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The Mathematics of Mobile Networks

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ABSTRACT

The mobile phones is used in our everyday life, the demand on mobile phone networks has increased steadily. Today, among other things like desktop laptops, mobile phones are used to access the internet, watch TV, read emails and use social media. The initial networks were designed to only transmit phone conversations. As technology improved, they had to be upgraded to deal with huge quantities of data. The Shannon-Hartley theorem is central to planning in the mobile phone industry. This formula relates the theoretical maximum bit-rate possible for a mobile phone user to the available bandwidth as licensed by the Government and the radio environment of the user. The radio environment of the user depends on a number of factors, most importantly the distance from the transmission tower, the power and frequency used by the transmission tower and the noise or interference from unwanted transmitters. In addition, it takes into account the ambient temperature of the day. Using this data, the mobile phone engineers are able to calculate the theoretical maximum bit-rate possible for information to be transmitted to the user. Then this result can then be compared with hardware limitations to evaluate the maximum transmission rate experienced by the user.

Keywords: *Bit-rate, Transmission tower, Network, Transmission rate, Shannon-Hartley theorem*

INTRODUCTION

The mobile network has improved lot as how the technology improves. Since when we started using the mobile phone many forgot about the post card, radio, calculator camera, watch etc., because they all are combined into the single MOBILE. So soon the mobile network was started as 2G, later developed into 3G and currently 4G network is ongoing. We have calculated the distance between bit-rate and transmission tower by using by using Shannon-Hartley Theorem.

CELL PLANNING ENGINEERING

Cell planning engineers work for all mobile phone companies in the country. Their aim is to ensure that we get good quality network coverage wherever we are. As a world leader in telecommunications equipment and data communication systems, Ericsson employs numerous engineers to research and study the latest technology and keep ahead of the competition.

MATHEMATICAL INFORMATION

Scientific notation is used for numbers which are too large or too small to be written using the standard decimal notation. For example, the distance between the Earth and the Sun is approximately 150,000,000 kilometers. This number has a lot of digits and will always be difficult to manipulate. Instead, it may be written using scientific notation in the form $a \times 10^b$ where a is a real number such that $1 \le |a| < 10$ and the exponent b is an integer. In the present situation, the distance between the Earth and the Sun is 150,000,000 kilometers or 1.5×10^8 kilometers.

MULTIPLICATION AND DIVISION OF NUMBERS IN SCIENTIFIC NOTATION

Two numbers in scientific notation, $x = a \times 10^b$ and $y = c \times 10^d$ may be multiplied or divided as follows:

$$x \times y = (a \times c) \times 10^{b+d},$$

$$\frac{x}{y} = \frac{a}{c}\, 10^{b-d}$$

Example

A big meteorite is observed at 20 times the distance between the Earth and the Sun (1.5×10^8km). Calculate the distance, D, between the sun and the meteorite.

$20 = 2 \times 10^1$

$D = 2 \times 10^1 \times 1.5 \times 10^8 = (2 \times 1.5) \times 10^{1+8} = 3 \times 10^9$km

If there are more than two numbers multiplied or divided, the rules stated previously still apply.

Ambient temperature: this is the temperature of the air surrounding us. It is measured in degrees Celsius, Fahrenheit or Kelvin.

Bit-rate: this represents the number of elementary blocks of information which can be transmitted during one second. Its unit is bits per second or bps.

Frequency: this is the number of times per second a radio wave reaches an antenna. Frequencies are measured in Hertz or Hz. Bandwidth: this is defined as the difference between the highest and the lowest frequencies in a radio signal, the unit is Hertz here again.

Example

Calculate the theoretical maximum bit-rate possible for a mobile phone user at a distance d = 5000 meters from the transmission tower on a 2G network using the Shannon-Hartley Theorem:

$$\text{Bit-rate} = \frac{B}{\log 2}\log\left(1 + \frac{p \times c^2}{(4 \times \pi \times d \times f)^2 \times k \times B \times T}\right)$$

where the constants are defined in the table 1.

Note: The above formula assumes "Free space" path loss between the user and mobile phone transmission tower and no interference from other transmission towers or networks. In realistic calculations these extra losses and interference are included, giving a more realistic result.

