International Journal of Trend in Scientific Research and Development (IJTSRD)
International Open Access Journal | www.ijtsrd.com
ISSN No: 2456 - 6470 | Volume - 2 | Issue – 6 | Sep – Oct 2018

Issues with Reliability of Fuzzy Logic

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ABSTRACT
Since 1973, fuzzy logic has been rejected by a majority of linguists as a theory for dealing with vagueness in natural language. However, control engineers apply fuzzy vagueness in natural language to control system problems. A real world example of automatic fan control has shown that: i) fuzzy logic can be applied ii) probability theory can better model the control system iii) Boolean algebra with decision processes can also be applied. In addition, a redundancy issue for fuzzy logic has been raised.

Moreover, a number of inherent flaws of fuzzy logic from an engineering viewpoint have been identified: i) fuzzy logic as probability ii) fuzzy conjunction iii) fuzzy disjunction iv) fuzzy inference engine v) fuzzy membership function. Fuzzy logic has been found to be an approximate model of probability, however, when viewed as probability fuzzy logical operations of OR and AND provide incorrect responses. Fuzzy inference will also fail when fuzzy logical operations are applied.

Some theorists have attempted to fix the flaws of fuzzy logic in a system termed „Compensatory Fuzzy Logic", but the rules of CFL are ad hoc. As a result of the reliability and robustness concerns in fuzzy systems, safety may be compromised.

Keyword: *fuzzy logic, control engineering, fuzzy membership function, probability, reliability, safety*

1. INTRODUCTION
Fuzzy logic is a form of many valued logic in which the truth values of variables may be any real number between zero and one, whereby one is true and zero is false and numbers in between are partial truths. Fuzzy logic and controllers have been applied internationally in a wide number of engineering systems and appliances in recent years. These systems include washing machines, cameras and video cameras, autonomous car control systems, auto-pilots for unmanned air-vehicles, automated trains and braking, home security systems and many more.

However, inherent flaws in fuzzy logic have been identified. According to Jakub Szymanik, Associate Professor in the Institute for Logic, Language and Computation at the University of Amsterdam, the majority of linguists and philosophers hold that there are considerable problems with fuzzy logic as a master theory for dealing with vagueness in natural language. The problems with fuzzy logic regarding language have been shown by a succession of academics including Rescher in 1969 [1], Lakoff in 1973 [2], Fine in 1975 [3], Kamp in 1975 [4], Klein in 1980 [5], Kamp and Partee in 1995 [6], Williamson in 1992 [7], Haack in 1996 [8], Lassiter in 2009 [9], Sauerland in 2011 [10] and Szymanik in 2018 [11]. Hence, academic consensus by linguists and philosophers has concluded that fuzzy logic does not apply correctly to vagueness in language.

Susan Haack is Distinguished Professor in the Humanities, Cooper Senior Scholar in Arts and Sciences, Professor of Philosophy and Professor of Law at the University of Miami. In her book of 1996 [8] *Deviant Logic, Fuzzy Logic: Beyond the Formalism*, Haack asserts that fuzzy logic is methodologically extravagant and linguistically incorrect. She analysed Zadeh"s motivations and finds that he relies on two main reasons for adopting fuzzy logic: a methodological and linguistic one. She found that Zadeh was wrong on both counts.

Kamp [4] found that fuzzy logic has contradicted the principle of non-contradiction! For a proposition, A, with a fuzzy truth value of ½ the negation, $\neg A = 1 - ½ = ½$. This means that the principle of non-contradiction can be applied to a minimum conjunction for fuzzy logic:

min (A ∩ ¬A) = ½

The outcome for the principle of non-contradiction (A ∩ ¬A = ∅) means that it has been contradicted for fuzzy logic. Kamp asks: "How could a logical contradiction be true to any degree?" and concludes that the result is absurd.

Uli Sauerland, a linguistic scholar of the Leibniz-Centre General Linguistics (ZAS), writes in 2011 [10]:

"When I started interacting with logicians, I was surprised to learn that fuzzy logic is still a big and active field of basic research. This surprise stemmed from my experience with fuzzy logic in my own field, linguistic semantics: In semantics, fuzzy logic was explored in the analysis of vagueness in the early seventies by Lakoff [10], but has been regarded as unsuitable for the analysis of language meaning at least since the influential work of Kamp in 1975 [8], which I summarize below. Therefore, I held the belief that fuzzy logic, though it has been useful in technical applications—I once possessed a Japanese rice-cooker that was advertised to use fuzzy logic—, was not useful for the analysis of vagueness."

