International Journal of Trend in Scientific Research and Development (IJTSRD)
International Open Access Journal | www.ijtsrd.com
ISSN No: 2456 - 6470 | Volume - 3 | Issue – 1 | Nov – Dec 2018

Exponential State Observer Design for a Class of Uncertain Chaotic and Non-Chaotic Systems

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ABSTRACT
In this paper, a class of uncertain chaotic and non-chaotic systems is firstly introduced and the state observation problem of such systems is explored. Based on the time-domain approach with integral and differential equalities, an exponential state observer for a class of uncertain nonlinear systems is established to guarantee the global exponential stability of the resulting error system. Besides, the guaranteed exponential convergence rate can be calculated correctly. Finally, numerical simulations are presented to exhibit the feasibility and effectiveness of the obtained results.

Key Words: *Chaotic system, state observer design, uncertain systems, exponential convergence rate*

1. INTRODUCTION
In recent years, various chaotic systems have been widely explored by scholars; see, for example, [1-8] and the references therein. Frequently, chaotic signals are often the main cause of system instability and violent oscillations. Moreover, chaos often occurs in various engineering systems and applied physics; for instance, ecological systems, secure communication, and system identification. Based on practical considerations, not all state variables of most systems can be measured. Furthermore, the design of the state estimator is an important work when the sensor fails. Undoubtedly, the state observer design of systems with both chaos and uncertainties tends to be more difficult than those without chaos and uncertainties. For the foregoing reasons, the observer design of uncertain chaotic systems is really significant and essential.

In this paper, the observability problem for a class of uncertain nonlinear chaotic or non-chaotic systems is investigated. By using the time-domain approach with integral and differential equalities, a new state observer for a class of uncertain nonlinear systems will be provided to ensure the global exponential stability of the resulting error system. In addition, the guaranteed exponential convergence rate can be precisely calculated. Finally, some numerical simulations will be given to demonstrate the effectiveness of the main result.

This paper is organized as follows. The problem formulation and main results are presented in Section 2. Several numerical simulations are given in Section 3 to illustrate the main result. Finally, conclusion remarks are drawn in Section 4. Throughout this paper, $\Re^n$ denotes the n-dimensional real space, $\|x\| := \sqrt{x^T \cdot x}$ denotes the Euclidean norm of the column vector *x*, and $|a|$ denotes the absolute value of a real number *a*.

2. PROBLEM FORMULATION AND MAIN RESULTS
In this paper, we explore the following uncertain nonlinear systems:

$$\dot{x}_1(t) = \Delta f_1(x_1(t), x_2(t), x_3(t)), \quad (1a)$$

$$\dot{x}_2(t) = a x_2(t) + f_2(x_1(t), x_3(t)), \quad (1b)$$

$$\dot{x}_3(t) = f_3(x_3(t)) + f_4(x_1(t)), \quad (1c)$$

$$y(t) = b x_3(t), \quad \forall\, t \ge 0, \quad (1d)$$

$$[x_1(0) \quad x_2(0) \quad x_3(0)]^T = [x_{10} \quad x_{20} \quad x_{30}]^T, \quad (1e)$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t)]^T \in \Re^3$ is the state vector, $y(t) \in \Re$ is the system output, $[x_{10} \quad x_{20} \quad x_{30}]^T$ is the initial value, Δf_1 is uncertain function, and $a, b \in \Re$ indicate the parameters of the systems, with $a < 0$ and $b \neq 0$. Besides, in order to ensure the existence and uniqueness of the solution, we assume that all the functions of Δf_1 and $f_i(\cdot), \forall\, i \in \{2,3,4\}$ are smooth and the inverse function of f_4 exists. The chaotic sprott E system is a special case of systems (1) with $\Delta f_1 = x_2 x_3$, $f_2 = x_1^2$, $f_3 = 1$, $f_4 = -4x_1$, and $a = -1$. It is a well-known fact that since states are not always available for direct measurement, particularly in the event of sensor failures, states must be estimated. The aim of this paper is to search a novel state observer for the uncertain nonlinear systems (1) such that the global exponential stability of the resulting error systems can be guaranteed.

Before presenting the main result, the state reconstructibility is provided as follows.

Definition 1

The uncertain nonlinear systems (1) are exponentially state reconstructible if there exist a state observer $f(z,\dot{z},y) = 0$ and positive numbers κ and α such that

$$\|e(t)\| := \|x(t) - z(t)\| \le \kappa \exp(-\alpha t), \quad \forall\, t \ge 0,$$

where $z(t)$ represents the reconstructed state of systems (1). In this case, the positive number α is called the exponential convergence rate.

