mixture. (Simon, et al., 1997)

3.5. Mathematical Model Validation

The polynomial models explain in sections 3.2.3 is fit to data using analysis of variance (ANOVA) and least squares techniques. Many statistical software packages have the ability to perform these analyses and fits. Once a model has been fit, it is needed to verify the adequacy of the chosen model quantitatively and graphically.

4. EXPERIMENTATION AND RESULTS

4.1 Selection of Components of Mixture System

In a mixture experiment, the total sum (mass or volume) of the mixture is settled, and the variables or segment settings are extents of the total sum for concrete, the whole of the volume portions is compelled to total to one. Since the volume divisions must whole to unity, the segment factors in a mixture experiment are not free.

4.2 Selection of Components of Mixture System

The components of concrete mixture system are cement, silica fume, water and aggregate. The proportions of component of concrete mixtures are expressed by volume. The unit of volume is m^3. The total volume of concrete mixture is unity i.e. 1 m3. The proportions of components of concrete mixture are input variables.

4.2.1 Cement

43 grade of Ordinary Portland Cement produced by BIRLA Cement limited conforming to IS: 12269-2013 is use in this research. The specific gravity of Portland cement is 3.15 which are measured using Le Chaterlier's flask. The Twenty Eight days compressive strength of Portland cement is measured As per IS: 4031 (Part 6)-1988. The initial and final setting time is measured according to IS: 4031 (Part 5)-1988.

Table4.1: Properties of cement.

S. No	Properties	Value
1.	Initial setting time (min)	92
2.	Final setting time (min)	408
3.	28 days Compressive strength (MPa)	57
4.	Specific gravity	3.15

4.2.2 Silica Fume

Silica fume is supplied by **Normet India Private limited, U.S Nagar Uttrakhand.** The specific gravity of silica fume is taken 2.2. (Holland, 2005)

2.2.3 Course Aggregate

The coarse aggregate is obtained from locally available resources conforming to IS: 383-1970. The maximum size of 20 mm is selected for this study. The properties of coarse aggregate, which are measured, are as follows:

Table4.2: Properties of Course Aggregate

S. No	Properties	Value
1.	Specific gravity	2.65
2.	Water absorption	1.15
3.	Aggregate impact value	7.070%
4.	Aggregate crushing value	19%

4.2.4 Fine Aggregate

Specific gravity and water absorption of fine aggregate are determined in this study.

Table 4.3: Properties of Fine Aggregate

S. No	Properties	Value
1.	Specific gravity	2.65
2.	Water absorption	0.80%

4.2.5 Water

Water conforming to IS code is used in this research

4.3 Selection of Responses

Responses are properties of mixture. In this research, following responses are selected for study:

1. 28 days Compressive strength of mixture.
2. Slump value of mixture.

4.4 Selection of Appropriate Ranges of Components of Mixture

In statistical mixture experiment, the sum of proportions of components is unity. The selection of maximum and minimum level of proportion of components is depending upon engineering knowledge about mixture.

The maximum and minimum level of proportion of each component is selected according to previous researches and conforming to IS: 10262-2009 and IS: 456-2000. These minimum and maximum levels are called as constraints. In addition to these constraints, linear constraints are also applied in this design to keep the water to cementitious ratio 0.4 to 0.5. the % level of replacement of silica fume are keep 5 to 25 %.The maximum and minimum level of proportion of components by volume is shown in Table 4.4, the minimum and maximum level of proportion of components by weight is shown in Table 4.5.

Table 4.4: Minimum and Maximum Volume Fraction of Components of Concrete Mixture.

Component	Nomenclature	Minimum volume fraction (m^3)	Maximum volume fraction (m^3)
Cement	X_1	0.1060	0.1342
Silica fume	X_2	0.01	0.05
Water	X_3	0.180	0.200
Aggregate	X_4	0.644	0.692

Table 4.5: Minimum and Maximum Weight Fraction of Components of Concrete Mixture.

