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Strong Regular L-Fuzzy Graphs

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ABSTRACT

In this paper, the notion of regular and strong regular L-fuzzy graph is introduced. The conditions for strong regularity of L-fuzzy graphs on cycle and star graph are derived.

Keywords: *L-fuzzy graph, strong L-fuzzy graph, regular L-fuzzy graph, strong regular L-fuzzy graph*

AMS Classification: 05C72, 03E72

1. INTRODUCTION

The first definition of fuzzy graph was proposed by Kaufmann [3] using fuzzy relations introduced by Zadeh[9]. The theory of fuzzy graphs was introduced by Azriel Rosenfeld [7] in 1975. Generalizing the notion of fuzzy sets, L. Goguen [2] introduced the notion of L-fuzzy sets. Pramada Ramachandran and Thomas [6] introduced the notion of L-fuzzy graphs and the degree of a vertex in a L-fuzzy graph. This motivated us to introduce the notion of regular L-fuzzy graphs and strong regular L-fuzzy graphs. Some preliminary, but primary work has been carried out. Comparison of set magic graphs and L-fuzzy structure introduces a new platform to work on fuzzy graph.

2. PRELIMINARIES

Definition 2.1: [4] A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu: V\, X\, V \rightarrow [0,1]$ with $\mu(u,v) \leq \sigma(u) \wedge \sigma(v), \forall\, u,v \in V,$ where V is a finite nonempty set and $\wedge$ denote minimum.

Definition 2.2: [4] The graph $G^* = (V, E)$ is called the underlying crisp graph of the fuzzy graph G, where $V = \{ u/\sigma(u) \neq o \}$ and $E = \{(u,v) \in V\, X\, V\, /\mu(u,v) \neq 0\}$.

Definition 2.3: [5] A fuzzy graph $G = (\sigma, \mu)$ is defined to be a strong fuzzy graph if

$$\mu(u,v) = \sigma(u) \wedge \sigma(v), \forall\, (u,v) \in E.$$

Definition 2.4: [5] A fuzzy graph $G = (\sigma, \mu)$ is defined to be a complete fuzzy graph if

$\mu(u, v) = \sigma(u) \wedge \sigma(v), \forall u, v \in V.$

Definition 2.5: [5] Let $G = (\sigma, \mu)$ be a fuzzy graph on $G^* = (V, E)$. The fuzzy degree of a node $u \in V$ is defined as $(fd)(u) = \sum_{u \neq v, v \in V} \mu(u, v)$. G is said to be regular fuzzy graph if each vertex has same fuzzy degree. If $(fd)(v) = k, \forall v \in V$, then G is said to be k −regular fuzzy graph.

Definition 2.6: [6] Let $(L, \vee, \wedge)$ be a complete lattice. A nonempty set V together with a pair of functions $\sigma : V \to L$ and $\mu: V X V \to L$ with $\mu(u, v) \leq \sigma(u) \wedge \sigma(v), \forall u, v \in V,$ is called an L-fuzzy graph. It is denoted by $G_L = (V, \sigma, \mu)$.

Definition 2.7: [6] Let $G_L = (V, \sigma, \mu)$ be an L-fuzzy graph. The L-fuzzy degree $d_L(u)$ of a vertex u in G_L is defined as $d_L(u) = \vee_{u \in V, u \neq v} \mu(u, v)$.

Definition 2.8 : [1] A graph G=(V,E) is said to have a set-magic labeling if its edges can be assigned distinct subsets of a set X such that for every vertex u of G, the union of the subsets assigned to the edges incident at u is X.
A graph is said to be a set-magic graph if it admits a set-magic labeling.

3. REGULAR L-FUZZY GRAPH

The above definition of degree in an L-fuzzy graph does not coincide with usual definition of degree in a fuzzy graph. Any fuzzy graph can be treated as an L-fuzzy graph taking the lattice $([0,1], \leq)$. Fuzzy degree of a vertex u, denoted as $f_d(u)$, defined as the sum of $\mu(u, v)$, where $u \in V, u \neq v$.

