



Optical Soliton Simulation by Symmetrized Split-Step Fourier Method

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ABSTRACT

Fiber optic telecommunication gives more benefits if compared to copper wire communication systems, however, group velocity dispersion (GVD), fiber loss and also self-phase modulation (SPM) limit the performance of fiber optic telecommunication. When there is a balance between GVD and SPM, there exists a stable wave (optical soliton) which can propagate for long distance undistorted. In this study, the optical soliton simulation will be carried out using symmetrized split-step Fourier method (SSFM).

Keyword: Optical soliton, symmetrized split-step Fourier method, SSFM, GVD, SPM

1.0 INTRODUCTION

Modern people today cannot live without the Internet. They require higher bandwidth and faster internet speed connection for job, educational, entertainment and social purposes. Therefore, a fiber optic is utilized in ultra-fast long-haul telecommunication. Fiber optic telecommunication uses light to transfer information through an optical fiber. The optical fiber is much smaller and lighter than copper wire. It provides a broader bandwidth, higher frequency transmission of signals with a greater bit rate over a long distance with lower loss and lower interference if compared to the copper wire communication systems. But, group velocity dispersion (GVD), fiber loss and also self-phase modulation (SPM) limit the performance of fiber optic telecommunication. [1]

GVD or Chromatic Dispersion in the optical fiber is due to the refractive index of the fiber is frequency dependent and hence light of different frequencies travel along the fiber at different velocities. Thus they arrive at the receiver at different times. This result in pulse spreading which cause intersymbol interference hence signals may not be detected correctly. [2]

The Kerr effect due to fiber nonlinearity which is dependent on refractive index variation due to the intensity of light causes SPM. SPM produces a change of spectrum without change of temporal distribution.

When there is a balance between dispersion broadening due to GVD and SPM in an optical fiber, a special kind of wave called temporal optical soliton (confinement of light occurs in time) is formed. When confinement of light happens in space, a spatial soliton is formed. Soliton can travel steadily for long distance (thousands of km) [3] undistorted in a lossless optical fiber. This property makes soliton suitable to be applied in telecommunication.

The optical soliton propagation in fiber optic is governed by the nonlinear Schrodinger (NLS) equation which can be derived by Maxwell equations. [4]

$$i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A + \frac{i}{2} \alpha A = 0 \quad (1)$$

2.0 Symmetrized Split Step Fourier method (SSFM)

The NLS Equation (1) will be solved numerically by numerical method namely Symmetrized SSFM.

$$\frac{\partial A}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i\gamma |A|^2 A - \frac{1}{2} \alpha A \quad (2)$$

Firstly, Equation (2) is divided into linear and nonlinear operator such that

$$\frac{\partial A}{\partial z} = (\hat{L} + \hat{N}) A \quad (3)$$

Where \hat{L} , \hat{N} are linear and nonlinear operators.

$$\text{In this case. } \hat{L} = -\frac{i\beta_2}{2} \frac{\partial^2}{\partial t^2} - \frac{1}{2} \alpha, \quad \hat{N} = i\gamma |A|^2$$

The solution of Equation (3) is

$$\begin{aligned} \frac{\partial A}{A} &= (\hat{L} + \hat{N}) \partial z \\ \ln(A) &= (\hat{L} + \hat{N}) z + c \\ A(z + \Delta z, t) &= \exp\left[(\hat{L} + \hat{N}) \Delta z\right] A(z, t) \end{aligned} \quad (4)$$

The Symmetrized SSFM method will be advanced using half step of a linear operator, then one step of the nonlinear operator and lastly half step of a linear operator as below:

$$A(z + \Delta z, t) = \exp\left(\frac{\Delta z}{2} \hat{L}\right) \exp\left(\int_z^{z+\Delta z} \hat{N}(z) \Delta z'\right) \exp\left(\frac{\Delta z}{2} \hat{L}\right) A(z, t) \quad (5)$$

Where Δz is the spatial step and the integral part is solved using the trapezoidal rule.