2. A Real World Control Problem

Does the fact that fuzzy logic not apply to linguistics mean that a fuzzy controller might not work properly?

2.1 Fuzzy Automatic Fan Control

Consider an example of the fuzzy „if-then" rules for an automatic ceiling fan control:

i. If temperature is „cold" then fan_speed is stop.
ii. If temperature is „less warm" then fan_speed is slow.
iii. If temperature is „warm" then fan_speed is moderate.
iv. If temperature is „hot" then fan_speed is high.

It is clear that fuzzy logic can offer a viable solution when applied to the fan control example. The temperatures can be given a range and the fan speed parameters can be given a single value (but variable) when both parameters are considered „fuzzy". The terms „temperature" and „fan_speed" are engineering terms allowing „crisp" decisions and control. Fuzzy logic will work in this case.

The assignment of numbers between zero and one [0, 1] to fuzzy variables is necessary for the logical operations of AND, OR and NOT. The fuzzy logic operations are defined as:

A AND B = min (A, B)

A OR B = max (A, B)

NOT (A) = ¬A = 1 - A

2.2 Probabilistic Automatic Fan Control

However, linguists have found that a natural language term is not a sliding scale between zero and one. For example, the temperature terms „hot", „warm" or „cold" cannot be assigned sliding scales between one and zero. This is because fuzzy logic holds that there are an infinite number of truth values that range between zero and one for a single term. The fuzzy truth values [0,1] cannot be applied to natural language terms.

The fan control problem can be solved with probability [9]. The input temperature „hot" can be defined as a probability between zero and one, where P(hot) = 1.0 when above 25°C and P(cold) = 1, is below 16° C, the input temperatures „hot", „warm", „less warm" or „cold" can each be modelled as a sliding scale [0,1]. Similarly, the output „fan_speed" could be assigned a probability where P (fan_speed) = 1.0 for fan speed at 210 rpm and P (fan_speed) = 0 for fan-speed at 0 rpm.

The ceiling fan control could be applied to a probabilistic decision process:

I. If the temperature is, P (cold), is assigned to temperatures, T: T ≤ 16° C, then the fan speed, P(fan_speed) = 0, is stopped.
II. If the temperature is, P („less warm"), assigned to temperatures, T: 16° C ≤ T < 20° C, then the fan speed, P(fan_speed) = 0.3, is set to 80 rpm.
III. If the temperature is, P(„warm") , assigned to temperatures, T: 20° C ≤ T ≤ 25 ° C, then the fan speed, P(fan_speed) = 0.6, is set to 140 rpm.
IV. If the temperature is „hot" assigned to temperatures, T: T > 25° C, If P(hot) = 1.0 then the fan speed, P(fan_speed) = 1, is set to 210 rpm.

It is suggested that this result, as an approximate model of probability, is the key to the „paradoxical success of fuzzy logic" [12]. This will be discussed in more detail in section three.

2.3 Boolean Automatic Fan Control

The problem is that the ceiling fan control can also be equally applied to Boolean algebra with an exact decision process:

I. If the temperature is „cold" assigned to temperatures less than or equal to 16° C, then the fan speed is stop.
II. If the temperature is „less warm" assigned to temperatures less than 20° C and greater than or equal to 16° C, then the fan speed is set to 80 rpm.
III. If the temperature is „warm" assigned to temperatures greater than or equal to 20° C and less than or equal to 25 ° C, then the fan speed is set to 140 rpm.
IV. If the temperature is „hot" assigned to temperatures greater than 25° C, then the fan speed is set to 210 rpm.

2.4 Summary

The control problem of the automatic fan is only one of many instances in which fuzzy logic could be applied, but equally well Boolean algebra with a decision process and probabilistic approach could be better applied. A redundancy issue for fuzzy logic has been raised.

This result would support Susan Haack"s title of a chapter in her book on fuzzy logic *„Do we need fuzzy logic? "*
[13]. The methodological extravagances of fuzzy logic are listed:

a. Fuzzy logic does not avoid complexities introduced by regimentation.
b. Fuzzy logic introduces enormous complexities.
c. Fuzzy logic still imposes artificial precision.

