International Journal of Trend in Scientific Research and Development (IJTSRD)
International Open Access Journal
ISSN No: 2456 - 6470 | www.ijtsrd.com | Volume - 2 | Issue – 2

Some new sets and a new decomposition of fuzzycontinuity, fuzzy almost strongI-continuity via idealization

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ABSTRACT

In this paper, we discuss the notion of fuzzy strong β -I -open sets. We introduce fuzzy almost I -open, fuzzy almost strong I -open sets, fuzzy SβI -set, fuzzy DI -sets closely related with fuzzy strong β -I -open sets. Additionally, we investigate some characterizations and properties of these sets. With the help of fuzzy strong β -I -open set and fuzzy DI -set to obtain a new decomposition of fuzzy continuity viaidealization.

***Keywords**: Fuzzy topological ideal; fuzzy strong β -I -open set; fuzzy DI -set; fuzzy SβI -set; fuzzy almost I -open; fuzzy almost strong I -open set*

1. Introduction and Preliminaries

Fuzziness is one of the most important and useful concepts in the modern scientific studies. In 1965, Zadeh[11] first introduced the notion of fuzzy sets. In 1945, Vaidyanathaswamy[9] introduced the concepts of ideal topological spaces. In 1990, Jankovic and Hamlett[4] have defined the concept of I -open set via localfunction in ideal topological space. In 1997, Mahmoud[6] and sarkar[8] independently presented some of the ideal concepts in the fuzzy trend and studied many other properties. Decomposition of fuzzy continuity is one of the many problems in the fuzzy topology. It becomes very interesting when decomposition is done via fuzzytopological ideals.

Throughout this paper, X represents a nonempty fuzzy set and fuzzy subset A of X , denoted by $A \leq X$, is characterized by a membership function in the sense of Zadeh[11]. The basic fuzzy sets are the empty set, the whole set and the class of all fuzzy subsets of X which will be denoted by 0, 1 and I^X respectively. A subfamily τ of I^X will denote topology of fuzzy sets on I^X as defined by Chang[3]. By (X,τ),we mean a fuzzy topological space in Chang's sense. A fuzzy point in X with support $x \in X$ and value $\alpha(0 < \alpha \leq 1)$is denoted by x_α . For a fuzzy subset A of X , Cl(A), Int(A) and 1 – Awill respectively, denote the fuzzy closure, fuzzy interior and fuzzy complement of A. A nonempty collection I of fuzzy subsets of X is called a fuzzyideal[8] if and only if

1. $B \in I$ and $A \leq B$, then $A \in I$ (heredity),
2. if $A \in I$ and $B \in I$ then $A \vee B \in I$ (finite additivity).

A fuzzy ideal topological space, denoted by (X,τ,I)means a fuzzy topological space with a fuzzy ideal I and fuzzy topology τ . For (X,τ,I), the fuzzy local function of $A \leq X$ with respect to τ and I is denoted by $A^*(\tau,I)$(briefly A^*)and is defined as $A^*(\tau,I) = \vee\{x \in X : A \wedge U \notin I$ for every $U \in \tau(x)\}$. While $A^*$is the union of the fuzzy points x such that if $U \in \tau(x)$ and $E \in I$,then there is at least one $y \in X$ for which $U(y) + A(y) - 1 > E(y)$. Fuzzy closure operator of a fuzzy set in$(X,\tau,I)$is defined as $Cl^*(A) = A \vee A^*$. In(X,τ,I),the collection $\tau^*(I)$means an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class

$\beta = \{U - E : U \in \tau, \in I\}$as a base[8]. This topology of fuzzy sets is considered as generalization of the ordinary one.

First, we shall recall some definitions used in the sequel.

Lemma 1.1. [8] Let(X,τ,I) be fuzzy ideal topological space and A, B subsets of X.

The following properties hold:

(a) If $A \leq B$, then $A^* \leq B^*$,
(b) $(A \vee B)^* = A^* \vee B^*$,
(c) $A^* = Cl(A^*) \leq Cl(A)$,
(d) if $U \in \tau$, then $U \wedge A^* \leq (U \wedge A)^*$,
(e) if $U \in \tau$, then $U \wedge Cl^*(A) \leq Cl^*(U \wedge A)$.