Component	Minimum weight fraction (kg/ m^3)	Maximum weight fraction (kg/ m^3)
Cement	333.90	422.73
Silica fume	22.22	111.32
Water	180	200
Aggregate	1706.6	1833.8

Table4.6: Linear constraints on proportions of components of concrete mixture

Water to cementitious ratio	Linear constraint
$0.4 \leq \frac{water}{3.15cement + 2.2\ silica\ fume}$	$water - 1.26\ cement - 0.88\ silica\ fume \leq 0$
$\frac{water}{3.15cement + 2.2\ silica\ fume} \leq 0.5$	$water - 1.575\ cement - 1.1\ silica\ fume \leq 0$

4.5 Design of Experiment

After selection of appropriate range of components, design points are generated. In this study, design points are generated according to d-optimal criteria by Design Expert Software. Design points are combinations on which the experiment is carried out.

There are 4 coefficients in mathematical model thus 15 design points are generated according to d-optimal criteria. The design points (combinations of components) by volume are shown in Table 4.6 and by weight in Table 4.8 Due to complexity of experiment and due to limited design points, the coarse aggregate and fine aggregate variables are considered as single variable "Aggregate". The main objective of study is focussed on three components cement, silica fume and water and the relative proportions of coarse aggregate and fine aggregate are deduced according to specifications given in IS:10262-2009. Design points in terms of cement, silica fume, water, coarse aggregate and fine aggregate are shown in Table 4.7

Table 4.7: design points by volume which is generated according to d-optimal design.

Run order	Cement (m^3)	Silica fume (m^3)	Water (m^3)	Aggregate (m^3)
1	0.1342	0.0101	0.2	0.6557
2	0.1314	0.0142	0.1852	0.6693
3	0.1286	0.0182	0.2	0.6532
4	0.1271	0.0202	0.1898	0.6628
5	0.1243	0.0243	0.18	0.6714
6	0.1229	0.0263	0.18	0.6708
7	0.1215	0.0283	0.1906	0.6596
8	0.1201	0.0303	0.194	0.6556
9	0.1158	0.0364	0.2	0.6477
10	0.1130	0.0405	0.18	0.6665
11	0.1116	0.0425	0.2	0.6459
12	0.1102	0.0445	0.18	0.6653
13	0.1088	0.0465	0.1914	0.6533
14	0.1074	0.0485	0.18	0.6641
15	0.1060	0.0506	0.2	0.6435

Table 4.8: Design points by volume in terms of cement, silica fume, water, coarse aggregate and fine aggregate.

Run Order	Cement (m^3)	Silica fume (m^3)	Water (m^3)	Coarse Aggregate (m^3)	Fine Aggregate (m^3)
1	0.1342	0.0101	0.2	0.4196	0.2360
2	0.1314	0.0142	0.1852	0.4283	0.2409
3	0.1286	0.0182	0.2	0.4181	0.2352
4	0.1271	0.0202	0.1898	0.4242	0.2386
5	0.1243	0.0243	0.18	0.4297	0.2417
6	0.1229	0.0263	0.18	0.4293	0.2415
7	0.1215	0.0283	0.1906	0.4221	0.2375
8	0.1201	0.0303	0.194	0.4196	0.2360
9	0.1158	0.0364	0.2	0.4146	0.2332
10	0.1130	0.0405	0.18	0.4266	0.2400
11	0.1116	0.0425	0.2	0.4134	0.2325
12	0.1102	0.0445	0.18	0.4258	0.2395
13	0.1088	0.0465	0.1914	0.4181	0.2352
14	0.1074	0.0485	0.18	0.4250	0.2391
15	0.1060	0.0506	0.2	0.4118	0.2317

Table 4.9: design points by weight in terms of cement, silica fume, water, coarse aggregate and fine aggregate.