3. Flaws in Fuzzy Control Methodology

The reliability of fuzzy logic controllers can be questioned in relation to four aspects:

i. Fuzzy Logic as Approximate Probability

Fuzzy engineer, Bart Kosko, in the paper *„Fuzziness vs. Probability"* [14] claims that probability theory is a sub-theory of fuzzy logic, even though many-valued logic and probability are independent theories. Probability should not be a sub-theory of fuzzy logic. Instead, the author has shown that fuzzy logic is based upon probability [15]. It has also been shown that probability can model vague control system variables and that the „success of fuzzy logic" may depend upon this.

a. The fuzzy logic definition ($A \cap B = \min(A, B)$) appears derived from the full probability formula, if events A and B are not mutually exclusive:
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
Where P is probability and ($A \cup B$) is union.
b. The fuzzy logic definition, ($A \cap B$) = min (A, B) is correct in fuzzy logic and also probability but applies to variables A and B if and only if they intersect one another (dependent) and are not mutually exclusive.
c. If two events A and B are mutually exclusive then the probability of both, $P(A \cap B)$, is zero and union is:
$P(A \cup B) = P(A) + P(B)$
d. For independent variables A and B:
$P(A \cap B) = P(A).P(B)$
e. For conditional probability given B:
$P(A \cap B) = P(A|B).\ P(B)$

The fuzzy logic definition $A \cap B = \min(A, B)$ applies only if the variables are not mutually exclusive and dependent (intersect one another). If variables A and B and not mutually exclusive, partially or totally independent, mutually exclusive or conditional then fuzzy logic will produce incorrect responses. However, input variables would typically be mutually exclusive and have a probability of zero, $P(A \cap B) = 0$. For independent variables, $P(A \cap B) = P(A).P(B)$. Therefore, the fuzzy logic operation of AND produces incorrect answers when applied as a probability. This is an intrinsic problem of fuzzy logic that has no remedy.

ii. Fuzzy Conjunction 'AND'

Fuzzy logic does not pass elementary logic tests for conjunction. Kamp has noted that fuzzy logic contradicts the principle of non-contradiction [4]. It might be added that the principle of non-contradiction is also the definition of paradox in fuzzy logic: $A = \neg A = 1/2$!

The author has also given an example to illustrate that fuzzy logic fails for traffic lights [16]. A red and amber light on (signalling stop) is interpreted by fuzzy logic to be amber, or proceed with caution or possibly stop. Fuzzy logic conjunction is incorrect and an intrinsic problem.

iii. Fuzzy Disjunction 'OR'

For probability, the logical OR operation has three possibilities:

a. A OR B = max(A, B) (dependent but not mutually exclusive)
b. A OR B = A + B (mutually exclusive)
c. A OR B = A + B - (A ∩ B) (not mutually exclusive and independent)

It should be noted that fuzzy logic OR has chosen only case (a). However, in the fan control example another input variable could be „humidity‟, that is not mutually exclusive to temperature. Other input variables would typically be independent that is case (c). The result is that the fuzzy logical OR operation gives incorrect answers.

iv. Fuzzy Inference Engine

The fuzzy inference engine has a finite number of instances (if A_n then B_n) as the decision system of an expert system [17]. However, real world applications may go outside or beyond normal operating conditions and demand a different response to the pre-programmed inference engine, causing failure. While, increasing the number of inferences will improve the reliability of operation of the fuzzy controller, chaos theory shows that real world applications may have an infinite number of possibilities that would be impossible to encode in a fuzzy controller [18]. This is a problem, however, for all control systems.

It is also clear that fuzzy inference may also fail when flawed fuzzy logic operations of OR and AND are applied. The flaws in fuzzy logic conjunction and disjunction from the fuzzy inference can lead to an incorrect output. The incorrect output may cause the control system to respond to new stimulus incorrectly and fail or crash. It is suggested that unplanned for inputs for fuzzy AND and OR operations could occur under non-standard operating conditions when it is most likely that the fuzzy system will respond incorrectly.

There are, however, limitations placed on the response of the fuzzy control system. The centroid method that calculates a weighted average of the outputs will also effectively average out errors, reducing the effect of incorrect responses in the system [17]. The centroid method will effectively reduce the likelihood of a crash in the fuzzy control system, but will not balance all situations.

v. Fuzzy Membership Function

Contrary to a probability function that is restricted between (0, 1), the fuzzy membership function employed in the fuzzy inference engine is not necessarily restricted between (0, 1) [15] [19]. If not restricted between zero and one, the fuzzy membership function may cause glitches or failures. In this case, a remedy is possible by restricting fuzzy membership functions between one and zero, and making the probabilities (fuzzy logic variables) add up to one in all cases.