For further discussion, L-fuzzy degree will be taken into account.

Definition 3.1: Let $G_L = (V, \sigma, \mu)$ be an L-fuzzy graph. G_L is said to be a regular L-fuzzy graph if each vertex has the same L-fuzzy degree. If $d_L(v) = k, \forall v \in V$, for some $k \in L$, then G_L is called a $k -$ regular L-fuzzy graph.

Example 3.2 : Let $S = \{a, b, c\}$ and L= $\wp(S)$, the power set of S.

Then $(L, \subseteq)$ is a complete lattice. Let $V = \{v_1, v_2, v_3, v_4, v_5\}$

Define $\sigma: V \to L$ and $\mu: V X V \to L$ by $\sigma(v_1) = \{a\}, \sigma(v_2) = \{a, b\} = \sigma(v_4)$,

$\sigma(v_3) = \sigma(v_5) = \{a, c\}$ and

$\mu(v_1, v_2) = \mu(v_1, v_3) = \mu(v_1, v_4) = \mu(v_2, v_3) = \mu(v_3, v_5) = \mu(v_3, v_4) = \{a\}.$

Then $d_L(u) = \vee_{v \in V, u \neq v} \mu(u, v) = \{a\}, \forall u \in V$. Then $G_L = (V, \sigma, \mu)$ is a regular L-fuzzy graph.

Example 3.3 : Consider the set S and the complete lattice L= $\wp(S)$ given as in Example 3.2.

Let $V = \{v_1, v_2, v_3\}$

Define $\sigma: V \rightarrow L$ and $\mu: VX\,V \rightarrow L$ by $\sigma(v_1) = \{a,b,c\}, \sigma(v_2) = \{a,b\}, \sigma(v_3) = \{a,b\}$ and $\mu(v_1, v_2) = \{b\}$, $\mu(v_2, v_3) = \{a,b\}$, $\mu(v_3, v_1) = \{a\}$. Then $G_L = (V, \sigma, \mu)$ is a L-fuzzy graph.

Example 3.4 : Consider the set S and $\wp(S)$ as given in example 3.2.

Let $V = \{ v_1, v_2, v_3, v_4, v_5 \}$

Define $\sigma: V \rightarrow L$ and $\mu: VX\,V \rightarrow L$ by

$\sigma(v_1) = \{a\}, \sigma(v_2) = \{a,b\}, \sigma(v_3) = \{a,c\}, \sigma(v_4) = \{a,b,c\}, \sigma(v_5) = \{b,c\}$ and

$\mu(v_1, v_2) = \mu(v_2, v_3) = \mu(v_1, v_3) = \mu(v_1, v_4) = \{a\}$,

$\mu(v_3, v_5) = \{c\}, \mu(v_3, v_4) = \{a,c\}, \mu(v_4, v_5) = \{b,c\}$

Then $G_L = (V, \sigma, \mu)$ is a non-regular L-fuzzy graph.

Remark : In general, an L-fuzzy graph, $G_L = (V, \sigma, \mu)$, where μ is a constant function, is a regular L-fuzzy graph.

Theorem 3.5 : Any set magic graph admits a regular L-fuzzy graph structure.

Proof : Let G=(V,E) be a set magic graph. Then G admits a set magic labeling l.

That is , the edges of G can be assigned distinct subsets of a set X such that for every vertex u of G, union of subsets assigned to the edges incident at u is X.

Let L= $\wp(X)$, the power set of X. Then $(L, \subseteq)$ is a complete lattice.

Define $\sigma: V \rightarrow L$ and $\mu: E \rightarrow L$ as follows:

$\sigma(v) = X, \forall\, v \in V$ and $\mu(e) = l(e)$, for all edges e of G and $l(e) \in L$.

Since, $l(e) \subseteq X, \mu(e) \leq \sigma(u) \wedge \sigma(v)$, for all $e = (u,v) \in E$.

Then, $G_L = (V, \sigma, \mu)$ is an L-fuzzy graph.

