International Journal of Trend in Scientific Research and Development (IJTSRD)
International Open Access Journal | www.ijtsrd.com
ISSN No: 2456 - 6470 | Volume - 3 | Issue – 1 | Nov – Dec 2018

Global Exponential Stabilization for a Class of Uncertain Nonlinear Control Systems Via Linear Static Control

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ABSTRACT
In this paper, the robust stabilization for a class of uncertain nonlinear systems is investigated. Based on the Lyapunov-like approach with differential inequalities, a simple linear static control is offered to realize the global exponential stability of such uncertain nonlinear systems. Meanwhile, the guaranteed exponential convergence rate can be correctly estimated. Finally, some numerical simulations are given to demonstrate the feasibility and effectiveness of the obtained results.

Key Words: *Global exponential stabilization, uncertain nonlinear systems, exponential convergence rate, linear static control*

1. INTRODUCTION
Control design with implementation of uncertain nonlinear dynamical systems is one of the most challenging areas in systems and control theory. It is well known that uncertainties and nonlinearities often appear in various physical systems and all physical systems are essentially nonlinear in nature. Nonlinear control has been active for many years, and many results concerning with nonlinear control have been proposed. However, it is still difficult to implement nonlinear controllers for practical systems.

Recently, there have several well-developed techniques and methodologies for analyzing uncertain nonlinear systems, such as back stepping approach, fuzzy adaptive control approach, feedback linearization, H-infinity control approach, sliding mode control methodology, LMI approach, singular perturbation method, Lyapunov approach, Riccati equation approach, center manifold theorem, adaptive sliding mode control, adaptive fuzzy-neural-network, and others; see, for example, [1-8] and the references therein.

In this paper, the stabilizability for a class of uncertain nonlinear systems will been considered. Based on the Lyapunoe-like approach with differential inequality, a linear static control will be established to realize the global exponential stability of such uncertain systems. Moreover, the guaranteed exponential convergence rate can be correctly calculated. Several numerical simulations will also be provided to illustrate the use of the main results.

The layout of the rest of this paper is organized as follows. The problem formulation, main result, and controller design are presented in Section 2. In Section 3, numerical simulations with circuit realization are given to illustrate the effectiveness of the developed results. Finally, some conclusions are drawn in Section 4. In what follows, $\Re^n$ denotes the n-dimensional real space, $\|x\|$ denotes the Euclidean norm of the vector $x \in \Re^n$, $|a|$ denotes the absolute value of a real number a, and A^T denotes the transport of the matrix *A*.

2. PROBLEM FORMULATION AND MAIN RESULTS
In this paper, we explore the following uncertain nonlinear systems:

$$\dot{x}_1 = \Delta a_1 x_1 + \Delta a_2 x_2 + \Delta a_3 x_4 + \Delta a_4 x_3 x_4, \quad (1a)$$

$$\dot{x}_2 = \Delta a_5 x_1 + \Delta a_6 x_2 + \Delta a_7 x_3 + \Delta a_8 x_4 + \Delta a_9 x_1 x_3 + \Delta a_{10} u_1, \quad (1b)$$

$$\dot{x}_3 = \Delta a_{11} x_2 + \Delta a_{12} x_3 - \Delta a_9 x_1 x_2, \quad (1c)$$

$$\dot{x}_4 = \Delta a_{13} x_1 + \Delta a_{14} x_2 - \Delta a_4 x_1 x_3 + \Delta a_{15} u_2, \quad (1d)$$

$$\begin{bmatrix} x_1(0) & x_2(0) & x_3(0) & x_4(0) \end{bmatrix}^T = \begin{bmatrix} x_{10} & x_{20} & x_{30} & x_{40} \end{bmatrix}^T, \quad (1e)$$

where $x(t) := \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) \end{bmatrix}^T \in \Re^4$ is the state vector, $u(t) := \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T \in \Re^2$ is the system control,

$[x_{10} \quad x_{20} \quad x_{30} \quad x_{40}]^T$ is the initial value, and $\Delta a_i, \forall\, i \in \{1,2,\cdots,15\}$ indicate uncertain parameters of the system. The hyper-chaotic Pan system is a special case of systems (1) with $u(t)=0$, $\Delta a_2 = -\Delta a_1 = 10$, $\Delta a_5 = 28$, $\Delta a_8 = -\Delta a_9 = 1$, $\Delta a_{12} = \frac{-8}{3}$, $\Delta a_{14} = -10$, and $\Delta a_i = 0, \quad \forall\, i \in \{3,4,6,7,10,11,13,15\}$.