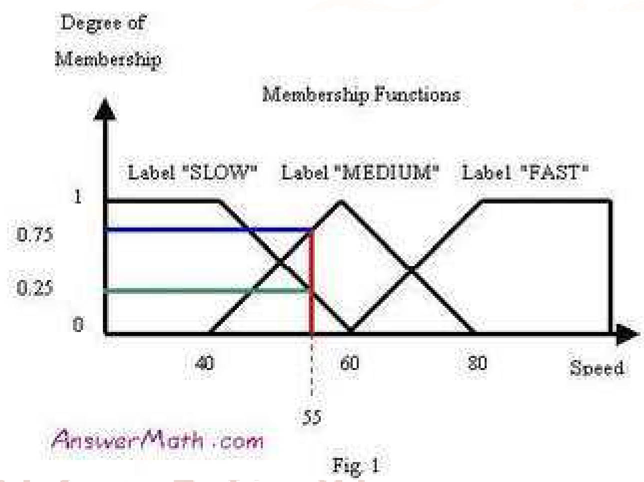

Figure1. Fuzzy Membership Function adding to one

4. Have Theorists tried to Compensate for Fuzzy Logic Flaws?

It would appear that theorists have also recognised flaws or limitations in fuzzy logic operations of conjunction and disjunction and offered improvements in a system termed „Compensatory fuzzy logic‟ (CFL) [20][21]. The ad hoc rules of CFL, however, support the view that it is trying to fix the unfixable.

The quote is from Wikipedia dated 10 July 2018:

„Compensatory fuzzy logic is a branch of fuzzy logic with modified rules for conjunction and disjunction. When the truth value of one component of a conjunction or disjunction is increased or decreased, the other component is decreased or increased to compensate. This increase or decrease in truth value may be offset by the increase or decrease in another component. An offset may be blocked when certain thresholds are met. Proponents claim that CFL allows for better computational semantic behaviours and mimic natural language. Compensatory fuzzy logic consists of four continuous operators: conjunction (c); disjunction (d); fuzzy strict order (or); and negation (n). The conjunction is the geometric mean and its dual as conjunctive and disjunctive operators.”

The geometric mean is defined as the n^{th} root of the product of n numbers, that is, for a set of numbers x_1, x_2, ..., x_n, the conjunction in CFL is equal to the geometric mean is defined as [22]:

$$x1 \cap x2, \ldots \cap xn = \sqrt[n]{x1.x2. \ldots xn}$$

The disjunction in CFL is defined as the complement of conjunction is:

$$x1 \cup x2, \ldots \cup xn = 1 - \sqrt[n]{x1.x2. \ldots xn}$$

5. Conclusion

Linguists and philosophers have found that fuzzy logic does not provide a model for linguistic semantics of vagueness. Despite academic consensus to the contrary, engineers have modelled fuzzy vagueness in natural language in control systems.

A real world example of a control system with automatic fan control has found that fuzzy logic can be applied. However, the vagueness of terms can be better modelled by probability and the automatic fan control can be solved probabilistically. It has been suggested that as an approximate model of probability the success of fuzzy logic can be found. Exactly the same result for the fan control system could also be achieved with Boolean algebra and decision processes, raising the question: „Do we need fuzzy logic?"

From an engineering viewpoint, a number of inherent flaws of fuzzy logic have been identified. The flaws have been elucidated: i) fuzzy logic as probability ii) fuzzy conjunction iii) fuzzy disjunction iv) fuzzy inference engine v) fuzzy membership function. Fuzzy logic has been found to be an approximate model of probability, however, when viewed as a probability the fuzzy logical operations of OR and AND provide incorrect answers. Fuzzy inference may also fail when these fuzzy logical operations are applied.

It appears that theorists have attempted to fix the flaws of fuzzy logic in a system termed „Compensatory fuzzy logic" but the ad hoc rules suggest that it is trying to fix the unfixable.

Inherent flaws in fuzzy control systems, especially when fuzzy logic OR and AND operations have been applied and or during non-standard operating conditions, may lead to failure in these systems. It would be anticipated there would be further instances of failure for fuzzy systems.

As a result of reliability and robustness concerns in fuzzy systems, safety may ultimately be compromised.

To quote Peter Clarke from an article in EE Times entitled 'Whatever happened to fuzzy logic?' in 2012:

'But perhaps fuzzy logic's time has come.'

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