Lemma 1.2. [5] Let(X,τ)be an ideal topological space with an arbitrary index Δ, I an ideal of subsets of X and $\rho(X)$the power set of X. If $\{A_\alpha : \alpha \in \Delta\} \leq \rho(X)$, then the following property holds:

$$\vee_{\alpha\in\Delta}(A_\alpha^*) \leq (\vee_{\alpha\in\Delta}A_\alpha)^*$$

Definition 1.1. A subset A of a space$(X,\tau,I)$is said to be

(a) fuzzy α-I -open[10] if $A \leq Int(Cl^*(Int(A)))$,
(b) fuzzy β -I -open[10] if $A \leq Cl(Int(Cl^*(A)))$.
(c) fuzzy I [2]-open if $A \leq Int(A^*)$.
(d) fuzzy pre-I -open[2] if $A \leq Int(Cl^*(A))$.
(e) fuzzy semi-I -open[1] if $A \leq Cl^*(Int(A))$.

Definition 1.2. [10] A subset A of a space$(X,\tau)$is said to be fuzzy β -open if $A \leq Cl(Int(Cl(A)))$.

Definition 1.3. [12] A subset A of a space$(X,\tau,I)$is said to be fuzzy $*$-I -dense in itself if $A \leq A^*$.

2. Fuzzy Strong β-I-open set and Almost Fuzzy Strong I-open set

Definition 2.1. A subset A of a space$(X,\tau,I)$is said to be fuzzy almost -I -open if $A \leq Cl(Int(A^*))$.

Definition 2.2. A subset A of a space$(X,\tau,I)$is said to be fuzzy strong β -I -open if $A \leq Cl^*(Int(Cl^*(A)))$. We denote the family of all fuzzy strong β -I -open set of(X,τ,I) by $Fs\beta IO(X)$.

Definition 2.3. A subset A of a fuzzy ideal topological space$(X,\tau,I)$is said to be almost fuzzy strong-I -open if $A \leq Cl^*(Int(A^*))$. We denote the family of all fuzzy almost strong-I -open set of (X,τ,I) by $FasIO(X)$.

Proposition 2.1. For a subset of a fuzzy ideal topological space(X,τ,I) the followinghold:

(a) Every fuzzy semi-I -open set is fuzzy strong β -I -open.
(b) Every fuzzy β -I -open is fuzzy β -open.

Proof: (a) Let A be a fuzzy semi-I -open set. Then $A \leq Cl^*(Int(A)) \leq Cl^*(Int(Cl^*(A)))$.
This shows that $A \in Fs\beta IO(X)$.

(b) Let A is fuzzy β -I -open. We have, $A \leq Cl(Int(Cl(A))) \leq Cl(Int(Cl(A)))$. Therefore, A is fuzzy β -open.

Remark 2.1. The converses of each statement in proposition 2.1 need not be true as the following examples show.

Example 2.1. Let X = {a,b,c} and A, B ,C be fuzzy subsets of X defined asfollows:

A(a) = 0.3, A(b) = 0.2, A(c) = 0.4,
B(a) = 0.8, B(b) = 0.8, B(c) = 0.4.

Let $\tau = \{0,B,1\}$. If we take $I = \{0\}$, then A is fuzzy strong β -I -open but A isnot fuzzy semi-I -open set.

Example 2.2. Let X = {a,b,c} and A, B, C be fuzzy subsets of X defined asfollows:

A(a) = 0.2, A(b) = 0.4, A(c) = 0.7,
B(a) = 0.2, B(b) = 0.6, B(c) = 0.7,

Let $\tau = \{0,B,1\}$. If we take $I = \rho(X)$, then A is fuzzy β -open but A is not fuzzy β -I -open set.

Proposition 2.2. For a subset of a fuzzy ideal topological space(X,τ,I)the followinghold:

(a) Every fuzzy I -open set is fuzzy almost strong-I -open.
(b) Every fuzzy almost strong-I -open is fuzzy strong β -I -open.

Proof : The proof is similar with the that Proposition 2.1.

Remark 2.2. The converses of each statement in proposition 2.2 need not be true as the following examples show.

Example 2.3. Let X = {a,b,c} and A, B be fuzzy subsets of X defined as follows:

A(a) = 0.2, A(b) = 0.3, A(c) = 0.7,
B(a) = 0.1, B(b) = 0.2, B(c) = 0.2.

Let $\tau = \{0,B,1\}$. If we take $I = \{0\}$, then A is fuzzy almost strong-I -open but A is not fuzzy I -open set.

Example 2.4. Let X = {a,b,c} and A, B be fuzzy subsets of X defined as follows:

A(a) = 0.5, A(b) = 0.4, A(c) = 0.6,
B(a) = 0.5, B(b) = 0.5, B(c) = 0.6.

Let $\tau = \{0,A,1\}$. If we take $I = \rho(X)$, then A is fuzzy strong-β -I -open but A isnot fuzzy almost strong-I -open set.