Run Order	Cement (Kg/M3)	Silica Fume (Kg/M3)	Water (Kg/M3)	Coarse Aggregate (Kg/M3)	Fine Aggregate (Kg/M3)
1	422.73	22.22	200	1111.94	625.4
2	413.91	31.24	185.2	1134.995	638.385
3	405.09	40.04	200	1107.965	623.28
4	400.365	44.44	189.8	1124.13	632.29
5	391.545	53.46	180	1138.705	640.505
6	387.135	57.86	180	1137.645	639.975
7	382.725	62.26	190.6	1118.565	629.375
8	378.315	66.66	194	1111.94	625.4
9	364.77	80.08	200	1098.69	617.98
10	355.95	89.1	180	1130.49	636
11	351.54	93.5	200	1095.51	616.125
12	347.13	97.9	180	1128.37	634.675
13	342.72	102.3	191.4	1107.965	623.28
14	338.31	106.7	180	1126.25	633.615
15	333.9	111.32	200	1091.27	614.005

4.6 Laboratory Experiment

Responses corresponding to design point are measured by conducting laboratory experiment on design points.
The responses measured are as follows:

1. 28 days compressive strength.
2. Slump value.

Three replicates of cubes of concrete are casted for each design point and after 28 days, compressive strength test is carry out as per IS: 516- 1959 to resolve compressive strength of concrete cubes for each design point. Compressive strength of cubes is measured on Compression Testing Machine (CTM). The unit of measurement of compressive strength is MPa. The rate of loading is 4 KN/sec.

The workability of concrete is determined on the basis of slump value. The slump value for each design point is calculated according to specifications given in IS: 7320-1974.

4.7 Response

The compressive strength at the age of 28 days and slump value is measured for each concrete mix and test results are shown in table.

Table 4.10: Responses of mixture which are measured in laboratory

Run No.	Average 28 days compressive strength (MPa)	Slump value (mm)
1	24.3	85
2	26.3	80
3	28.42	72
4	30.62	65
5	33.32	58
6	37.52	55
7	34	50
8	32	45
9	30	40
10	28.42	35
11	27	32
12	26	25
13	25	20
14	24	18
15	24	15

4.8 Mathematical Model Fitting.

The mathematical model for 28 days compressive strength is linear Scheffe's cannonical polynomial as shown in equation.

$$Y_1 = \beta_1^{(1)} X_1 + \beta_2^{(1)} X_2 + \beta_3^{(1)} X_3 + \beta_4^{(1)} X_4 + e_1 \qquad (4.1)$$

Where,

Y_1 = 28 days compressive strength in MPa.

X_1 = Proportion of cement by volume (m^3).

X_2 = Proportion of silica fume by volume (m^3).

X_3 = Proportion of water by volume (m^3).

X_4 = Proportion of aggregate by volume (m^3).

e_1 = Error of model

And $\beta_1^{(1)}, \beta_2^{(1)}, \beta_3^{(1)}, \beta_4^{(1)}$ are coefficients of Scheffe's cannonical polynomial equation of 28 days compressive strength.
The mathematical model for slump value is also linear Scheffe's cannonical polynomial as shown in equation.

$$Y_2 = \beta_1^{(2)}X_1 + \beta_2^{(2)}X_2 + \beta_3^{(2)}X_3 + \beta_4^{(2)}X_4 + e_2 \qquad (4.2)$$

Where,
Y_2 = Slump value in mm.
X_1 — Proportion of Cement by volume (m3).
X_2 = Proportion of silica fume by volume (m3).
X_3 = Proportion of water by volume (m3).
X_4 = Proportion of aggregate by volume (m3).
e_2 = Error of model.

And $\beta_1^{(2)}, \beta_2^{(2)}, \beta_3^{(2)}, \beta_4^{(2)}$ are coefficients of Scheffe's cannonical polynomial equation of slump value.
The data is obtained from test results and fitted to Scheffe's cannonical polynomial equation by method of least square. The least square estimator can be shown in matrix form in equation.

$$\beta = (X^TX)^{-1}X^TY \qquad (4.3)$$

Where, $\beta = \begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\end{bmatrix}$, $\quad Y = \begin{bmatrix}Y^{(1)}\\ Y^{(2)}\\ \vdots\\ Y^{(15)}\end{bmatrix}$,

$$X = \begin{bmatrix} X_1^{(1)} & X_2^{(1)} & X_3^{(1)} & X_4^{(1)} \\ X_1^{(2)} & X_2^{(2)} & X_3^{(2)} & X_4^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ X_1^{(15)} & X_2^{(15)} & X_3^{(15)} & X_4^{(15)} \end{bmatrix}$$

The Y and X matrix are shown in Table 5.14.