Table1: Constant definitions

Term	Symbol	Definition	Value	Unit
Power of the Tower	P	Energy /time	10	Watt (W)
Speed of light	C	Distance /time	3×10^{8}	Metre per second (m/s)
Boltzmann's Constant	K	Energy/ Temperature	1.38×10^{-23}	Joule per Kelvin (J/K)
Temperature	T	Temperature	2.9×10^{2}	Kelvin (K)

Table 2: The characteristics of the different networks

Network	Bandwidth (B)(Hz)	Frequency (f)(Hz)	Hardware Maximum Bit-rate (bps)
2G (EDGE)	2×10^{5}	9×10^{8}	3.84×10^{5}
3G (UMTS)	5×10^{6}	2.1×10^{9}	4.2×10^{7}
4G (LTE)	2×10^{7}	8×10^{8}	1.5×10^{8}

Replacing each letter with its value and using a calculator, for the 2G network, the theoretical maximum bit-rate possible for a mobile phone user may be calculated as follows:

$P \times c^2 = 10 \times (3 \times 10^8)^2$
$= 3 \times 3 \times 10 \times 10^8 \times 10^8 = 9 \times 10^{17}$

$4 \times \pi \times d \times f = 4 \times \pi \times (5 \times 10^3) \times (9 \times 10^8)$
$= 4 \times \pi \times 5 \times 9 \times 10^3 \times 10^8$
$= 565.48 \times 10^{11} = 5.6548 \times 10^{13}$

$(4 \times \pi \times d \times f)^2 = 5.6548 \times 10^{13} \times 5.6548 \times 10^{13}$
$= 5.6548 \times 5.6548 \times 10^{13} \times 10^{13}$
$= 31.9767 \times 10^{26} = 3.19 \times 10^{27}$

$kBT = (1.38 \times 10^{-23}) \times (2 \times 10^5) \times (2.9 \times 10^2)$

$= 1.38 \times 2 \times 2.9 \times 10^{-23} \times 10^5 \times 10^2 = 8 \times 10^{-16}$

Therefore,

$$\frac{p \times c^2}{(4 \times \pi \times d \times f)^2 \times k \times B \times T} = \frac{9 \times 10^{17}}{3.19 \times 10^{27} \times 8 \times 10^{-16}}$$
$$= \frac{9}{3.19 \times 8} \times \frac{10^{17}}{10^{27} \times 10^{-16}}$$
$$= 0353 \times 10^6 = 3.53 \times 10^5$$

$$\text{Bit-rate} = 2 \times 10^5 \times \frac{\log(1+3.53 \times 10^5)}{\log 2} = 3.68 \times 10^6 \text{bps}$$

The theoretical maximum bit-rate possible for a 2G user 5,000 metres away from the tower is 3.68 mega bits per second (note 106 bits per second = 1 mega bit per second). Note that this value is above the hardware maximum bit-rate: the user will therefore experience a maximum transmission rate of 3.84×10^5 bits per second. It should be remembered that the actual bit rate experienced by the user will be lower than this due to other factors not included in these calculations.

Example 1

Evaluate the theoretical maximum bit-rate possible for a 4G (LTE) user using the Shannon-Hartley Theorem as a function of the distance d, between the user and the mobile phone transmission tower.

D	PC^2	$4\pi df$	$(4\pi df)^2$	kBT	$\frac{Pc^2}{(4\pi df)^2 kBT}$	Bit-rate
5,000						
10,000						
15,000						
20,000						
30,000						
40,000						
50,000						