Proposition 2.3. Let(X,τ,I)be a fuzzy ideal topological space with an arbitrary index set Δ. Then the following properties hold:

(a) if $\{A_\alpha: \alpha \in \Delta\} \leq Fs\beta IO(X)$, then $\vee\{A_\alpha : \alpha \in \Delta\} \in Fs\beta IO(X)$.
(b) if $A \in Fs\beta IO(X)$ and $U \in \tau$, then $(U \wedge V) \in Fs\beta IO(X)$.

Proof :

(a) Since $\{A_\alpha : \alpha \in \Delta\} \leq Fs\beta IO(X)$, $A_\alpha \leq Cl^*(Int(Cl^*(A_\alpha)))$ for every $\alpha \in \Delta$.

Thus by using Lemma1.1,1.2, we have

$\vee_{\alpha\in\Delta}A_\alpha \leq \vee_{\alpha\in\Delta}Cl^*(Int(Cl^*(A_\alpha))) = \vee_{\alpha\in\Delta}[Int(Cl^*(A_\alpha)) \vee (Int(Cl^*(A_\alpha)))^*]$

$= [\vee_{\alpha\in\Delta}(Int(Cl^*(A_\alpha)))] \vee [\vee_{\alpha\in\Delta}(Int(Cl^*(A_\alpha)))^*]$

$\leq [Int(\vee_{\alpha\in\Delta}(Cl^*(A_\alpha)))] \vee [Int(\vee_{\alpha\in\Delta}(Cl^*(A_\alpha)))]^*$

$= ([Int(\vee_{\alpha\in\Delta}(A_\alpha \vee A_\alpha^*))] \vee [Int(\vee_{\alpha\in\Delta}(A_\alpha \vee A_\alpha^*))]^*)$

$\leq [Int((\vee_{\alpha\in\Delta}A_\alpha) \vee (\vee_{\alpha\in\Delta}A_\alpha)^*)] \vee [Int((\vee_{\alpha\in\Delta}A_\alpha) \vee (\vee_{\alpha\in\Delta}A_\alpha)^*)]^*$

$= Int(Cl^*(\vee_{\alpha\in\Delta}A_\alpha)) \vee (Int(Cl^*(\vee_{\alpha\in\Delta}A_\alpha)))^*$

$= Cl^*(Int(Cl^*(\vee_{\alpha\in\Delta}A_\alpha)))$.

(b) By the hypothesis, $A \leq Cl^*(Int(Cl^*(A)))$ and $U \leq Int(U)$. Thus

$(A \wedge U) \leq Cl^*(Int(Cl^*(A))) \wedge Int(U)$

$= [Int(Cl^*(A)) \vee (Int(Cl^*(A)))^*] \wedge Int(U)$

$= [Int(Cl^*(A)) \wedge Int(U)] \vee [(Int(Cl^*(A)))^* \wedge Int(U)]$

$\leq ([Int(Cl^*A) \wedge U)] \vee [(Int(Cl^*(A))) \wedge Int(U)]^*)$

$\leq ([Int((A \vee A^*) \wedge U)] \vee [Int((A \vee A^*) \wedge U)]^*)$

$\leq Int((A \wedge U) \vee (A \wedge U)^*) \vee [Int((A \wedge U) \vee (A \wedge U)^*)]^*$

$= Int(Cl^*(A \wedge U)) \vee [Int(Cl^*(A \wedge U))]^*$

$= Cl^*(Int(Cl^*(A \wedge U)))$. Thus $A \wedge U \in Fs\beta IO(X)$.

Proposition 2.4. Let(X,τ,I)be a fuzzy ideal topological space with an arbitrary indexΔ.

Then the following properties hold:

(a) if $\{A_\alpha : \alpha \in \Delta\} \leq FasIO(X)$, then $\vee\{A_\alpha : \alpha \in \Delta\} \in FasIO(X)$.
(b) if $A \in FasIO(X)$ and $U \in \tau$, then $(U \wedge V) \in FasIO(X)$.

Proof : The proof is similar with Proposition 2.3.

Proposition 2.5. Let(X,τ,I)be a fuzzy ideal topological space and N be the ideal of nowhere dense sets of X . Then the following properties hold:

(a)If I = {0} or N, then the five properties fuzzy almost strong I -open, fuzzy strong β -I -open, fuzzy almost-I -open fuzzy β -I -open and fuzzy β -open are all equivalent of one another.

(b) If I = ρ(X), then

(1) the five properties fuzzy open, fuzzy α-I -open, fuzzy pre-I -open, fuzzy semi-I open and fuzzy strong β -I -open are all equivalent of one another.

(2) a subset A of X is fuzzy β -I -open if and only if A is fuzzy semi-I -open.

