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Strong Semiclosed Sets in Topological Spaces

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ABSTRACT
In this paper, we introduce and study the notion of strong semiclosed sets in topological spaces.

Keyword: *Strong semiclosed sets*

I. INTRODUCTION AND PRELIMINARIES

Stone[30] and Velicko[34] introduced and investigated regular open sets and αopen sets both are stronger forms of open sets, respectively. Levine[19], Mash hour et al[24], Njastad[26], Andrijevic[3], Palaniappan and Rao[27], Maki et al [23]and Gnanambal[14] introduced and studied semi open and generalized open sets, pre open sets, αopen sets, semipreopen sets, regular generalized open sets, generalized pre open sets and generalized pre regular open sets, respectively, which are some weaker forms of open sets and the complements of these sets are called their respective closed sets. Biswas[5] introduced and studied simply open sets which are weaker than semi open sets. Cameron [8] introduced regular semiopen sets which are weaker than regular open sets. Tong ([32], [33]), Dontchev[10], Przemski[28], Hatir et al[15] and Dontchev and Przemski[12] introduced A-sets and t-sets and B-sets, generalised semipreclosed sets, D(c,α) and D(c,p), and D(α,p) sets, α*-sets and D(c,s) and related sets, respectively. Generalized semipreclosed sets, t -sets and α*- sets are weaker forms of closed sets. A- Sets, B-sets, D(c, α) sets.

D(c, p) sets, D(α, p) sets, D(c, s) and related sets are weak forms of open sets. In 1921, Kuratowski and Sierpinski [17] considered the difference of two closed subsets of an n - dimensional Euclidean space. The notion of locally closed sets was implicit in their work. Bourbaki[7] introduced it to topological spaces. Stone[29] used the term FG for a locally closed sub set. Balachandran, Sundaram and Maki[4] generalized this notion and introduced generalized locally closed sets which are weaker than both open sets and closed sets. Now, we recall the following definitions which are used in this paper.

Definition 1.1
A subset S of X is called

A. Regular open[30] if S = int(cl(S) and regular closed if S = cl (int (S)),
B. Semi open[19] if there exists an open set G such that G⊆S⊆cl (G) and semi closed[6] if there exists a closed set F such that int(F) ⊆ S ⊆ F,
C. α-open[26] if S ⊆int (cl(int(S))) and α – closed if S ⊇cl(int(cl(S))),
D. pre open [24] if S ⊆int (cl(S)) and preclosed if S ⊇ cl(int(S)),
E. β-open [1] (=semi pre open) [3])if S ⊆ cl(int(cl(S))) and β-closed [1] (= semi preclosed[3])
If S⊇int(cl(int(S))),
F. Simply open([5], [25]) if S = G∪W, where G is open in X and W is nowhere dense in X.

Definition 1.2
For a subset S of X, the semiclosure of S, denoted by scl (S), is defined as the intersection of all semi closed sets containing S in X and the semi interior of S, denoted by s int(S), is defined as the union of all semi open sets contained in S in X [9]. The preclosure of S [3] denoted by p cl(S), the pre interior of S [3] denoted by pint (S), the α-closure of S [2] denoted by αcl(S), the α interior of S[3] denoted by α int(S), the semi pre closure of S [3] denoted by s pcl (S) and the semi pre interior of S [3] denoted by s pint (S) are defined similarly.

Result 1.3
For a subset S of X

A. scl (S) = S∪int (cl(S))[16],
B. scl (S) = S∩ cl(int (S)) [3],
C. pcl (S) = S∪ cl (int(S)) [3],
D. pint(S) = S∩int (cl(S)) [3],
E. αcl(S) = S ∪ cl (int(cl(S))) [2],
F. αint(S) = S∩int (cl(int(S))) [3],
G. spcl(S) = S ∪int (cl(int(S))) [3],
H. spint(S) = S∩ cl(int(cl(S))) [3].

Definition 1.4
A subset S of X is called
A. generalized closed (g-closed)[20], if cl(S)⊆ G whenever S⊆G and G is open in X,
B. α generalized closed (αg-closed)[22], if αcl(S)⊆ G whenever S⊆G and G is open in X,
C. gα**-closed[21] if αcl(S) ⊆int(cl(G)) whenever S⊆G and G is α open in X,
D. regular generalized closed (rg-closed)[27] if cl(S)⊆G whenever S ⊆G and G is regular open in X,
E. generalized pre closed (gp-closed) [23]if pcl (S) ⊆G whenever S ⊆G and G is open in X,
F. generalized pre regular closed (gpr-closed) [14]if pcl (S) ⊆G whenever S ⊆G and G is regular open in X.