Table 4.11: X and Y matrix.

X matrix				*Y* matrix	
X_1	X_2	X_3	X_4	Y_1	Y_2
0.1342	0.0101	0.2	0.6557	24.3	85
0.1314	0.0142	0.1852	0.6693	26.3	80
0.1286	0.0182	0.2	0.6532	28.42	72
0.1271	0.0202	0.1898	0.6628	30.62	65
0.1243	0.0243	0.18	0.6714	33.32	58
0.1229	0.0263	0.18	0.6708	37.52	55
0.1215	0.0283	0.1906	0.6596	34	50
0.1201	0.0303	0.194	0.6556	32	45
0.1158	0.0364	0.2	0.6477	30	40
0.1130	0.0405	0.18	0.6665	28.42	35
0.1116	0.0425	0.2	0.6459	27	32
0.1102	0.0445	0.18	0.6653	26	25
0.1088	0.0465	0.1914	0.6533	25	20
0.1074	0.0485	0.18	0.6641	24	18
0.1060	0.0506	0.2	0.6435	24	15

By using equation the least square technique, coefficients are calculated. The coefficients of Scheffe's cannonical polynomial equation of 28 days compressive strength is shown in Table 4.11.

Table 4.12: Coefficient of Scheffe's Canonical Polynomial Equation of 28 Days Compressive Strength

$\beta_1^{(1)}$	$B_2^{(1)}$	$B_3^{(1)}$	$B_4^{(1)}$
-143.88	-146.92	-77.97	99.80

The mathematical model of compressive strength is

$$Y_1 = -143.88 * X1 - 146.92 * X_2 - 77.97 * X_3 + 99.80 * X4 + e_1$$

Where, e_1 is the error of model.

The coefficients of Scheffe's cannonical polynomial equation of slump value are shown in Table. 4.12

Table 4.13: Coefficient of Scheffe's Cannonical Polynomial Equation of Slump Value

$\beta_1^{(2)}$	$B_2^{(2)}$	$B_3^{(2)}$	$B_4^{(2)}$
-526.46	-2266.93	226	210

The mathematical model of slump value is

$$Y_2 = -526.46 * X_1 - 2266.93 * X_2 + 226 * X_3 + 210 * X_4 + e_2$$

Where, e_2 is the error of the model.

Table 4.14 Error value for compressive strength and slump value

Run order	For compressive strength model		For slump model	
	Error	Error value	Error	Error value
1	e_1^1	-4.75	e_2^1	-4.35
2	e_1^2	-5.06	e_2^2	-1.04
3	e_1^3	0	e_2^3	-1.41
4	e_1^4	.62	e_2^4	-4.37
5	e_1^5	1.81	e_2^5	-3.14
6	e_1^6	6.16	e_2^6	-2.22
7	e_1^7	4.68	e_2^7	-3.47
8	e_1^8	3.43	e_2^8	-4.60
9	e_1^9	3	e_2^9	2.27
10	e_1^{10}	-1.85	e_2^{10}	5.66
11	e_1^{11}	0.43	e_2^{11}	6.25
12	e_1^{12}	-3.96	e_2^{12}	3.51
13	e_1^{13}	-2.78	e_2^{13}	2.24
14	e_1^{14}	-4.66	e_2^{14}	4.35
15	e_1^{15}	-1.94	e_2^{15}	5.18

