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Oscillation of Even Order Nonlinear Neutral Differential Equations of E

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ABSTRACT

This paper presently exhibits about the oscillation of even order nonlinear neutral differential equations of E of the form

$$\left(e(t)z^{(n-1)}(t)\right)' + r(t)f(h(\gamma(t)) + v(t)f(\delta(t)) = 0$$

Where $z(t) = x(t) + p(t)x(\rho(t)), n \geq 2$, is a even integer. The output we considered $\int_{t_o}^{\infty} e^{-1}(t)dt = \infty$, and $\int_{t_o}^{\infty} e^{-1}(t)dt < \infty$. This canon here extracted enhanced and developed a few known results in literature. Some model are given to embellish our main results.

INTRODUCTION

We apprehensive with the oscillation theorems for the following half-linear even order neutral delay differential equation

$$\left(e(t)z^{(n-1)}(t)\right)' + r(t)f(h(\gamma(t)) + v(t)f(\delta(t)) = 0, t \geq t_{0,} \qquad (1)$$

Where$z(t) = x(t) + p(t)x(\rho(t)), n \geq 2$, is a even integer .Every part of this paper, we assume that:

(E_1) $e \in C([t_0,\infty), E), e(t) > 0, e'(t) \geq 0;$

(E_2) $p, q \in C([t_0,\infty), E)$,

$0 \leq p(t) \leq p_0 < \infty, q(t) > 0$, where p_0 is a constant;

(E_3) $\rho \in C^1([t_0,\infty), E), \gamma \in C([t_0,\infty), E)$,

$\delta \in C([t_0,\infty), E), \rho'(t) \geq \rho_0 > 0$,

$\gamma(t) \leq t,\ \delta(t) \leq t, \rho \circ \gamma = \gamma \circ \rho$,

$\rho \circ \delta = \delta \circ \rho$,

$\lim_{t\to\infty} \gamma(t) = \infty$, $\lim_{t\to\infty} \delta(t) = \infty$, where ρ_0 is a constant.

(E_4) $f \in C(E, E)$ and

$f(x)/x \geq M_1, M_2 > 0$, for $x \neq 0$, where M_1, M_2 is constant.

Then the two cases are

$$\int_{t_0}^{\infty} \frac{1}{e(t)} dt = \infty \qquad (2)$$

$$\int_{t_0}^{\infty} \frac{1}{e(t)} dt < \infty, \qquad (3)$$

By a solution z of (1) a function be

$e \in C^{m-1}([t_x,\infty), E)$ for some $t_z \geq t_0$,

Where $z(t) = x(t) + a(t)x(\rho(t))$, has a property $ez^{n-1} \in C^1([t_x,\infty), E)$ and satisfies (1) on (t_z,∞). Then (1) satisfies $sup\{|x)t): t \geq T|\} > 0$ for all $T \geq t_x$ is called oscillatory.

In certain case when $n = 2$ the equation (1) lessen to the following equations

$(e(t)\left(x(t)+p(t)x(\rho(t))'\right)' + \quad r(t)f(h(\gamma(t)) + v(t)f(\delta(t)) = 0, t \geq t_{0,}$ (4)

Where $\int_{t_0}^{x} e^{-1}(t)dt = \infty$,

$$\rho(t) \leq t, \gamma(t) \leq t, \delta(t) \leq t,$$

$0 \leq p(t) \leq p_0 < \infty$.

Then the oscillatory behavior of the solutions of the neutral differential equations of the second order

$(e(t)\left(x(t)+p(t)x(\rho(t))'\right)' + r(t)(h(\gamma(t)) + v(t)(\delta(t)) = 0, t \geq t_0$ (5)

Where $\int_{t_0}^{x} e^{-1}(s)ds = \infty$,

$0 \leq p(t) \leq p_0 < \infty$.

The usual limitations on the coefficient of (5) be $\rho(t) \leq t, \gamma(t) \leq \rho(t), \delta(t) \leq \rho(t)$,

$\gamma(t) \leq t, \delta(t) \leq t, 0 \leq p(t) < 1$, are not assumed.