The complements of the above mentioned closed sets are called their respective open sets.

Definition 1.5
A subset S of X is called
A. a locally closed set (LC-set)[7] if S = G∩F where G is open and F is closed in X,
B. an A-set[32] if S = G∩F where G is open and F is regular closed in X,
C. a t-set [100] if int(S) = int(cl(S)),
D. a B-set[33] if S = G∩F where G is open and F is a t-set in X,
E. an α*-set[15] if int(S) = int(cl(int(S))),
F. a C-set (Due to sundaram)[31] is S = G∩F where G is g-open and F is a t-set in X,
G. a C-set (Due to Hatir, Noiri and Yuksel)[15], if S = G∩F where G is open and F is an α-set in X,
H. interior closed (= ic-set)[13] if int(S) is closed in S,
I. a D(c,p)-set [86] if S ∈ D (c,p) = {S⊆X | int (S) = pint (S) },
J. a D(c, α)-set[28] if S ∈ D (c, α) = {S⊆X | int (S) = αint (S) },
K. a D(α, p)-set[28] if S ∈ D (α, p) = {S⊆X | αint (S) = pint (S) },
L. a D(c, s)-set [12]if S ∈ D (c, s) = {S⊆X | int (S) = sint (S) },
M. a D(c, ps)-set[12] if S ∈ D (c, ps) = {S⊆X | int (S) = spint (S) },
N. a D(α, s)-set [12]if S ∈ D (α, s) = {S⊆X | αint (S) = sint (S) },
O. a D(α, ps)-set[12] if S ∈ D (α, ps) = {S⊆X | αint (S) = spint (S) },
P. a strong B-set[11] if S = G∩F where G is open and F is semiclosed and int(cl(F)) = cl(int(F)) in X.

II. STRONG SEMI CLOSED SETS

In this section, we introduce and study the notion of strong semi closed sets in topological spaces.

Definition 2.1
A subset S of X is called strong semi closed set in X if S = A∩F, where A is regular open and F is closed in X.

Proposition 2.2
Let S be a subset of X.
A. if S regular open in X, then S is strong semi closed in X,
B. if S is closed in X, then S is strong semi closed in X .

Proof:
Trivial.
However, the converses need not be true as seen from the following example.

Example 2.3
Let X = {a, b, c} and τ = {ϕ, {a}, {b}, {a,b},X}. In (X, τ), the set {c} is strong semi closed but not regular open. Also, in (X, τ), the set {a} is strong semi closed but not closed.

Remark 2.4
A. the complement of a strong semi closed set in X need not be strong semi closed in X and
B. finite union of strong semi closed sets need not be strong semi closed as seen from the following example.

Example 2.5
Let X = {a, b, c} and τ = {ϕ, {a}, {b}, {a,b},X}. In (X, τ), the sets {a}, {b}, {c} are strong semi closed. But X – {c} = {a, b} = {a}∪ {b} is not strong semi closed in (X, τ).

Lemma 2.6
A subset S of X is simply open in X if and only if S is the intersection of a semi open and a semi closed set in X.

Lemma 2.7
Let S be a subset of X. Then S is semi closed if and only if int(cl(S)) = int(S).

Lemma 2.8
Let A and B be subsets of X. If either A is semi open or B is semi open,
then int(cl (A∩B) = int (cl(A)) ∩int (cl(B)).

Preposition 2.9
Let S and T be subsets of X.

A. If S and T are strong semi closed in X, then S∩T is strong semi closed in X,
B. If S is strong semi closed in X, then S is semi closed in X ,
C. If S is strong semi closed in X, then S is β-closed in X ,
D. If S is strong semi closed in X, then S is an LC-set in X,
E. If S is strong semi closed in X, then S is a D(c,p)-set in X,
F. If S is strong semi closed in X, then S is simply open in X.

Proof :
Proof of (a) : LetS = A ∩B and T = C∩D, where A, C, are regular open and B, D are closed in X. Using Lemma 2.8.we obtain.int(cl(A ∩C) = A ∩C.