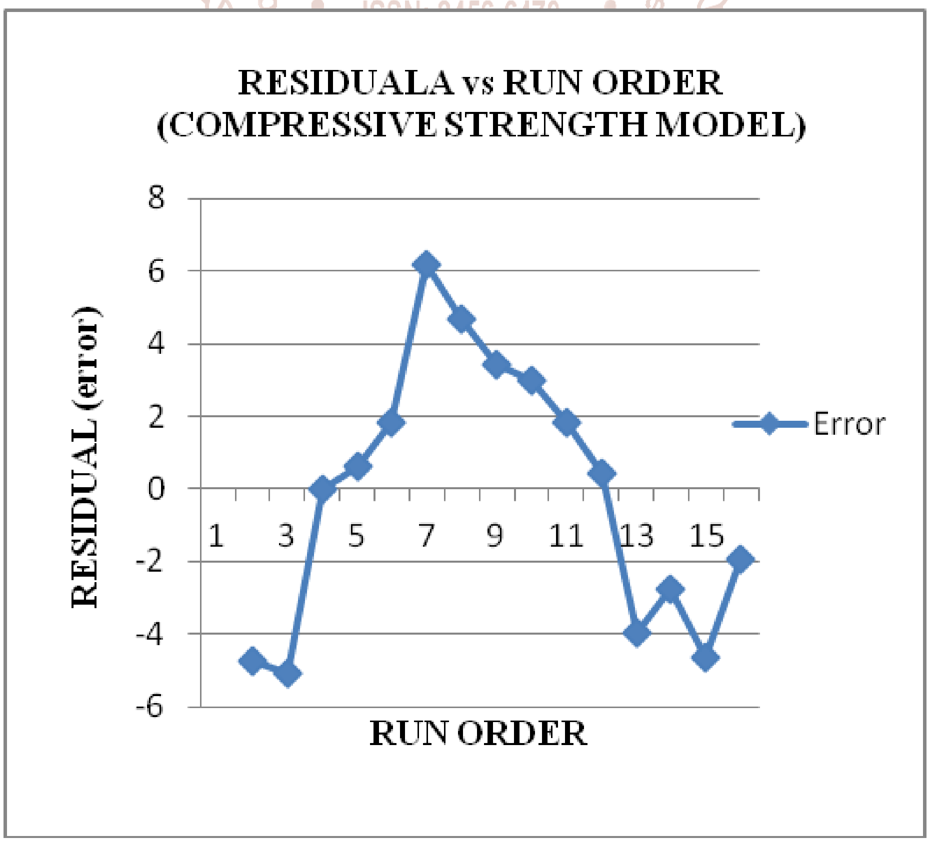

Figure 4.1: Response trace plot of residual value (Error) in compressive strength model

Fig 4.1 Shows the graph between residual (Error) and run order for compressive strength model. Maximum error is found in run order 06.

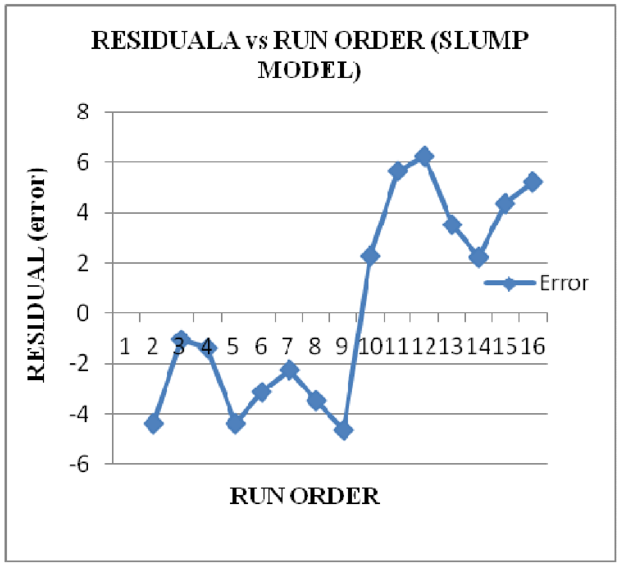

Figure 4.2: Response trace plot of residual value (Error) in slump model

Fig 4.2 Shows the graph between residual (Error) and run order for slump model. Maximum error is found in run order 11.

4.9 MATHEMATICAL MODEL VALIDATION

Validation of mathematical model is done by Analysis of Variance (ANOVA). The analysis of variance (ANOVA) details are shown in table.