Solution

Formula

For 4G(LTE) networks,

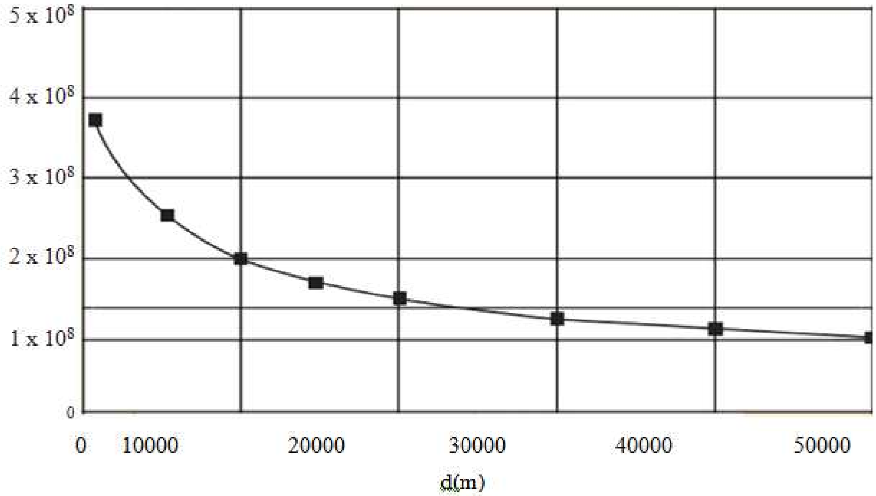
$B = 2 \times 10^7$ Hz

And

$f = 8 \times 10^8$

D	PC^2	$4\pi df$	$(4\pi df)^2$	kBT	$\frac{Pc^2}{(4\pi df)^2 kBT}$	Bit-rate
1,000	9×10^{17}	1×10^{13}	1×10^{26}	8×10^{-14}	1.11×10^{5}	3.4×10^{8}
5,000	9×10^{17}	5.02×10^{13}	2.52×10^{27}	8×10^{-14}	4.45×10^{3}	2.4×10^{8}
10,000	9×10^{17}	1×10^{14}	1×10^{28}	8×10^{-14}	1.12×10^{3}	2.0×10^{8}
15,000	9×10^{17}	1.5×10^{14}	2.25×10^{28}	8×10^{-14}	4.94×10^{2}	1.8×10^{8}
20,000	9×10^{17}	2.01×10^{14}	4.04×10^{28}	8×10^{-14}	2.78×10^{2}	1.6×10^{8}
30,000	9×10^{17}	3.02×10^{14}	9.12×10^{28}	8×10^{-14}	1.23×10^{2}	1.4×10^{8}
40,000	9×10^{17}	4.02×10^{14}	1.62×10^{29}	8×10^{-14}	6.94×10	1.2×10^{8}
50,000	9×10^{17}	5.03×10^{14}	2.53×10^{29}	8×10^{-14}	4.45×10	1.1×10^{8}

Let us plot the graph from the above information which is representing the variation of the bit-rate as a function of the distance d on the grid paper.



Example 3

Using the curve from example 1, we work out up to what distance the maximum rate at which the transmission of information is limited by the hardware maximum bit-rate. Once this is done, we have evaluated the time it would take to download 1×10^9 bits of information when you are 10,000 meters away from the tower. Then evaluate the time would take to download 10×10^9 bits of information when you are 40,000 meters away from the tower.

Solution

The maximum rate at which information may be transmitted is provided by the Shannon-Hartley theorem and the limitations of the mobile phone. For a 4G mobile phone, the hardware maximum bit-rate is 1.5×10^8 bps. When the result provided by the Shannon-Hartley theorem is above this value, it should be substituted with 1.5×10^8 bps.

To determine at what distance from the tower it occurs, draw the line corresponding to 1.5×10^8 bits per second on the graph. It crosses the curve obtained using the Shannon-Hartley theorem at the distance d ≈ 25,000 metres. Therefore, when the distance to the tower is smaller than 25,000 metres, the maximum bit-rate at which information is transmitted is 1.5×10^8 bps. If the distance to the tower is above 25,000 metres, the maximum bit-rate at which information is transmitted is provided by the Shannon-Hartley theorem

At d = 10,000 metres from the tower, the theoretical maximum bit-rate for a user as calculated with Shannon-Hartley theorem is 2.02×10^8 bps. This is above the hardware maximum bit-rate, so the bit-rate at which at which information is transmitted to the user is 1.5×10^8 bps. Consequently, it takes

$$t_1 = \frac{1\times10^9}{1.22\times10^8} = 6.6s$$

to download 1×10^9 bits of data.

At d = 40,000 metres from the tower, the theoretical maximum bit-rate for a user as calculated with Shannon-Hartley theorem is 1.22×10^8 bps. This is below the hardware maximum bit-rate, so the bit-rate at which information is trasmitted to the user is 1.22×10^8 bps. Consequently, it takes

$$t_2 = \frac{1\times10^9}{1.22\times10^8} = 8.2s$$

To conclude, as suggested by the curve, the further away you are from the transmission tower, the longer it will take to download information. In the case considered here, the difference remains small, less than two seconds, which will hardly be noticed. When more complex formulas are used which include extra path losses and interference from other transmission towers and networks, this difference becomes more evident.

CONCLUSION

In this paper we have discussed the concept cell planning engineering & Shannon-Hartley theorem.

So the distance between bit-rate and transmission tower is calculated in order to calculate the mobile phone user by using 2G network of transmission tower from Shannon-Hartley theorem. In future 5G network may be applied to this concept, then the speed of mobile network may be increased, users may be increased. As the technology develops the usage of mobile phone is increased in the past 10 years. For the next 10 years the usage of mobile may be increased doubled than the past year.

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