Now, we are in a position to present the main results for the exponential state observer of uncertain systems (1).

Theorem 1.

The uncertain systems (1) are exponentially state reconstructible. Moreover, a suitable state observer is given by

$$z_1(t) = f_4^{-1}\left(\frac{1}{b}\dot{y}(t) - f_3\left(\frac{1}{b}y(t)\right)\right), \qquad (2a)$$

$$\dot{z}_2(t) = a z_2(t) + f_2(z_1(t), z_3(t)), \qquad (2b)$$

$$z_3(t) = \frac{1}{b}y(t), \quad \forall\, t \ge 0. \qquad (2c)$$

In this case, the guaranteed exponential convergence rate is given by $\alpha := -a$.

Proof. For brevity, let us define the observer error $e_i(t) := x_i(t) - z_i(t), \forall\, i \in \{1,2,3\}$ and $t \ge 0$. (3)

From (1d) and (2c), one has

$$\begin{aligned} e_3(t) &= x_3(t) - z_3(t) \\ &= \frac{1}{b}y(t) - \frac{1}{b}y(t) \\ &= 0, \quad \forall\, t \ge 0. \end{aligned} \qquad (4)$$

From (1c), it can be readily obtained that

$$f_4(x_1(t)) = \dot{x}_3(t) - f_3(x_3(t)).$$

It results that

$$\begin{aligned} x_1(t) &= f_4^{-1}(\dot{x}_3(t) - f_3(x_3(t))) \\ &= f_4^{-1}\left(\frac{1}{b}\dot{y}(t) - f_3\left(\frac{1}{b}y(t)\right)\right) \end{aligned} \qquad (5)$$

Thus, one has

$$\begin{aligned} e_1(t) &= x_1(t) - z_1(t) \\ &= f_4^{-1}\left(\frac{1}{b}\dot{y}(t) - f_3\left(\frac{1}{b}y(t)\right)\right) \\ &\quad - f_4^{-1}\left(\frac{1}{b}\dot{y}(t) - f_3\left(\frac{1}{b}y(t)\right)\right) \\ &= 0, \quad \forall\, t \ge 0, \end{aligned} \qquad (6)$$

in view of (5) and (2a). In addition, from (1b), (1c), (4), and (6), it is easy to see that

$$\begin{aligned} \dot{e}_2(t) &= \dot{x}_2(t) - \dot{z}_2(t) \\ &= [a x_2(t) + f_2(x_1(t), x_3(t))] \\ &\quad - [a z_2(t) + f_2(z_1(t), z_3(t))] \\ &= [a x_2(t) + f_2(x_1(t), x_3(t))] \\ &\quad - [a z_2(t) + f_2(x_1(t), x_3(t))] \\ &= a[x_2(t) - z_2(t)] \\ &= a e_2(t), \quad \forall\, t \ge 0. \end{aligned}$$

It follows that

$$\dot{e}_2(t) - a e_2(t) = 0, \quad \forall\, t \ge 0.$$

Multiplying with $\exp(-at)$ yields

$$\dot{e}_2(t)\cdot\exp(-at) - a e_2(t)\cdot\exp(-at) = 0, \quad \forall\, t \ge 0.$$

Hence, it can be readily obtained that

$$\frac{d\,[e_2(t)\cdot\exp(-at)]}{dt} = 0, \quad \forall\, t \ge 0.$$

Integrating the bounds from 0 and t, it results

$$\int_0^t \frac{d\,[e_2(t)\cdot\exp(-at)]}{dt}\,dt = \int_0^t 0\,dt = 0, \quad \forall\, t \ge 0.$$

This implies that

$$e_2(t) = e_2(0)\cdot\exp(at), \quad \forall\, t \ge 0. \qquad (7)$$

Thus, from (4), (6), and (7), we have

$$\begin{aligned} \|e(t)\| &= \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)} \\ &= |e_2(0)|\cdot\exp(at), \quad \forall\, t \ge 0. \end{aligned}$$

Consquently, we conclude that the system (2) is a suitable state observer with the guaranted exponential convergence rate $\alpha = -a$. This completes the proof. □