$$\int_z^{z+\Delta z} \hat{N}(z) \Delta z' \approx \frac{\Delta z}{2} [\hat{N}(z) + \hat{N}(z + \Delta z)] \quad (6)$$

Sub (6) in (5) yields

$$A(z + \Delta z, t) = \exp\left(\frac{\Delta z}{2} \hat{L}\right) \exp\left(\frac{\Delta z}{2} \hat{N}(z)\right) \exp\left(\frac{\Delta z}{2} \hat{N}(z + \Delta z)\right) \exp\left(\frac{\Delta z}{2} \hat{L}\right) A(z, t) \quad (7)$$

Simplifying it

$$A(z + \Delta z, t) = \exp\left(\frac{\Delta z}{2} \hat{L}\right) \exp\left(\frac{\Delta z [\hat{N}(z) + \hat{N}(z + \Delta z)]}{2}\right) \exp\left(\frac{\Delta z}{2} \hat{L}\right) A(z, t) \quad (8)$$

$$A(z + \Delta z, t) = \exp\left(\frac{\Delta z}{2} \hat{L}\right) \exp\left(\Delta z \hat{N}\left(z + \frac{\Delta z}{2}\right)\right) \exp\left(\frac{\Delta z}{2} \hat{L}\right) A(z, t)$$

By taking the Fourier transform (FFT) on Equation (8), we get

$$A(z + \Delta z, t) = F^{-1} \left\{ \exp \left(-\frac{\Delta z}{2} \left[\frac{i\beta_2}{2} (i\omega)^2 + \frac{\alpha}{2} \right] \right) F \left\{ \exp(\Delta z i\gamma |A_{half}|^2) F^{-1} \left\{ \exp \left(-\frac{\Delta z}{2} \left[\frac{i\beta_2}{2} (i\omega)^2 + \frac{\alpha}{2} \right] \right) F \{A(z, t)\} \right\} \right\} \right\} \quad (9)$$

where $A_{half} = F^{-1} \left\{ \exp \left(-\frac{\Delta z}{2} \left[\frac{i\beta_2}{2} (i\omega)^2 + \frac{\alpha}{2} \right] \right) F \{A(z, t)\} \right\}$, ω is the Fourier frequency.

3.0 Methodology

In this optical fiber communication link, single mode fiber (SMF) is used to propagate the signal with wavelength, $\lambda = 1550$ nm. The attenuation, α of single mode fiber is 0 dB/km, the dispersion, β_2 of this single mode fiber is $-20 \text{ ps}^2/\text{km}$. and the parameter for nonlinearity, γ is $1.317 \text{ W}^{-1}\text{km}^{-1}$. At 40 GB/s bit rate, the bit slot is 25ps as shown below [5]:

$$\begin{aligned} \text{Time window (bit slot)} &= \text{Sequence length} \times \frac{1}{\text{Bit rate}} & (10) \\ &= 1 \times \frac{1}{40G} \\ &= 25 \text{ ps.} \end{aligned}$$

The pulse full width half maximum (FWHM), T_{FWHM} is 12.5 ps. The relation between the temporal characteristic value of the initial pulse of T_0 parameter and T_{FWHM} for the sech pulses is

$$\begin{aligned} T_{FWHM} &= 2 \ln(1 + \sqrt{2}) T_0 & (11) \\ &\cong 1.763 T_0 & (12) \end{aligned}$$

Hence initial pulse of T_0 is

$$\begin{aligned} T_0 &= \frac{T_{FWHM}}{1.763} & (13) \\ &= \frac{12.5 \text{ ps}}{1.763} \\ &= 7.0902 \text{ ps.} \end{aligned}$$

The formula of power is:

$$\begin{aligned} P &= \frac{|\beta_2|}{N^2 \gamma T_0^2} & (14) \\ &= \frac{(20)}{(1.317)(7.0902)^2} \\ &= 0.30208 \text{ N}^2 [\text{W}] \end{aligned}$$

For fundamental soliton, the soliton number N is equal to one.