The global exponential stabilization and the exponential convergence rate of the system (1) are defined as follows.

Definition 1.

The uncertain systems (1) are said to be globally exponentially stable if there exist a control u and positive number α satisfying

$$\|x(t)\| \le \|x(0)\| \cdot e^{-\alpha t}, \quad \forall\, t \ge 0.$$

In this case, the positive number α is called the exponential convergence rate.

The aim of this paper is to find a simple linear static control such that the global exponential stabilization of uncertain systems (1) can be guaranteed. Meanwhile, an estimate of the exponential convergence rate of such stable systems is also explored.

Throughout this paper, we make the following assumption:

(A1) There exist constants $\overline{a}_i$ and $\underline{a}_i$ such that $\underline{a}_i \le \Delta a_i \le \overline{a}_i, \ \forall\, i \in \{1,2,\cdots,15\}$, with $b_i := \max\{|\overline{a}_i|, |\underline{a}_i|\}$, $\underline{a}_i > 0, \ \forall\, i \in \{10, 15\}$ and $\overline{a}_i < 0, \ \forall\, i \in \{1, 12\}$.

Now we present the main result for the globally exponential stabilization of uncertain systems (1) via Lyapunov-like theorem with the differential and integral inequalities.

Theorem 1

The uncertain systems (1) with (A1) realize the globally exponential stabilization under the linear static control

$$u = [u_1 \quad u_2]^T = [-k_1 x_2 \quad -k_2 x_4]^T, \tag{2a}$$

where

$$k_1 \ge \frac{1}{\underline{a}_{10}} \times \left[\frac{-3(b_2+b_5)^2}{4\overline{a}_1} + \frac{-(b_7+b_{11})^2}{2\overline{a}_{12}} + b_6 + \delta_1 + 1\right], \tag{2b}$$

$$k_2 \ge \frac{1}{\underline{a}_{15}} \left[\frac{-3(b_3+b_{13})^2}{4\overline{a}_1} + \frac{(b_8+b_{14})^2}{4} + \delta_2\right], \tag{2c}$$

with $\delta_1 > 0$ and $\delta_2 > 0$. In this case, the guaranteed exponential convergence rate is given by

$$\alpha := \min\left\{\frac{-\overline{a}_1}{3}, \ \delta_1, \ \frac{-\overline{a}_{12}}{2}, \ \delta_2\right\}. \tag{3}$$

Proof. Let

$$V(x(t)) := x_1^2(t) + x_2^2(t) + x_3^2(t) + x_4^2(t). \tag{4}$$

The time derivative of $V(x(t))$ along the trajectories of the closed-loop systems (1) with (2) and (A1), is given by