3. NUMERICAL SIMULATIONS

Consider the uncertain nonlinear systems (1) with

$$\Delta f_1 = \Delta c \cdot x_2 x_3,\ f_2 = x_1^2,\ f_3 = 1, \quad (8a)$$

$$f_4 = -4x_1,\ a = -1,\ b = 2,\ -1 \le \Delta c \le 1. \quad (8b)$$

By Theorem 1, we conclude that the uncertain systems (1) with (8) is exponentially state reconstructible by the state observer

$$z_1(t) = \frac{-1}{8}\dot{y}(t) + \frac{1}{4}, \quad (9a)$$

$$\dot{z}_2(t) = -z_2(t) + z_1^2(t), \quad (9b)$$

$$z_3(t) = \frac{1}{2}y(t), \quad \forall t \ge 0. \quad (9c)$$

The typical state trajectory of the uncertain systems (1) with (8) is depicted in Figure 1. Furthermore, the time response of error states is depicted in Figure 2. From the foregoing simulations results, it is seen that the uncertain systems (1) with (8) are exponentially state reconstructible by the state observer of (9), with the guaranted exponential convergence rate $\alpha = 1$.

4. CONCLUSION

In this paper, a class of uncertain chaotic and non-chaotic systems has been introduced and the state observation problem of such systems has been studied. Based on the time-domain approach with integral and differential equalities, a novel state observer for a class of uncertain nonlinear systems has been constructed to ensure the global exponential stability of the resulting error system. Moreover, the guaranteed exponential convergence rate can be precisely calculated. Finally, numerical simulations have been presented to exhibit the effectiveness and feasibility of the obtained results.

ACKNOWLEDGEMENT

The author thanks the Ministry of Science and Technology of Republic of China for supporting this work under grants MOST 106-2221-E-214-007, MOST 106-2813-C-214-025-E, and MOST 107-2221-E-214-030. Besides, the author is grateful to Chair Professor Jer-Guang Hsieh for the useful comments.

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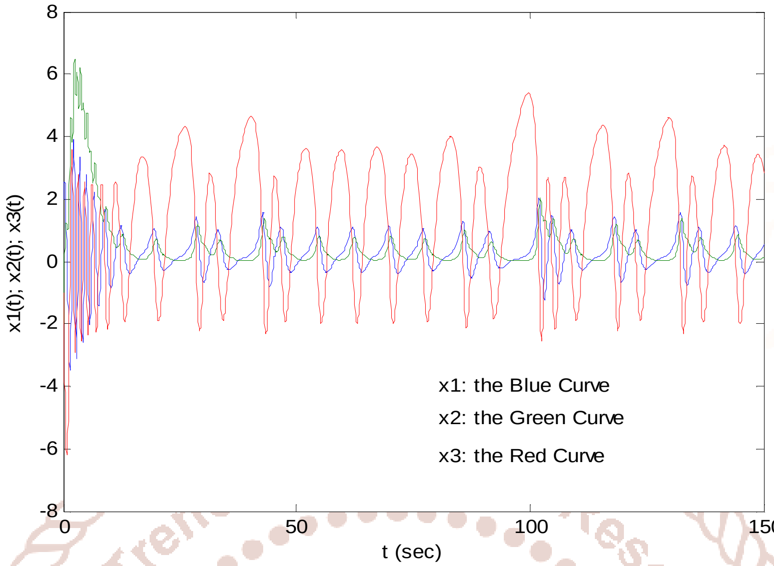

Figure 1: Typical state trajectory es of the uncertain nonlinear systems (1) with (8).

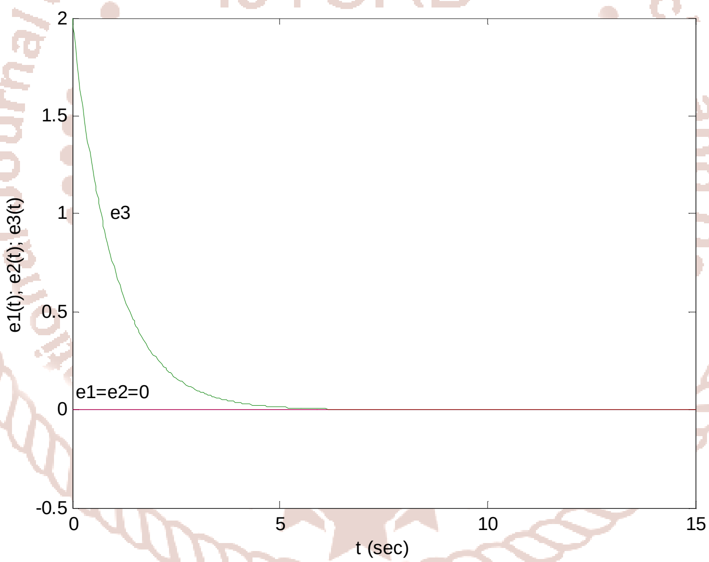

Figure 2: The time response of error states.