For all u $\in V$, $d_L(u) = \bigvee_{v \in V, v \neq u} \mu(u,v)$

$= \bigvee_{e=(u,v), v \neq u} \mu(e)$

$= \bigvee_{e=(u,v), v \neq u} l(e)$

=X, (since l is a set magic labeling)

Hence, G_L is a regular L-fuzzy graph.

4. STRONG REGULAR L-FUZZY GRAPH

Definition 4.1 : An L-fuzzy graph $G_L = (V, \sigma, \mu)$ is said to be a strong regular L-fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$, for all edges (u,v) of G_L and the L-fuzzy degree $d_L(v)$ is constant, for all $v \in V$.

Example 4.2 : Let $S = \{a, b, c\}$ and L= $\wp(S)$, the power set of S. Then $(L, \subseteq)$ is a complete lattice. Let $V = \{ v_1, v_2, v_3, v_4 \}$

Define $\sigma: V \to L$ by $\sigma(v_1) = \sigma(v_3) = \{a, b, c\}, \sigma(v_2) = \sigma(v_4) = \{a, b\}$ and

$\mu(v_1, v_2) = \mu(v_2, v_3) = \mu(v_3, v_4) = \mu(v_4, v_1) = \{a, b, c\}$.

Then $G_L = (V, \sigma, \mu)$ is a strong regular L-fuzzy graph.

Example 4.3 : If σ and μ are constants in a L-fuzzy graphG_L, then G_L is strong regular L-fuzzy graph.

Remark : If G_L is a strong regular L-fuzzy graph, then G_L is a regular L-fuzzy graph. However, the converse need not be true, in general. This is seen from Example 3.3.

Note In [8], it has been proved that no fuzzy graph on a star graph with at least two spokes is strong regular. But in case of L-fuzzy graph, there exist a strong regular L-fuzzy structure on star graph.

Example 4.3 : Let L= {1,2,3}. $(L, \leq)$ is a complete lattice. Let $V = \{ v, v_1, v_2, v_3, v_4 \}$

Define $\sigma: V \to L$ by $\sigma(v) = 1, \sigma(v_1) = 1, \sigma(v_2) = 2, \sigma(v_3) = 3, \sigma(v_4) = 4$

and $\mu(v, v_i) = 1,\ i = 1,2,3,4$. Then, $d_L(v_1) = d_L(v_2) = d_L(v_3) = d_L(v_4) = 1$.

Hence, $G_L = (V, \sigma, \mu)$ is a strong regular L-fuzzy graph on a star graph.

Example 4.4 :Let $S = \{a, b,$c$\}$ and L= $\wp(S)$, the power set of S. $(L, \subseteq)$ is a lattice.

1. Let $= \{ v_1, v_2, v_3, v_4 \}$.
   Define $\sigma: V \to L$ by $\sigma(v_1) = \{a, b\}, \sigma(v_2) = \{a\}, \sigma(v_3) = \{a, c\}, \sigma(v_4) = \{a\}$ and
   $\mu(v_1, v_2) = \mu(v_2, v_3) = \mu(v_3, v_4) = \mu(v_4, v_1) = \{a\}$.
   Then, $G_L = (V, \sigma, \mu)$ is strong regular L-fuzzy graph.
2. Let $= \{ v, v_1, v_2, v_3, v_4 \}$. Define $\sigma: V \to L$ by
   $$\sigma(v_1) = \{a, b, c\}, \sigma(v_2) = \{a\}, \sigma(v_3) = \{a, c\}, \sigma(v_4) = \{a, b\}, \sigma(v) = \{a\}$$
   and $\mu(v, v_1) = \mu(v, v_2) = \mu(v, v_3) = \mu(v, v_4) = \{a\}$.
   Then, $G_L = (V, \sigma, \mu)$ is a strong regular L-fuzzy graph.

Theorem 4.5 Let $G_L = (V, \sigma, \mu)$ be the L-fuzzy graph.