In ANOVA, hypothesis testing is done. The null hypothesis is that there is no relationship between input variables and output variables and the alternative hypothesis is that there is relationship between input variables and output variables.
The null hypothesis H_0 is

$$H_o : \beta_1 = \beta_2 = \cdots = \beta_4 = 0$$

And
The alternative hypothesis H_A is
$H_A : \beta_j \neq 0$ for atleast one j, $\quad j = 1,2,..,4$

If there is relationship between dependent and independent variables, we reject null hypothesis and accept alternative hypothesis that there is relationship between response variable and input variables i.e. proportions of mixture.
The null hypothesis is rejected when mean of sum of square of model is relatively large than mean of sum of squares of error because total sum of square is explained more by sum of squares of model than sum of square of error and if sum of square of error is large than model is not adequate. The ratio of mean of sum of square of model to mean of sum of squares of error follows F distribution.

The means of total sum of square, model sum of square and error sum of square are given as

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Where,

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \text{total sum of square} = TSS$$

$$\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 = \text{model sum of square} = MSS$$

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \text{error sum of square} = ESS$$

Mean of sum of square due to model $MSS_M = SS_M / k$

Mean of sum of square due to error $MSS_E = SS_E / (n - 1 - k)$

Where,
n = number of observations
k = number of coefficients in regression model

The ratio of mean of model sum of square to mean of error sum of square F is given as

$$F = (\frac{MSS}{k})/(\frac{RSS}{n-k-1})$$

If MSS is relatively large than ESS then F value becomes large and if F value is so large that it.

4.10 VALIDATION OF TWENTY EIGHT DAYS COMPRESSIVE STRENGTH MODEL

The calculation of ANOVA for twenty eight days compressive strength model is shown in Table 4.13

The F value calculated is 1.56 which is greater than $F_{critical}$ value 0.2749 thus null hypothesis is rejected and model is adequate.

Table4.15: Analysis of variance calculations for mathematical model of compressive strength.

Source	Sum of Squares	Degree of Freedom	Mean of Sum of Square	F Value	F Critical For 95% Confidence Interval
Model	186.73	3	62.24	1.56	0.2547
Residual	439.07	11	39.92		

5. GRAPHICAL INTERPRETATION

Response trace plot is plot between mean deviation of proportions from centroid of design points and response of mixture. So as to have a better understanding of effect of each component on the response, a trace plot is used. The slope of plot indicates the sensitivity of response to change in proportion of component.

Table 5.1: The value of compressive strength & slump for different run order

Run order	Cement (m^3)	Silica fume (m^3)	Compressive Strength (Mpa)	Slump Value
1	0.1342	0.0101	24.3	85
2	0.1314	0.0142	26.3	80
3	0.1286	0.0182	28.42	72
4	0.1271	0.0202	30.62	65
5	0.1243	0.0243	33.32	58
6	0.1229	0.0263	37.52	55
7	0.1215	0.0283	34	50
8	0.1201	0.0303	32	45
9	0.1158	0.0364	30	40
10	0.1130	0.0405	28.42	35
11	0.1116	0.0425	27	32
12	0.1102	0.0445	26	25
13	0.1088	0.0465	25	20
14	0.1074	0.0485	24	18
15	0.1060	0.0506	24	15