ρ Could be a advanced argument and γ, δ could be a delay argument ,

Some known expand results are seen in [1,5]. Then the

Even-order nonlinear neutral functional differential equations

$(x(t)+p(t)x(\rho(t)))^{(n)} + r(t)f(h(\gamma(t)) + v(t)f(\delta(t)) = 0, t \geq t_{0,}$ (6)

Where n is even $0 \leq p(t) < 1$ and $\rho(t) \leq t$.

(A) Lemma. 1

The oscillatory behavior of solutions of the following linear differential inequality

$$w'(t) + r(t)h(\gamma)) + v(t)g(\delta(t)) \leq 0$$

Where $r, v, \gamma, \delta \in C([t_0, \infty))$,

$\gamma(t)t, \delta(t) \leq t$

$$\lim_{t\to\infty} \gamma(t) = \infty \quad , \lim_{t\to\infty} \delta(t) = \infty$$

Now integrating from $\gamma(t)$ to t and $\delta(t)$ to t

If

$$\lim_{t\to\infty} inf \int_{\gamma(t)}^{t} r(s)ds + \lim_{t\to\infty} inf \int_{\delta(t)}^{t} v(s)ds > \frac{1}{e},$$

Then it has no finally positive solutions.

Main results

The main results which covenant that every solution of (1) is oscillatory

(1) $\lim_{t\to\infty} inf \int_{\gamma(t)}^{t} J(s)ds > \frac{1}{e}$

(2) $\lim_{t\to\infty} sup \int_{\delta(t)}^{t} J(s)ds > 1$

B.Theorem. 2.1

Assume that $\int_{t_0}^{\infty} \frac{1}{e(t)} dt = \infty$ holds. If

$$\int_{t_0}^{\infty} S_1(t) + \int_{t_0}^{\infty} S_2(t)\, dt = \infty$$

Where $S_1(t)$= min $\{r(t), r(\rho(t))\}$ $S_2(t)$= min$\{v(t), v(\rho(t))\}$, then every solutions of (1) is oscillatory.

Proof

Suppose, on the contradictory,x is a nonoscillatory solutions of (1). Without loss of generality, we may assume that there exists a constant $t_1 \geq t_0$, such that

$x(t) > 0, x(\rho(t)) > 0$ and $(\gamma(t)) > 0$,

$x(\delta(t)) > 0$ for all $t \geq t_1$. Using the definitions of z and x is a eventually positive solution of (1). Then there exists $t_1 \geq t_0$, such that

$z(t) > 0, z'(t) > 0, z^{(n-1)}(t) > 0$ and $z^n(t) \leq 0$ for all $t \geq t_1$.

.Hencelim$_{t\to\infty} z(t) \neq 0$.

Applying (E_4) and (1) we get

$$\left(e(t)z^{(n-1)}(t)\right)' \leq -M_1 r(t)h(\gamma(t)) < 0, \qquad t \geq t_1$$

$$\left(e(t)z^{(n-1)}(t)\right)' \leq -M_2 v(t)g(\delta(t)) < 0, \; t \geq t_1.$$

Therefore $(e(t)z^{(n-1)}(t))$ is a nonincreasing function. Besides, from the above inequality and the definition of z, we get

Case (1)

$$\left(e(t)z^{(n-1)}(t)\right)' + M_1 r(t)h\left(\gamma(t)\right) + \frac{p_0}{\rho'(t)}\left(e\left(\rho(t)\right)z^{(n-1)}\left(\rho(t)\right)' + M_1 p_0 r((\rho(t))h(\gamma((\rho(t))) \le 0$$

$$(e(t)z^{(n-1)}(t))' + M_1 R(t)z\left(\gamma(t)\right) + \frac{p_0}{\rho_0}(e\left(\rho(t)\right)z^{(n-1)}(\rho(t)))' \le 0 \qquad (7)$$