Proof :(a)

(1) Let I = {0}. Then $A^*= Cl(A)$ for any subset of X and $Cl^*(A) = A\vee A^*= Cl(A)$. Therefore, we have $A \leq Cl(Int(Cl(A))) = Cl(Int(A^*)) = Cl^*(Int(A^*))$. This shows that every fuzzy β -open set is fuzzy almost strong I -open.

(2) Let I = N . Then $A^*= Cl(Int(Cl(A)))$for any subset A of X.If A is fuzzy β -open, $A \leq Cl(Int(Cl(A)))$ and $Cl(A) = Cl(Int(Cl(A))) = A^*$.Therefore, we have $Cl(A) = A \vee A^*= Cl^*(A)$for each fuzzy β -open set Aand hence $A \leq Cl(Int(Cl(A))) = Cl(Int(A^*)) = Cl^*(Int(A^*))$. This shows that every fuzzy β -open set is fuzzy almost strong I -open.

(b) Let I = ρ(X). Then $A^*= \{0\}$ and $Cl^*(A) = A \vee A^*= A$for every subset A of X .

(1) If A is a fuzzy strong β -I -open set, then we have $A \leq Cl^*(Int(Cl^*(A))) = Cl^*(Int(A)) = Int(A)$. This shows that every fuzzy strong β -I -open set is fuzzy open.

(2) If A is a fuzzy β -I -open set, then $A \leq Cl(Int(Cl^*(A))) = Cl(Int(A))$. This shows that every fuzzy β -I -open set is fuzzy semi-open.

Proposition 2.6. For a subset A of a fuzzy ideal topological space(X,τ,I) the following statements are equivalent.

(a) A is fuzzy almost strong I -open.

(b) A is fuzzy strong β -I -open and fuzzy ∗-dense-in-itself.

Proof :(a) ⇒ (b) Every fuzzy almost strong I -open is fuzzy strong β -I -open from Proposition 2.2. On the other hand, $A \leq Cl^*(Int(A^*)) = Int(A^*) \vee (Int(A^*))^* \leq Int(A^*) \vee Cl(Int(A^*)) \leq Cl(Int(A^*)) \leq Cl(A^*) = A^*$by Lemma 1.1.

(b) ⇒ (a) By the assumption, $A \leq Cl^*(Int(CL^*(A))) = Cl^*(Int(A\vee A^*)) \leq Cl^*(Int(A^*))$.

This shows that A ∈FasIO(X).

Remark 2.3. fuzzy strong I -open set and fuzzy ∗-I -dense in itself are independent of each other as shown in the following example.

Example 2.5. Let X = {a,b,c} and A, B be fuzzy subsets of X defined as follows:

A(a) = 0.2, A(b) = 0.3, A(c) = 0.7,
B(a) = 0.1, B(b) = 0.2, B(c) = 0.2.

Let τ = {0,B,1}. If we take I = ρ(X), then A is fuzzy ∗-I -dense in itself but A is not fuzzy strong-β -I -open.

Example 2.6. Let X = {a,b,c} and A, B be fuzzy subsets of X defined as follows:

A(a) = 0.5, A(b) = 0.4, A(c) = 0.6,
B(a) = 0.5, B(b) = 0.5, B(c) = 0.6.

Let τ = {0,A,1}. If we take I = ρ(X), then A is fuzzy strong-β -I -open but A is not fuzzy ∗-I -dense in itself.

3. $FS_{\beta I}$-set and FD_I-set

Definition 3.1. A subset of a fuzzy ideal topological space(X,τ,I) is called

(a) a fuzzy $S_{\beta I}$ -set if $Cl^*(Int(Cl^*(A))) = Int(A)$.
(b) a fuzzy t-I -set[7] if $Int(Cl^*(A)) = Int(A)$.

Proposition 3.1. Every fuzzy $S_{\beta I}$ -set is a fuzzy t-I -set.

Proof : Let A be a fuzzy $S_{\beta I}$ -set.

Then $Cl^*(Int(Cl^*(A))) = Int(A)$.

Since$Int(A) \leq Int(Cl^*(A))$ and $Int(Cl^*(A)) \leq Cl^*(Int(Cl^*(A))) = Int(A)$. Therefore

$Int(A) = Int(Cl^*(A))$ and hence A is fuzzy t-I -set.

Remark 3.1. The converse of Proposition 3.1 need not be true is shown by the following example.

Example 3.1. Let $X = \{a,b,c\}$ and A, B be fuzzy subsets of X defined as follows:

$A(a) = 0.2$, $A(b) = 0.3$, $A(c) = 0.7$,
$B(a) = 0.1$, $B(b) = 0.2$, $B(c) = 0.2$.