Hence, S∩T = (A ∩C) ∩ (B ∩D). Therefore S∩T is strong semi closed in X.

Proof of (b): Let S = A ∩B, where A is regular open and B is closed in X. Using Lemma 2.8. we get, Int(cl(S)) = int(cl(A)) ∩int (cl(B)) = A ∩int (B) = int (A) ∩int (B) = int (S).

Therefore by Lemma 2.7. S is semi closed.

Since every semi closed set is β-closed, the proof of (c) follows.
Proof of (d) is trivial.

Proof of (e): Let S be strong semi closed in X. Then by (b), int (cl(S)) = int (S). This implies that pint (S) = int (S). Therefore S∈ D (c,p). The proof of (f) follows from the definition of strong semi closed set and from Lemma 2.6. However, the converses need not be true as seen from the following example.

Example 2.10
Let X = {a, b, c} and τ = {ϕ,{a}, {a, b}, X}. The set S = {b} is semi closed, β-closed, simply open, a D(c,p)-set and an LC-set in (X, τ) but not strong semi closed in (X, τ). In (X, τ), {c} is strong semi closed and {c} – {a, c } ∩ {b, c}, where {a, c, } is not strong semi closed in (X, τ).

Proposition 2.11
For a subset S of X, the following are equivalent.

A. S is strong semi closed in X,
B. X – S is the union of a regular closed set and an open set in X,
C. S = A ∩ cl (S) where A is regular open in X .

Proof:
The proof of (a) → (b) and (b) → (a) are trivial.
Let S = A∩F, where A is regular open and F is closed in X. Then S⊆A and S⊆F. So, cl(S) ⊆F.But S⊆cl (S) always. Therefore, S⊆A∩cl(S) ⊆ A∩F = S.

Thus S = A∩cl(S). Hence (a) → (c) and (c) → (a) is obvious.

Remark 2.12
The notion of strong semiclosed set is independent of the notions of A-set, strong B-set, D(c,ps)-set D(c,s)-set, α-closed set, preclosed set, g-closed set and rg-closed set as seen from the following examples.

Example 2.13
Let X = {a, b, c} and τ= {ϕ,{a},X}. In (X, τ), the set {a} is an A-set, D(c,ps)-set,D(c,s)-set and a strong B-set but not a strong semiclosed set. Again, in (X, τ),the set {b, c} is strong semi closed but not an A-set.

Example 2.14
Let X = {a, b, c, d} and τ= {ϕ,{c}, {d}, {c, d}, {a, c}, {a, c, d}, X}. In (X, τ), the set {b, d} is strong semi closed but is neithera D(c, ps)-set nor a D(c, s)-set.

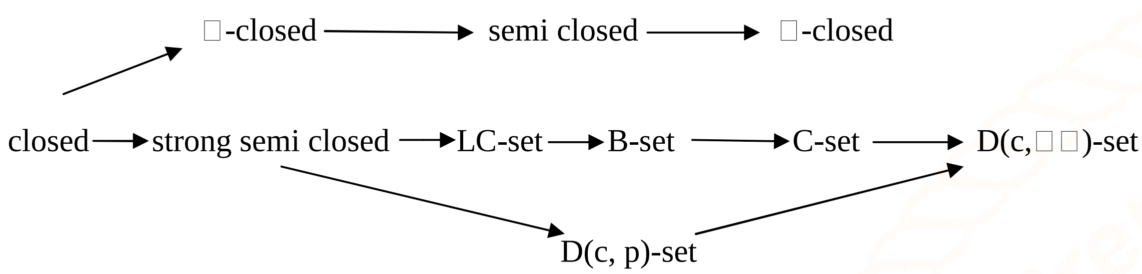
Example 2.15
Let X = {a, b, c} and τ = {ϕ,{a}, {b}, {a, b}, X}. In (X,τ), the set {b, c} is strong semi closed but not a strong B set. Again, in (X,τ), the set {a} is strong semi closed but is neither preclosed nor g-closed nor Rg-closed.

Example 2.16

Let X = {a, b, c,} and τ = {ϕ,{a},X}. In (X, τ), the set {b} is α-closed, g-closed and rg-closed but not strong semiclosed.

III. REMARK

From Tong ([32], [33]), Hatir, Noiri and Yuksel ([15]) Dontchev and Przemski ([12]) and from the results and examples, we have the following implications. None of them is reversible.



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