The dispersion length is given as:

$$\begin{aligned} L_D &= \frac{T_0^2}{|\beta_2|} & (15) \\ &= \frac{7.0902 \text{ ps}^2}{20 \text{ ps}^2/\text{km}} \\ &= 2.5135 \text{ km} \end{aligned}$$

Whereas the nonlinearity length is defined as:

$$\begin{aligned} L_{NL} &= \frac{1}{\gamma P_0} & (16) \\ &= \frac{1}{(1.317 \text{ W}^{-1}\text{km}^{-1})(0.30208)} \\ &= 2.5135 \text{ km} \end{aligned}$$

The soliton period is given by

$$\begin{aligned} z_0 &= \frac{\pi}{2} (L_D) & (17) \\ &= \frac{\pi}{2} (2.5135) \\ &= 3.9482 \text{ km.} \end{aligned}$$

4.0 Results and Discussions

The initial wave at $z=0\text{km}$ of the optical soliton was inputted into the symmetrized SSFM scheme (9) as given in Figure 1. It is noticed that the amplitude is given by P and the pulse width is given by $\frac{t}{T_0}$. The initial pulse is in sech form. The optical soliton propagates along the optical fiber at one soliton period of $z_0 = 3.9483\text{km}$ and it was round up to 4km .

The 3D-visualization of the optical soliton propagation is depicted in Figure 1. The results of optical soliton propagation at $z=1\text{km}$, $z=2\text{km}$, $z=3\text{km}$ and $z=4\text{km}$ are shown in Figures 2 - 5 respectively. While Figure 7 gives a series of 2D-graph at each distance of z obtained from Simbiology in Matlab. It can be seen that the optical soliton propagates undistorted (amplitude and shape maintain) along the optical fiber.