Let $\tau = \{0,B,1\}$. If we take $I = \{0\}$, then A is fuzzy t-I -set but A is not fuzzy$S_{\beta I}$ -open.

Definition 3.2. A subset A of a fuzzy ideal topological space(X,τ,I) is called fuzzy D_I -set(resp. fuzzy B_I -set[2]) if $A = U \wedge V$, where $U \in \tau$ and V is fuzzy $S_{\beta I}$ -set(resp. fuzzy t-I -set).
We denote the family of all fuzzy D_I -sets(resp. fuzzy $S_{\beta I}$ -sets, fuzzy B_I -sets) by $FD_I(X)$(resp. $FS_{\beta I}(X)$, $FB_I(X)$-set).

Proposition 3.2. In a fuzzy ideal topological space(X,τ,I), $FS_{\beta I}(X) \leq FD_I(X), \tau \leq FD_I(X)$ and $FD_I(X) \leq FB_I(X)$.

Proof: Since $X \in \tau \wedge FS_{\beta I}(X)$ and every $FS_{\beta I}$ -set is a fuzzy t-I -set by proposition 3.1, the inclusions are obvious.

Proposition 3.3. For a subset A of a fuzzy ideal topological space(X,τ,I), the following properties are equivalent:

(a) A is fuzzy open.
(b) A is fuzzy strong β -I -open and fuzzy D_I -set.

Proof :
(a) ⇒ (b) This is obvious.
(b) ⇒ (a) Let A be a fuzzy strong β -I -open and fuzzy D_I -set. Then

$A \leq Cl^*(Int(Cl^*(A))) = Cl^*(Int(Cl^*(U \wedge V))) \leq Cl^*(Int(Cl^*(U) \wedge Cl^*(V))) =$

$Cl^*(Int(Cl^*(U)) \wedge Int(Cl^*(V))) \leq Cl^*(Int(Cl^*(U))) \wedge Cl^*(Int(Cl^*(V))) = Cl^*(Int(Cl^*(U))) \wedge$

$Int(V)$, where $V \in FS_{\beta I}(X)$ and $U \in \tau$. Hence $A \leq U$ and $Int(A) \leq A \leq U \wedge Int(V) = Int(A)$.

Thus $A \in \tau$.

Remark 3.2. The following Examples show that fuzzy strong β -I -open set and fuzzy D_I -sets are independent notions.

Example 3.2. Let X = {a,b,c} and A, B ,C be fuzzy subsets of X defined asfollows:

A(a) = 0.6, A(b) = 0.7, A(c) = 0.5,
B(a) = 0.4, B(b) = 0.3, B(c) = 0.2.
C(a) = 0.5, C(b) = 0.7, C(c) = 0.4.

Let $\tau = \{0,C,1\}$. If we take I = {0}, then $B = B \wedge C$ is fuzzy D_I -set but B is notfuzzy strong β -I -open set.

Example 3.3. Let X = {a,b,c} and A, B be fuzzy subsets of X defined as follows:

A(a) = 0.5, A(b) = 0.4, A(c) = 0.6,
B(a) = 0.5, B(b) = 0.5, B(c) = 0.6.

Let $\tau = \{0,A,1\}$. If we take I = {0}, then A is fuzzy strong-β -I -open but $A = A \wedge B$ is not fuzzy D_I -set.

4. Decomposition of fuzzy almost strong I-continuity and fuzzy continuity

Definition 4.1. A function $f : (X,\tau,I) \rightarrow (Y,\sigma)$ is said to be fuzzy strongly β I -continuous(resp. fuzzy D_I -continuous, fuzzy B_I -continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is fuzzy strong β -I -open(resp. fuzzy D_I -set, fuzzy B_I -set)of (X,τ,I).

Definition 4.2. A function $f : (X,\tau,I) \rightarrow (Y,\sigma)$ is said to be fuzzy almost strong-I continuous if for every $V \in \sigma$, $f^{-1}(V)$ is fuzzy strong almost-I -open set of (X,τ,I).

Theorem 4.1. For a function $f : (X,\tau,I) \rightarrow (Y,\sigma)$, the following statements areequivalent:

(a) A is fuzzy almost strong I -continuous.
(b) A is fuzzy strong β -I -continuous and fuzzy $*$-I -continuous.

Proof : This follows from Proposition 2.6.

Theorem 4.2. For a function $f : (X,\tau,I) \rightarrow (Y,\sigma)$, the following statements areequivalent:

(a) A is fuzzy continuous.
(b) A is fuzzy strong β -I -continuous and fuzzy D_I -continuous.

Proof : This follows from Proposition 3.3.

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