Integrating (7) from t_1 *to* t, we have

$$\int_{t_1}^{t}(e(s)z^{(n-1)}(s))'ds + M_1\int_{t_1}^{t}R(s)z\left(\gamma(s)\right)ds + \frac{p_0}{\rho_0}\int_{t_1}^{t}(e\left(\rho(s)\right)z^{(n-1)}(\rho(s)))'ds \le 0$$

Pointing that $\rho'(t) \ge \rho_0 > 0$

$$M_1\int_{t_1}^{t}R(s)z\left(\gamma(s)\right)ds \le -\int_{t_1}^{t}e(s)z^{(n-1)}(s))'ds - \frac{p_0}{\rho_0}\int_{t_1}^{t}\frac{1}{\rho'(s)}(e\left(\rho(s)\right)z^{(n-1)}(\rho(s)))'ds\, d(\rho(s)$$

$$\le e(t_1)z^{(n-1)}t_1 - (e(t)z^{(n-1)}(t)) + \frac{p_0}{\rho_0^2}(e\left(\rho(t_1)\right)z^{(n-1)}(\rho(t_1) - (e(\rho(t))z^{(n-1)}(e(t))) \qquad (8)$$

Since $z'(t) > 0$ for $t \ge t_1$. We can find a invariable $c > 0$ such that

$$z\left(\gamma(t)\right) \ge c, t \ge t_1.$$

Then from (8) and the fact that $(e(t)z^{(n-1)}(t))$ is non increasing, we obtain

$$\int_{t_0}^{\infty}S_1(t) < \infty \qquad (9)$$

Case (2)

$$(e(t)z^{(n-1)}(t))' + M_2 v(t)g\left(\delta(t)\right) + \frac{p_0}{\rho'(t)}\left(e\left(\rho(t)\right)z^{(n-1)}\left(\rho(t)\right)' + M_2 p_0 v((\rho(t))g(\delta((\rho(t))) \le 0$$

$$\left(e(t)z^{(n-1)}(t)\right)' + M_2 V(t)z\left(\delta(t)\right) + \frac{p_0}{\rho_0}\left(e\left(\rho(t)\right)z^{(n-1)}\left(\rho(t)\right)' \qquad (10)$$

Integrating (10) from t_1 *to* t, we have

$$\int_{t_1}^{t}e(s)z^{(n-1)}(s))'ds + M_2\int_{t_1}^{t}V(s)z\left(\delta(s)\right)ds + \frac{p_0}{\rho_0}\int_{t_1}^{t}\left(e\left(\rho(s)\right)z^{(n-1)}\left(\rho(s)\right)'ds \le 0$$

Pointing that $\rho'(t) \ge \rho_0 > 0$

$$M_2\int_{t_1}^{t}V(s)z\left(\delta(s)\right)ds \le -\int_{t_1}^{t}e(s)z^{(n-1)}(s))'ds - \frac{p_0}{\rho_0}\int_{t_1}^{t}\frac{1}{\rho'(s)}(e\left(\rho(s)\right)z^{(n-1)}(\rho(s)))'ds\, d(\rho(s))$$

$$\le (e(t_1)z^{(n-1)}t_1 - (e(t)z^{(n-1)}(t)) + \frac{p_0}{\rho_0^2}(e\left(\rho(t_1)\right)z^{(n-1)}(\rho(t_1) - \left(e\left(\rho(t)\right)z^{(n-1)}(e(t))\right) \qquad (11)$$

Since $z'(t) > 0$ for $t \ge t_1$. We can find a invariable $c > 0$ such that

$$z\left(\delta(t)\right) \ge c, t \ge t_1.$$

Then from (8) and the fact that $(e(t)z^{(n-1)}(t))$ is nonincreasing, we obtain $\int_{t_0}^{\infty}S_2(t) < \infty$ (12)

We get inconsistency with (9),(12).