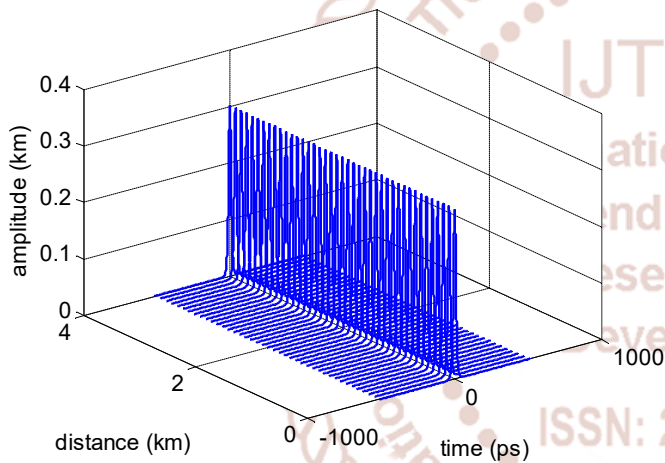


Figure 1: 3D-Optical soliton

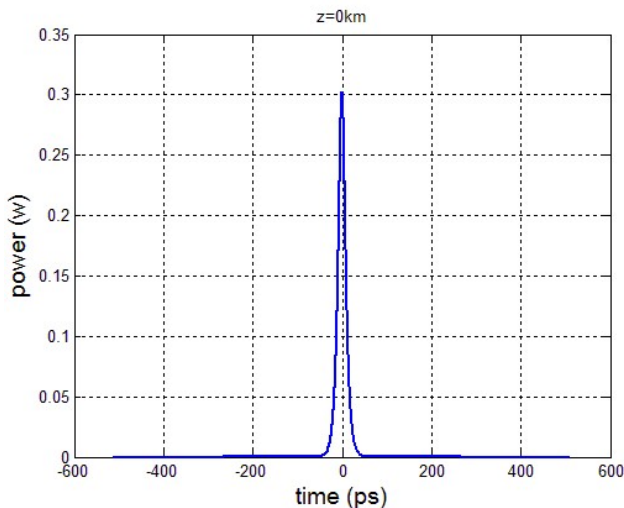


Figure 2: Initial optical soliton at $z = 0\text{km}$

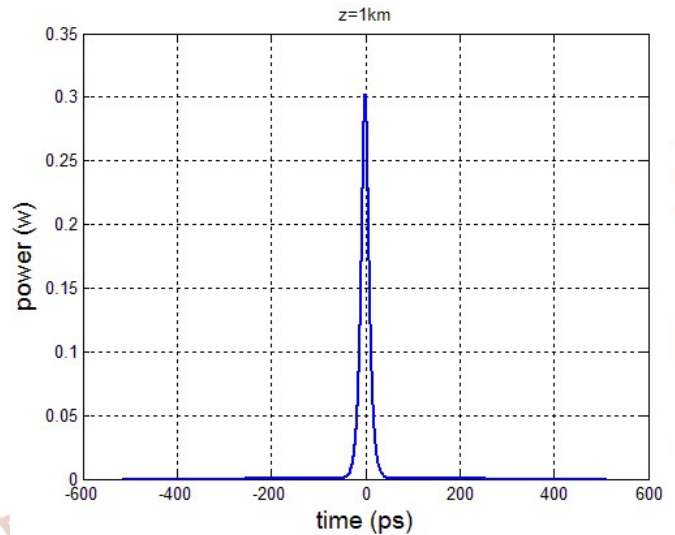


Figure 3: Optical soliton at $z = 1\text{km}$

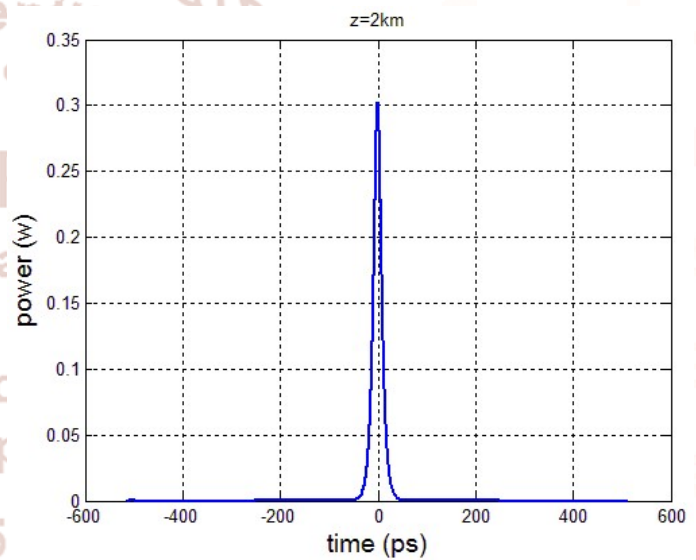


Figure 4: Optical soliton at $z = 2\text{km}$

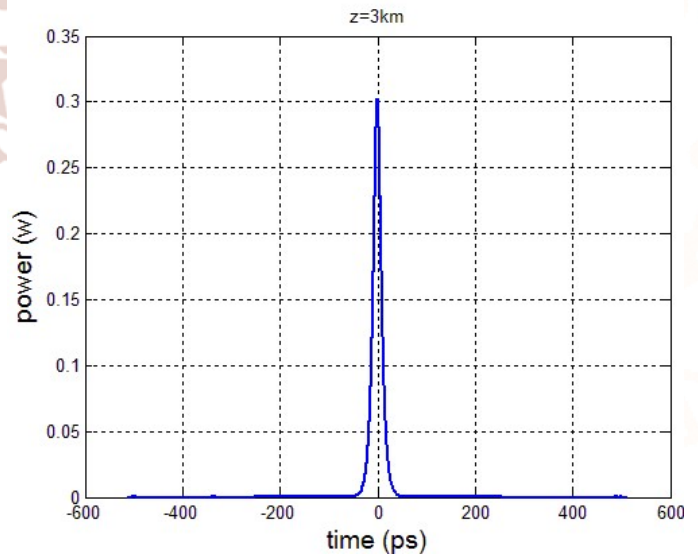


Figure 5: Optical soliton at $z = 3\text{km}$

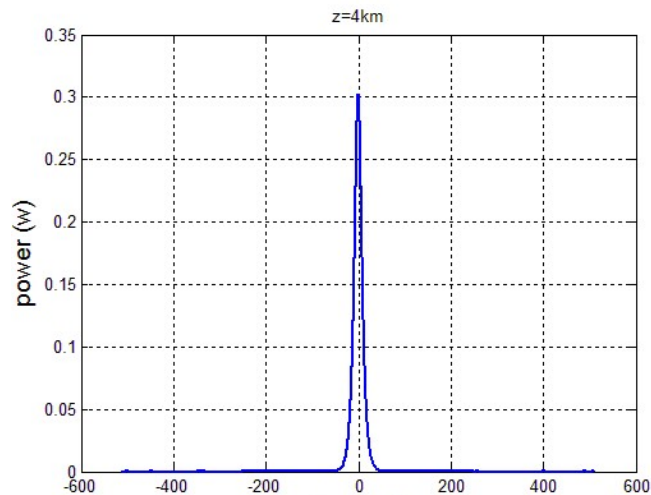
