



Simple Exponential Observer Design for the Generalized Liu Chaotic System

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ABSTRACT

In this paper, the generalized Liu chaotic system is firstly introduced and the state observation problem of such a system is investigated. Based on the time-domain approach with differential and integral equalities, a novel state observer for the generalized Liu chaotic system is constructed to ensure the global exponential stability of the resulting error system. Besides, the guaranteed exponential convergence rate can be precisely calculated. Finally, numerical simulations are presented to exhibit the effectiveness and feasibility of the obtained results.

Keywords: Generalized Liu chaotic system, observer design, chaotic system, exponential convergence rate

1. INTRODUCTION

In recent years, numerous chaotic systems have been generally explored; see, for example, [1-10] and the references therein. Regularly, chaos in many dynamic systems is an origin of the generation of oscillation and an origin of instability. Chaos commonly appeared in various fields of application; for instance, ecological systems, secure communication, and system identification. Based on the practical considerations, it is either impossible or inappropriate to measure all the elements of the state vector. Furthermore, a state observer can be used to take the place of sensor signals, in the event of sensor failures. Undoubtedly, the state observer design of dynamical systems with chaos is not as easy as that without chaos. Owing to the foregoing reasons, the observer design of chaotic systems is really essential and significant.

In this paper, the observability problem for the generalized Liu chaotic system is investigated. By

using the time-domain approach with differential and integral equalities, a new state observer for the generalized Liu chaotic system will be provided to ensure the global exponential stability of the resulting error system. In addition, the guaranteed exponential convergence rate can be precisely calculated. Finally, some numerical simulations will be given to demonstrate the effectiveness of the main result.

2. PROBLEM FORMULATION AND MAIN RESULTS

Nomenclature

\mathbb{R}^n the n -dimensional real space

$\|x\|$ the Euclidean norm of the vector $x \in \mathbb{R}^n$

In this paper, we explore the following generalized Liu chaotic system:

$$\dot{x}_1(t) = a_1 x_1(t) + a_2 x_2(t), \quad (1a)$$

$$\dot{x}_2(t) = a_3 x_1(t) + a_4 x_1(t) x_3(t), \quad (1b)$$

$$\dot{x}_3(t) = a_5 x_1(t) + a_6 x_2(t) + a_7 x_3(t) + a_8 x_1^2(t), \quad (1c)$$

$$y(t) = b x_1, \quad \forall t \geq 0, \quad (1d)$$

$$[x_1(0) \ x_2(0) \ x_3(0)]^T = [x_{10} \ x_{20} \ x_{30}]^T, \quad (1e)$$

where $x(t) := [x_1(t) \ x_2(t) \ x_3(t)]^T \in \mathbb{R}^3$ is the state vector, $y(t) \in \mathbb{R}$ is the system output, $[x_{10} \ x_{20} \ x_{30}]^T$ is the initial value, and $a_i, b \in \mathbb{R}$ indicate the parameters of the

system with $a_2 b \neq 0$ and $a_7 < 0$. The original Liu chaotic system is a special case of system (1) with $a_1 = -a_2 = 10$, $a_3 = 40$, $a_4 = -1$, $a_5 = a_6 = 0$, $a_7 = -2.5$, and $a_8 = 4$. It is a well-known fact that since states are not always available for direct measurement, particularly in the event of sensor failures, states must be estimated. The aim of this paper is to search a novel state observer for the system (1) such that the global exponential stability of the resulting error systems can be guaranteed.

Before presenting the main result, the state reconstructibility is provided as follows.

Definition 1

The system (1) is exponentially state reconstructible if there exist a state observer $f(z, \dot{z}, y) = 0$ and positive numbers κ and α such that

$$\|e(t)\| := \|x(t) - z(t)\| \leq \kappa \exp(-\alpha t), \quad \forall t \geq 0,$$

where $z(t)$ represents the reconstructed state of the system (1). In this case, the positive number α is called the exponential convergence rate.