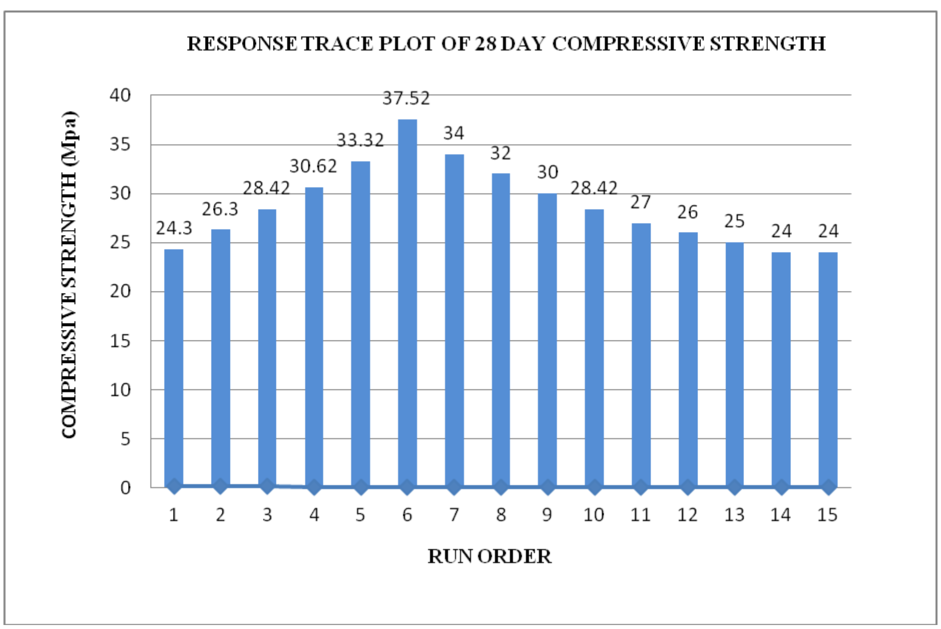

Figure 5.1: Response trace plot of 28 days compressive strength

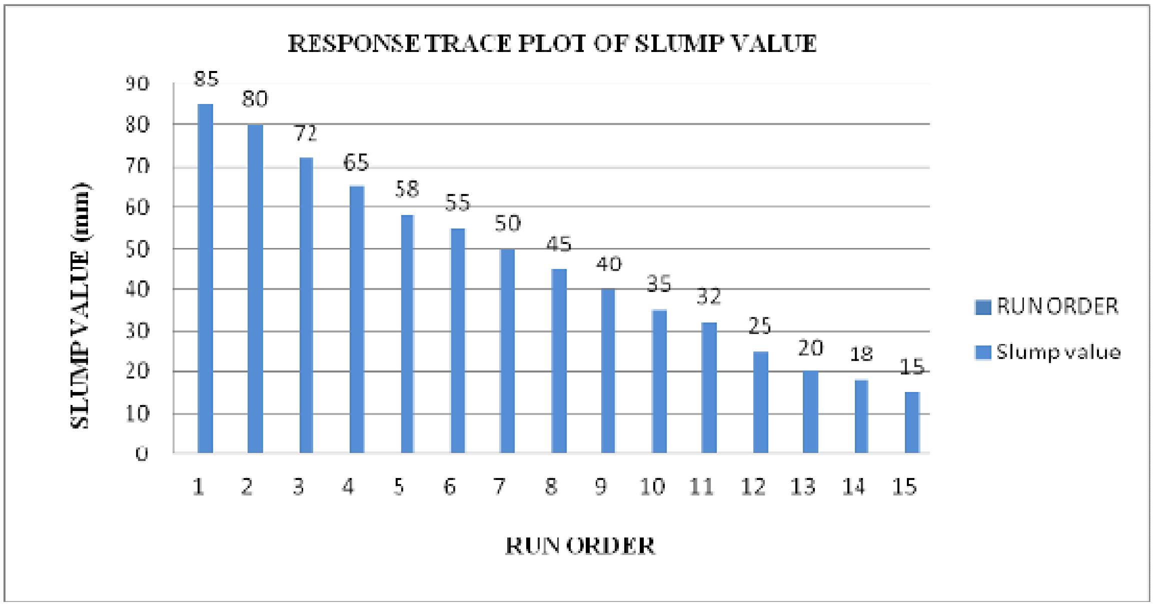

Figure 5.2: Response trace plot slump value

The response trace plot of twenty eight days compressive strength for different run is shown in Figure 5.1. The Plot shows that compressive strength increases gradually up to 6th run. After that the compressive strength is decreased. The maximum compressive strength is found in model no 06. The response trace plot of slump value is shown in Figure 5.2. The Plot shows that the slump value decrease when proportion of silica fume is increased.

Similarly response trace plot of compressive strength for % of replacement of silica fume shown in Figure 5.3. The plot show that the compressive strength is increased gradually up to 15%. And after that the value of compressive strength is decreased.