$$\begin{aligned}
&\dot{V}(x(t)) \\
&= 2x_1\dot{x}_1 + 2x_2\dot{x}_2 + 2x_3\dot{x}_3 + 2x_4\dot{x}_4 \\
&= 2x_1(\Delta a_1 x_1 + \Delta a_2 x_2 + \Delta a_3 x_4 + \Delta a_4 x_3 x_4) \\
&\quad + 2x_2(\Delta a_5 x_1 + \Delta a_6 x_2 + \Delta a_7 x_3 + \Delta a_8 x_4 \\
&\quad + \Delta a_9 x_1 x_3 - \Delta a_{10} k_1 x_2) \\
&\quad + 2x_3(\Delta a_{11} x_2 + \Delta a_{12} x_3 - \Delta a_9 x_1 x_2) \\
&\quad + 2x_4(\Delta a_{13} x_1 + \Delta a_{14} x_2 - \Delta a_4 x_1 x_3 - \Delta a_{15} k_2 x_4) \\
&\le 2\overline{a}_1 x_1^2 + 2b_2|x_1||x_2| + 2b_3|x_1||x_4| + 2\Delta a_4 x_1 x_3 x_4 \\
&\quad + 2b_5|x_1||x_2| + 2b_6 x_2^2 + 2b_7|x_2||x_3| + 2b_8|x_2||x_4| \\
&\quad + 2\Delta a_9 x_1 x_2 x_3 - 2\underline{a}_{10} k_1 x_2^2 \\
&\quad + 2b_{11}|x_2||x_3| + 2\overline{a}_{12} x_3^2 - 2\Delta a_9 x_1 x_2 x_3 \\
&\quad + 2b_{13}|x_1||x_4| + 2b_{14}|x_2||x_4| - 2\Delta a_4 x_1 x_3 x_4 \\
&\quad - 2\underline{a}_{15} k_2 x_4^2 \\
&\le 2\left(\frac{\overline{a}_1}{3} + \frac{\overline{a}_1}{3} + \frac{\overline{a}_1}{3}\right)x_1^2 + 2(b_2+b_5)|x_1||x_2| \\
&\quad + 2(b_3+b_{13})|x_1||x_4| + 2b_6 x_2^2 + 2(b_7+b_{11})|x_2||x_3| \\
&\quad + 2(b_8+b_{14})|x_2||x_4| + 2\left(\frac{\overline{a}_{12}}{2} + \frac{\overline{a}_{12}}{2}\right)x_3^2 \\
&\quad - 2\left[\frac{-3(b_2+b_5)^2}{4\overline{a}_1} + \frac{-(b_7+b_{11})^2}{2\overline{a}_{12}} + b_6 + \delta_1 + 1\right]x_2^2 \\
&\quad - 2\left[\frac{-3(b_3+b_{13})^2}{4\overline{a}_1} + \frac{(b_8+b_{14})^2}{4} + \delta_2\right]x_4^2 \\
&= -2\left[\frac{-\overline{a}_1}{3}x_1^2 - (b_2+b_5)|x_1||x_2| + \frac{-3(b_2+b_5)^2}{4\overline{a}_1}x_2^2\right] \\
&\quad - 2\left[\frac{-\overline{a}_1}{3}x_1^2 - (b_3+b_{13})|x_1||x_4| + \frac{-3(b_3+b_{13})^2}{4\overline{a}_1}x_4^2\right] \\
&\quad - 2\left[\frac{-\overline{a}_{12}}{2}x_3^2 - (b_7+b_{11})|x_2||x_3| + \frac{-(b_7+b_{11})^2}{2\overline{a}_{12}}x_2^2\right] \\
&\quad - 2\left[x_2^2 - (b_8+b_{14})|x_2||x_4| + \frac{(b_8+b_{14})^2}{4}x_2^2\right] \\
&\quad - 2\left(\frac{-\overline{a}_1}{3}\right)x_1^2 - 2\left(\frac{-\overline{a}_{12}}{2}\right)x_3^2 - 2\delta_1 x_2^2 - 2\delta_2 x_4^2 \\
&\le -2\left(\frac{-\overline{a}_1}{3}\right)x_1^2 - 2\left(\frac{-\overline{a}_{12}}{2}\right)x_3^2 - 2\delta_1 x_2^2 - 2\delta_2 x_4^2 \\
&\le -2(\alpha x_1^2 + \alpha x_2^2 + \alpha x_3^2 + \alpha x_4^2) \\
&= -2\alpha V, \quad \forall\, t \ge 0.
\end{aligned}$$

Hence, one has

$$e^{2\alpha t}\cdot\dot{V}+e^{2\alpha t}\cdot 2\alpha V=\frac{d}{dt}\left[e^{2\alpha t}\cdot V\right] \le 0,\quad \forall\, t\ge 0.$$