$$\int_{t_0}^{\infty}S_1(t) + \int_{t_0}^{\infty}S_2(t) = \infty$$

Theorem. 2.2

Assume that $\int_{t_0}^{\infty}\frac{1}{e(t)}dt = \infty$ holds and $\rho(t) \ge t$. if either

$$\lim_{t\to\infty} inf\left(\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}ds + \int_{\delta(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))}ds\right) > \frac{(p_0+\rho_0)(n-1)!}{\rho_0^e}, \qquad (13)$$

Or when γ , δ is increasing,

$\lim_{t\to\infty} sup(\int_{\gamma(t)}^{t} \frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))} ds + \int_{\delta(t)}^{t} \frac{\delta^{n-1}(s)K(s)}{e(\delta(s))} ds) > \frac{(p_0+\rho_0)(n-1)!}{\rho_0}$, (14)

Where $(t) = \min\{M_1 j(t), M_1 j(\rho(t))\}$,

then every solution of (1) is oscillatory .

Proof

Suppose, on the contrary x is a oscillatory solution of (1). Without loss of generality, we may assume that there exists a constant $t_1 \geq t_0$, such that t $x(t) > 0$,

$x(\rho(t)) > 0$ and $x(\gamma(t)) > 0$, $x(\delta(t)) > 0$ for all $t \geq t_1$.

Assume that $x^{(n)}(t)$ is not consonantly zero on any interval $[t_0, \infty)$, and there exists a $t_1 > t_0$. Such that$x^{(n-1)}(t)u^{(n)}(t) \leq 0$ for all $t \geq t_1$. If $\lim_{t\to\infty} x(t) \neq 0$,then for every λ, $0 < \lambda < 1$, there exists$T \geq t_1$, such that for all $t \geq T$,

$x(t) \geq \frac{\lambda}{(n-1)!} t^{n-1} u^{(n-1)}$(t) forever

$0 < \lambda < 1$, we obtain

$(e(t)z^{(n-1)}(t))' + \frac{p_0}{\rho_0}(e(\rho(t))z^{(n-1)}(\rho(t))' + \frac{\lambda}{(n-1)!}\gamma^{n-1}(t)J(t)z^{(n-1)}(\gamma((t)) + \frac{\lambda}{(n-1)!}\delta^{n-1}(t)K(t)z^{(n-1)}(\gamma((t)) \leq 0$,

For every t sufficiently large.

Let $x(t) = \left(e(t)z^{(n-1)}(t)\right) > 0$. Then for all t large enough, we have

$$\left(x(t) + \frac{p_0}{\rho_0} x(\rho(t))\right)' + \frac{\lambda}{(n-1)!} \frac{\gamma^{n-1}(t)J(t)}{e(\gamma(t))} x(\gamma(t)) + \frac{\lambda}{(n-1)!} \frac{\delta^{n-1}(t)K(t)}{e(\delta(t))} x(\delta(t)) \leq 0 \quad (15)$$

Next , let us denote

$y(t) = x(t) + \frac{p_0}{\rho_0} x(\rho(t))$. Since x is non increasing, it follows from $\rho(t) \geq t$ that

$y(t) \leq \left(1 + \frac{p_0}{\rho_0}\right) x(t)$. (16)

By combining (15) and (16), we get

$y'(t) + \frac{\rho_0}{p_0+\rho_0} \frac{\lambda}{(n-1)!}\left(\frac{\gamma^{n-1}(t)J(t)}{e(\gamma(t))} y(\gamma(t)) + \frac{\delta^{n-1}(t)K(t)}{e(\delta(t))} y(\delta(t))\right) \leq 0$ (17)

Therefore , y is a non negative solutions of (17).

Then there will be two cases

Case (1)

If $\lim_{t\to\infty} inf(\int_{\gamma(t)}^{t} \frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))} ds + \int_{\delta(t)}^{t} \frac{\delta^{n-1}(s)K(s)}{e(\delta(s))} ds) > \frac{(p_0+\rho_0)(n-1)!}{\rho_0^e}$ holds then a constant be $0 < \lambda_0 < 1$, such that

$\lim_{t\to\infty} inf(\int_{\gamma(t)}^{t} \frac{\lambda_0}{(n-1)!}(\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))} ds + \int_{\delta(t)}^{t} \frac{\delta^{n-1}(s)K(s)}{e(\delta(s))} ds)) > \frac{1}{e}$, (18)

By lemma (1), (18) holds that (17) has negative solutions, which is contradictory.