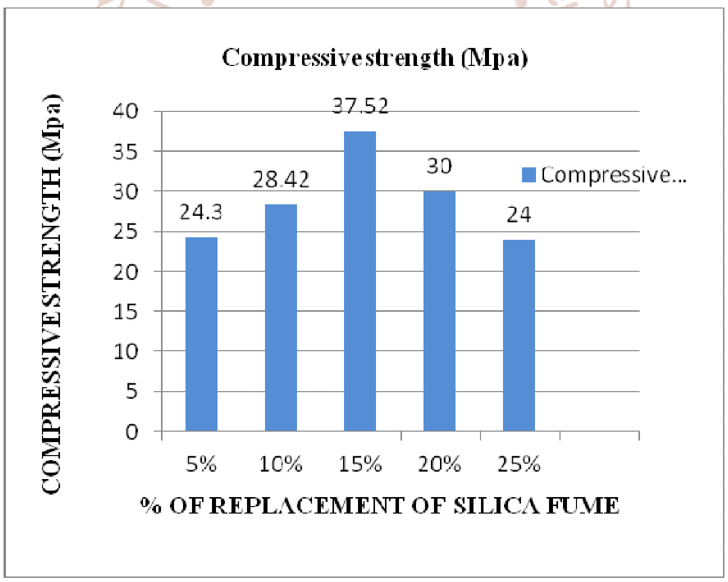

Figure 5.3: Response trace plot of 28 days compressive strength.

5.1 Optimization using desirability function method

The maximum 28 days compressive strength is 37.52 MPa. For obtaining the optimization, desirability function method is used. The criteria are to maximize the compressive strength, slump value, silica fume and minimize the cement. The calculations of optimization are shown in Table 5.2 and the solution of optimization is shown in Table 5.3

Table 5.2: Goals and criteria of optimization of 28 days compressive strength

Name	Goal	Lower limit	Upper Limit
Cement	Minimize	0.1016	0.140
Silica fume	Maximize	0.01	0.05
Water	In range	0.180	0.200
Aggregate	In range	0.644	0.692
Compressive strength	Maximize	24	37.52
Slump value	In rang	15	85

Table 5.3: Solution of optimization of 28 days compressive strength using desirability function method

Cement (cu- m)	Silica fume (cu-m)	Water (cu-m)	Aggregate (cu-m)	Compressive strength (Mpa)	Slump value (mm)
0.1229	0.0263	0.180	0.6708	37.52	55

6. CONCLUSION

1. Statistical mixture experiment method can be used for mixture proportioning of Silica fume blended concrete and normal concrete.
2. In statistical mixture experiment, smaller number of experiments is required to obtain meaningful data about concrete mixture than traditional trial and error method.
3. Simplex lattice and simplex centroid designs are not suitable for mixture proportioning of concrete because concrete ingredients usually vary between minimum level greater than 0 and maximum level less than 1.
4. D-optimal designs are suitable for concrete mixture proportioning.
5. The Mathematical model of 28 days compressive strength which is obtained in study is

$$Y_1 = -143.88 * X1 - 146.92 * X_2 - 77.97 * X_3 + 99.80 * X4 + e_1$$

Where,
Y_1 = 28 days compressive strength in MPa.
X_1 = Proportion of Cement by volume (m^3).
X_2 = Proportion of silica fume by volume (m^3).
X_3 = Proportion of water by volume (m^3).
X_4 = Proportion of aggregate by volume (m^3).
e_1 = Error of model.
This model is used to predict and optimize the 28 days compressive strength of silica fume blended concrete.

6. The mathematical model of slump value which is obtained in study is

$$Y_2 = -526.46 * X_1 - 2266.93 * X_2 + 226 * X_3 + 232.5542 * X_4 + e_2$$

Where,
Y_2 = Slump value in mm.
X_1 = Proportion of Cement by volume (m^3).
X_2 = Proportion of silica fume by volume (m^3).
X_3 = Proportion of water by volume (m^3).
X_4 = Proportion of aggregate by volume (m^3).
e_2 = Error of model

This model is used to predict and optimize the slump value of silica fume blended concrete.

7. The maximum compressive strength of 37.52 MPa is obtained for silica fume blended concrete.
8. The optimum level of silica fume is 15%.

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