It results

$$\int_0^t \frac{d}{d\tau}\left[e^{2\alpha\tau}\cdot V(x(t))\right]d\tau = e^{2\alpha t}\cdot V(x(t))-V(x(0)) \le \int_0^t 0\, d\tau = 0,\quad \forall\, t\ge 0. \qquad (5)$$

From (4) and (5), it follows

$$\|x(t)\|^2 = V(x(t))\le e^{-2\alpha t}V(x(0)) = e^{-2\alpha t}\|x(0)\|^2,\quad \forall\, t\ge 0.$$

As a consequence, we conclude that

$$\|x(t)\|\le e^{-\alpha t}\|x(0)\|,\quad \forall\, t\ge 0.$$

This completes the proof. □

3. NUMERICAL SIMULATIONS

Consider the uncertain systems (1) with

$$-11\le \Delta a_1\le -10,\quad 9\le \Delta a_2\le 10, \qquad (6a)$$

$$27\le \Delta a_5\le 28,\quad 0\le \Delta a_8\le 1, \qquad (6b)$$

$$-1\le \Delta a_9\le 1,\quad -3\le \Delta a_{12}\le \frac{-8}{3}, \qquad (6c)$$

$$-10\le \Delta a_{14}\le -9,\quad 1\le \Delta a_{10}, \Delta a_{15}\le 2, \qquad (6d)$$

$$\Delta a_i = 0,\quad \forall\, i\in\{3,4,6,7,11,13\}. \qquad (6e)$$

By comparing (A1) and (6) with selecting the parameters of

$$(\overline{a}_1, \underline{a}_{10}, \overline{a}_{12}, \underline{a}_{15}) = \left(-10, 1, \frac{-8}{3}, 1\right),$$

$$(b_2, b_5, b_7, b_{11}, b_6) = (10,28,0,0,0),$$

$$(b_3, b_{13}, b_8, b_{14}) = (0,0,1,10),$$

(A1) is evidently satisfied. With the choice $\delta_1=\delta_2=1$ in (2), it can be obtained that

$$u=\begin{bmatrix}u_1 & u_2\end{bmatrix}^T=\begin{bmatrix}-111x_2 & -32x_4\end{bmatrix}^T. \qquad (7)$$

As a consequence, by Theorem 1, we conclude that the uncertain systems (1) with (6) are globally exponentially stable under the linear static control of (7). Besides, from (3), the guaranteed exponential convergence rate is given by $\alpha=1$. The typical state trajectories of the uncontrolled system and the feedback-controlled system are depicted in Figure 1 and Figure 2, respectively. In addition, the control signals and the electronic circuit to realize such a control law are depicted in Figure 3 and Figure 4, respectively.

4. CONCLUSION

In this paper, the robust stabilization for a class of uncertain nonlinear systems has been explored. Based on the Lyapunov-like approach with differential inequalities, a simple linear static control has been presented to realize the global exponential stability of such uncertain nonlinear systems. Meanwhile, the guaranteed exponential convergence rate can be correctly calculated. Finally, some numerical simulations have been offered to demonstrate the feasibility and effectiveness of the obtained results.

ACKNOWLEDGEMENT

The author thanks the Ministry of Science and Technology of Republic of China for supporting this work under grants MOST 106-2221-E-214-007, MOST 106-2813-C-214-025-E, and MOST 107-2221-E-214-030. Furthermore, the author is grateful to Chair Professor Jer-Guang Hsieh for the useful comments.

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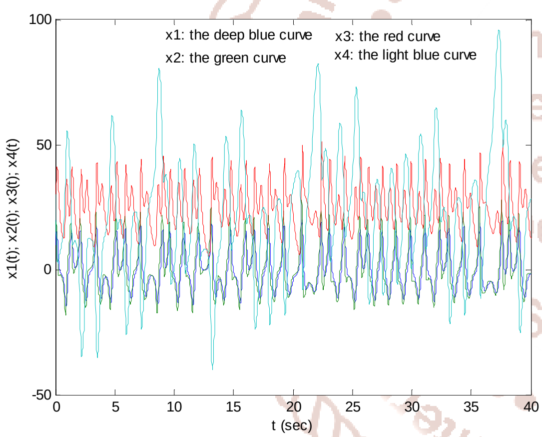

Figure 1: Typical state trajectories of the system (1) with (6) and $u = 0$.