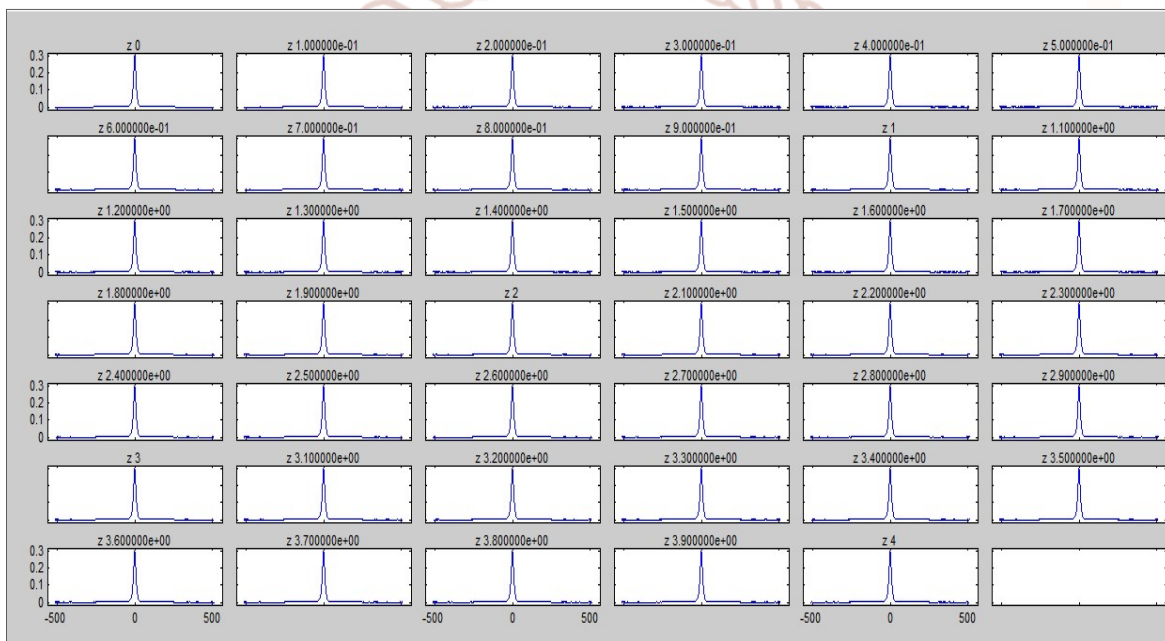
Figure 6: Optical soliton at $z = 4\text{km}$ 

Figure 7: Series of 2D Optical soliton for one soliton period

5.0 Conclusion

The optical soliton propagation along the optical fiber with $\beta_2 = -20 \text{ ps}^2/\text{km}$, $\gamma = 1.317 \text{ W}^{-1}\text{km}^{-1}$ and $\alpha = \text{dB}/\text{km}$ was simulated. The optical soliton can propagate for long distance while maintaining its shape and amplitude.

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