Now, we are in a position to present the main results for the exponential state observer of system (1).

Theorem 1

The system (1) is exponentially state reconstructible. Besides, a suitable state observer is given by

$$z_1(t) = \frac{1}{b} y(t), \tag{2a}$$

$$z_2(t) = \frac{1}{a_2} \dot{z}_1(t) - \frac{a_1}{a_2} z_1(t), \tag{2b}$$

$$\begin{aligned} \dot{z}_3(t) = & a_5 z_1(t) + a_6 z_2(t) + a_7 z_3(t) \\ & + a_8 z_1^2(t), \quad \forall t \geq 0. \end{aligned} \tag{2c}$$

In this case, the guaranteed exponential convergence rate is given by $\alpha := -a_7$.

Proof. For brevity, let us define the observer error

$$e_i(t) := x_i(t) - z_i(t), \quad \forall i \in \{1, 2, 3\} \text{ and } t \geq 0. \tag{3}$$

From (1a), (1d), and (2a), one has

$$\begin{aligned} e_1(t) &= x_1(t) - z_1(t) \\ &= x_1(t) - \frac{1}{b} y(t) \\ &= x_1(t) - \frac{1}{b} [bx_1(t)] \\ &= 0, \quad \forall t \geq 0. \end{aligned} \tag{4}$$

It is easy to see that

$$\begin{aligned} e_2(t) &= x_2(t) - z_2(t) \\ &= \left[\frac{1}{a_2} \dot{x}_1(t) - \frac{a_1}{a_2} x_1(t) \right] \\ &\quad - \left[\frac{1}{a_2} \dot{z}_1(t) - \frac{a_1}{a_2} z_1(t) \right] \\ &= \frac{1}{a_2} [\dot{x}_1(t) - \dot{z}_1(t)] - \frac{a_1}{a_2} [x_1(t) - z_1(t)] \\ &= 0, \quad \forall t \geq 0, \end{aligned} \tag{5}$$

in view of (1a), (2b), (3), and (4). Besides, from (1c), (2c), (4), and (5), it can be readily obtained that

$$\begin{aligned} \dot{e}_3(t) &= \dot{x}_3(t) - \dot{z}_3(t) \\ &= [a_5 x_1(t) + a_6 x_2(t) + a_7 x_3(t) + a_8 x_1^2(t)] \\ &\quad - [a_5 z_1(t) + a_6 z_2(t) + a_7 z_3(t) + a_8 z_1^2(t)] \\ &= a_5 [x_1(t) - z_1(t)] + a_6 [x_2(t) - z_2(t)] \\ &\quad + a_7 [x_3(t) - z_3(t)] + a_8 [x_1^2(t) - z_1^2(t)] \\ &= a_5 e_1(t) + a_6 e_2(t) + a_7 e_3(t) \\ &\quad + a_8 [x_1^2(t) - z_1^2(t)] \\ &= a_7 e_3(t), \quad \forall t \geq 0. \end{aligned}$$

It follows that

$$\dot{e}_3(t) - a_7 e_3(t) = 0, \quad \forall t \geq 0.$$

Multiplying with $\exp(a_7 t)$ yields

$$\dot{e}_3(t) \cdot \exp(a_7 t) - a_7 e_3(t) \cdot \exp(-a_7 t) = 0, \quad \forall t \geq 0.$$

Hence, it can be readily obtained that

$$\frac{d [e_3(t) \cdot \exp(-a_7 t)]}{dt} = 0, \quad \forall t \geq 0.$$

Integrating the bounds from 0 and t , it results

$$\int_0^t \frac{d [e_3(t) \cdot \exp(-a_7 t)]}{dt} dt = \int_0^t 0 dt = 0, \quad \forall t \geq 0.$$