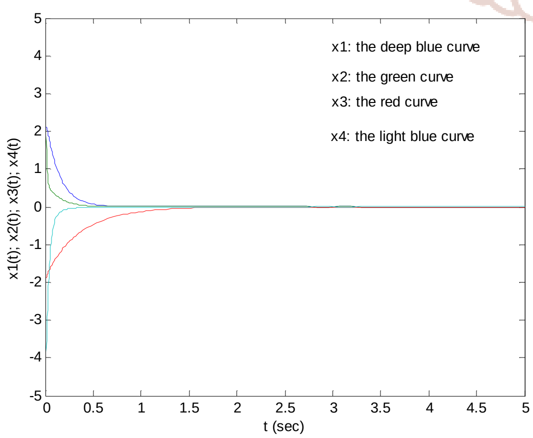

Figure 2: Typical state trajectories of the feedback-controlled system of (1) with (6) and (7).

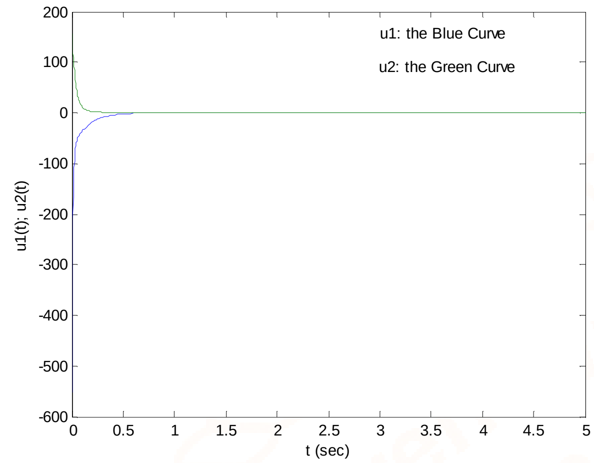

Figure 3: Control signals of $u_1(t)$ and $u_2(t)$.

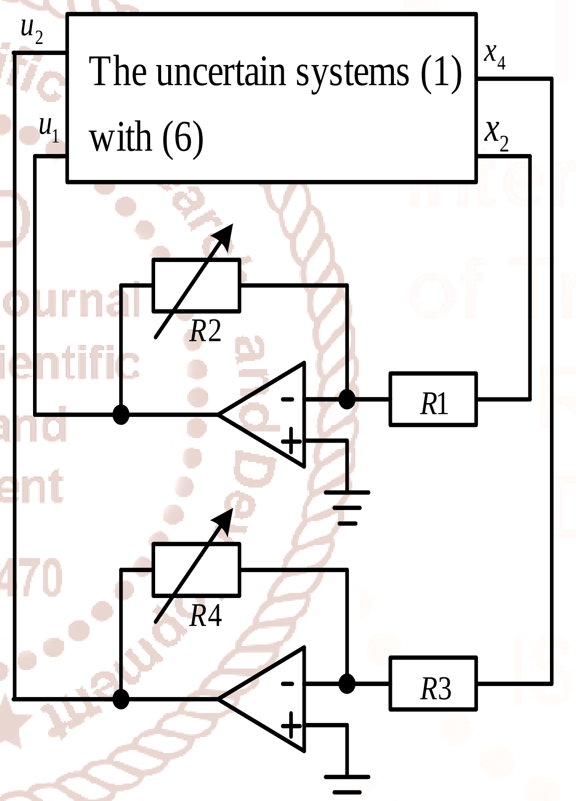

Figure 4: The diagram of implementation of controller, where $R1 = R3 = 1\,k\Omega$, $R2 = 111\,k\Omega$, and $R4 = 32\,k\Omega$.