Case (2)

By the definition of y and

$(e(t)z^{(n-1)}(t))' + M_1 R(t)z(\gamma(t)) + \frac{p_0}{\rho_0}(e(\rho(t))z^{(n-1)}(\rho(t)))' \leq 0$

we get ,

$y'(t) = x'(t) + \frac{p_0}{\rho_0}\left(x(\rho(t))\right)' \leq -J(t)z(\gamma(t)) - K(t)z(\delta(t)) < 0$ (19)

pointing that $\gamma(t) \leq t, \delta(t) \leq t$, there exists $t_2 \geq t_1$, such that

$y(\gamma(t)) \geq y(t), y(\delta(t)) \geq y(t), t \geq t_2$. (20)

Integrating (17) from $\gamma(t) to\ t$ and $\delta(t)$ to t and applying γ , δ is increasing, we have

$$y(t)-(y(\gamma(t))+y(\delta(t)))+\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}(\,y(\gamma(s))+\int_{\gamma(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))}\,y(\delta(s)))ds\leq 0, t\geq t_2\ .$$

Thus

$$y(t)-(y(\gamma(t))+y(\delta(t)))+\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}((y(\gamma(t))\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}+y(\delta(t)))\int_{\gamma(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))})\,ds\leq 0, t\geq t_2$$

From the above the inequality,we get

$$\frac{y(t)}{(y(\gamma(t))+y(\delta(t)))}-1+\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}+\int_{\gamma(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))})\,ds\leq 0$$

From (20), we have

$$\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}+\int_{\gamma(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))})\,ds\leq 1, t\geq t_2 \qquad (21)$$

Taking upper limits as $t\to\infty$ in (21) we get

$$\lim_{t\to\infty} sup(\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}ds+\int_{\delta(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))}ds)\leq\frac{(p_0+\rho_0)(n-1)!}{\lambda\rho_0}, \qquad (22)$$

If (14) holds, we choose a constant

$0<\lambda_0<1$ such that

$$\lim_{t\to\infty} sup(\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}ds+\int_{\delta(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))}ds)>\frac{(p_0+\rho_0)(n-1)!}{\lambda_0\rho_0},$$

Which is in contrary with (22).

Theorem 2.3

Assume that $\int_{t_0}^{\infty}\frac{1}{e(t)}dt=\infty$ holds and $\gamma(t)\leq\rho(t)\leq t, \delta(t)\leq\rho(t)\leq t.$ If either

$$\lim_{t\to\infty} inf(\int_{\rho^{-1}\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}ds+\int_{\rho^{-1}\delta(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))}ds)>\frac{(p_0+\rho_0)(n-1)!}{\rho_0^{e}}, \qquad (23)$$

Or when $\rho^{-1}o\gamma$, $\rho^{-1}o\delta$ is increasing,

$$\lim_{t\to\infty} sup(\int_{\rho^{-1}\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}ds+\int_{\rho^{-1}\delta(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))}ds)>\frac{(p_0+\rho_0)(n-1)!}{\rho_0} \qquad (24)$$

Where J, K is defined as in theorem (2.2), then every solution of (1) is oscillatory.