This implies that

$$e_3(t) = e_3(0) \cdot \exp(a_7 t), \quad \forall t \geq 0. \quad (6)$$

Thus, from (4)-(6), we have

$$\begin{aligned} \|e(t)\| &= \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)} \\ &= |e_3(0)| \cdot \exp(a_7 t), \quad \forall t \geq 0. \end{aligned}$$

Consequently, we conclude that the system (2) is a suitable state observer with the guaranteed exponential convergence rate $\alpha = -a_7$. This completes the proof. \square

3. NUMERICAL SIMULATIONS

Consider the generalized Liu chaotic system (1) with $a_1 = -a_2 = 10$, $a_3 = 40$, $a_4 = -1$, $a_5 = a_6 = 0$, $a_7 = -2.5$, $a_8 = 4$, and $b = 0.5$. By Theorem 1, we conclude that the system (1) is exponentially state reconstructible by the state observer

$$z_1(t) = 2y(t), \quad (7a)$$

$$z_2(t) = -0.1\dot{z}_1(t) + z_1(t), \quad (7b)$$

$$\dot{z}_3(t) = -2.5z_3(t) + 4z_1^2(t), \quad \forall t \geq 0. \quad (7c)$$

The typical state trajectory of the generalized Liu chaotic system (1) with $a_1 = -a_2 = 10$, $a_3 = 40$, $a_4 = -1$, $a_5 = a_6 = 0$, $a_7 = -2.5$, and $a_8 = 4$, is depicted in Figure 1. In addition, the time response of error states is depicted in Figure 2. From the foregoing simulations results, it is seen that the system (1), with $a_1 = -a_2 = 10$, $a_3 = 40$, $a_4 = -1$, $a_5 = a_6 = 0$, $a_7 = -2.5$, $a_8 = 4$, and $b = 0.5$, is exponentially state reconstructible by the state observer of (7), with the guaranteed exponential convergence rate $\alpha = 2.5$.

4. CONCLUSION

In this paper, the generalized Liu chaotic system has been introduced and the state observation problem of such a system has been studied. Based on the time-domain approach with differential and integral equalities, a novel state observer for the generalized Liu chaotic system has been constructed to ensure the global exponential stability of the resulting error system. Moreover, the guaranteed exponential convergence rate can be precisely calculated. Finally, numerical simulations have been presented to exhibit the effectiveness and feasibility of the obtained results.

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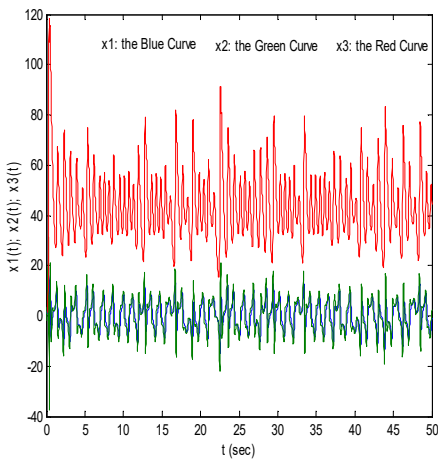


Figure 1: Typical state trajectories of the system (1) with $a_1 = -a_2 = 10$, $a_3 = 40$, $a_4 = -1$, $a_5 = a_6 = 0$, $a_7 = -2.5$, and $a_8 = 4$.

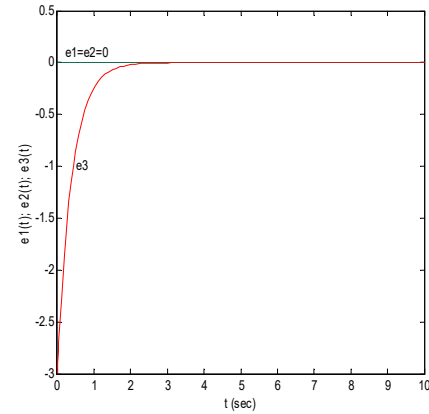


Figure 2: The time response of error states.