Proof

Suppose, on the contrary x is a oscillatory solution of (1). Without loss of generality, we may assume that there exists a constant $t_1\geq t_0$, such that

$x(t)>0, x(\rho(t))>0$ and

$(\gamma(t))>0x(\delta(t))>0$ for all $t\geq t_1$. Continuing as in the proof of the theorem (2.2), we have

$$\left(x(t)+\frac{p_0}{\rho_0}x(\rho(t))\right)'+\frac{\lambda}{(n-1)!}\frac{\gamma^{n-1}(t)J(t)}{e(\gamma(t))}\,x(\gamma(t))+\frac{\lambda}{(n-1)!}\frac{\delta^{n-1}(t)K(t)}{e(\delta(t))}\,x(\delta(t))\leq 0.$$ Let

$y(t)=x(t)+\frac{p_0}{\rho_0}x(\rho(t))$ again. Since x is non increasing, it follows from $\rho(t))\leq 0$ that

$$y(t)\leq(1+\frac{p_0}{\rho_0})x(\rho(t)) \qquad (25)$$

By combining (15) and (24), we get

$$y'(t)+\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}\left(\frac{\gamma^{n-1}(t)J(t)}{e(\gamma(t))}y\left(\rho^{-1}(\gamma(t))\right)+\frac{\delta^{n-1}(t)K(t)}{e(\delta(t))}\ y\left(\rho^{-1}(\delta(t))\right)\right)\leq 0 \qquad (26)$$

Therefore, y is a positive solution of (26). Now, we consider the following two cases , on (23) and (24) holds .

Case (1)

If

$$\lim_{t\to\infty} inf(\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}ds+\int_{\delta(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))}ds)>\frac{(p_0+\rho_0)(n-1)!}{\rho_0^{e}}$$ holds then a constant be $0<\lambda_0<1$, such that

$\lim_{t\to\infty} inf(\int_{\gamma(t)}^{t} \frac{\lambda_0}{(n-1)!}(\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}ds +$
δttδn− 1sKseδsds))>1e (27)

Therefore (27) holds (26) has no positive solutions which is a contradiction.

Case (2)

From (19) and the condition

$\gamma(t) \leq \rho(t), \delta(t) \leq \rho(t)$, there exists

$t_2 \geq t_1$, such that $y\left(\rho^{-1}\left(\gamma(t)\right)\right) \geq y(t)$,

$y\left(\rho^{-1}\left(\delta(t)\right)\right) \geq y(t) \quad t \geq t_2.$ (28)

Integrating (26) from $\rho^{-1}\left(\gamma(t)\right)$tot, $\rho^{-1}\left(\delta(t)\right)$ and applying $\rho^{-1}o\gamma$ is nondecreasing, then we get

$y(t) - y\left(\rho^{-1}\left(\gamma(t)\right)\right) +$
$\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}\int_{\rho^{-1}\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}y\left(\rho^{-1}\left(\gamma(t)\right)\right) +$
ρ−1δ(t)tδn−1sK(s)e(δs)yρ−1δtds≤ 0 , t≥t2.

Thus

$y(t) - y\left(\rho^{-1}\left(\gamma(t)\right)\right) +$
$\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}y\left(\rho^{-1}\left(\gamma(t)\right)\right)\int_{\rho^{-1}\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))} +$
yρ−1δtρ−1δ(t)tδn−1sK(s)e(δs)ds≤0, t≥t2.

From the inequality , we obtain

$$\frac{y(t)}{y\left(\rho^{-1}\left(\gamma(t)\right)\right)} - 1\,\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}\int_{\rho^{-1}\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))} + \int_{\rho^{-1}\delta(t)}^{t}\frac{\delta^{n-1}(s)K(s)}{e(\delta(s))}ds \leq 0\,.$$

From (28) we get

$\frac{\rho_0}{p_0+\rho_0}\frac{\lambda}{(n-1)!}\int_{\rho^{-1}\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))} +$
ρ−1δ(t)tδn−1sK(s)e(δs)ds≤1, t≥t2. (29)

Taking the upper limit as $t \to \infty$ in (29), we get

$\lim_{t\to\infty} sup(\int_{\gamma(t)}^{t}\frac{\gamma^{n-1}(s)J(s)}{e(\gamma(s))}ds +$
δ(t)tδn−1sK(s)e(δs)ds)≤(p0+ρ0)(n−1)!λρ0,
(30)

Then the proof is similar to that of the theorem (2.2) then it is contradiction to (24).

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