1. If the underlying graph G_L^* is a cycle and if μ is a constant say k, for all edges (u,v) and $\sigma(u) \geq k$, for all $u \in V$, then G_L is a strong regular L-fuzzy graph.
2. If the underlying graph G_L^* is a star graph with $V = \{ v, v_1, v_2, \ldots\ldots v_n\}$ and
   if $\sigma(v) \leq \sigma(v_i), \forall i = 1,2,3, \ldots\ldots n$ and $\mu(v, v_i) = \sigma(v)$, for all the edges (v, v_i), then
   G_L is a strong regular L-fuzzy graph.

Proof

1. Let μ = constant = k(say).
   Since, $\sigma(u) \ \geq k,$ for all $u \ \in V, \sigma(u) \wedge \sigma(v) \ \geq k = \ \mu(u,v)$.
   From the definition of L-fuzzy graph, $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.
   Then, $\mu(u,v) = \sigma(u) \wedge \sigma(v)$.
   Also, $d_L(u) = \ \vee_{u \in V, u \neq v} \mu(u,v) = \vee_{u \in V, u \neq v} k = k, \forall u \ \in V$.
   Hence, G_L is strong regular L-fuzzy graph.

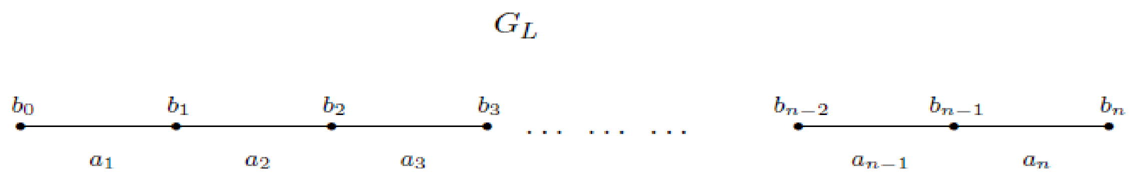
2. Let $\sigma(v) \leq \sigma(v_i), \forall i = 1,2,3, \dots . . n$.
   Therefore, $\sigma(v_i) \wedge \sigma(v) \leq \ \sigma(v) = \mu(v, v_i)$.
   From the definition of L-fuzzy graph, $\mu(v, v_i) \ \leq \sigma(v_i) \wedge \sigma(v)$.
   Hence, $\mu(v, v_i) = \sigma(v) \wedge \sigma(v_i), \forall i = 1,2,3, \dots . . n$.
   Now, $d_L(v_i) = \ \vee_{v \in V} \mu(v_i, v), \forall i = 1,2,3, \dots . . n$.
   $= \vee \ \sigma(v)$
   $= \sigma(v)$
   and $d_L(v) = \ \vee_{v_i \in V} \mu(v, v_i), \forall i = 1,2,3, \dots . . n$.
   $= \vee \ \sigma(v)$
   $= \sigma(v)$
   Thus, G_L is strong regular L-fuzzy graph.

Theorem 4.6 : For any strong regular L-fuzzy graph $G_L = (V, \sigma, \mu)$ whose crisp graph G_L^* is a path, μ is a constant function.

Proof Let $V = \{ v_0, v_1, v_2, \dots \dots , v_n \}. E(G_L^*) = \{v_0v_1, v_1v_2, v_2v_3, \dots \dots \dots . . , v_{n-1}v_n \}$

$\sigma(v_i) = b_i, for \ i = 0,1,2, \dots \dots \dots . n$ and

$\mu(v_{i-1}, \ v_i) = a_i$, for $i = 1,2, \dots \dots \dots . n$, where $a_i , b_i \in (L, \leq)$



Since, G_L is a strong regular L-fuzzy graph, we have

$a_1 = k. a_i \vee \ a_{i+1} = k, for \ i = 1,2, \dots \dots \dots n-2, a_n = k, where \ k \ \in L$ ------------------------(1)

$b_{i-1} \wedge \ b_i = \ a_i, for \ i = 1,2, \dots \dots \dots n$ --(2)

(1) $\Rightarrow \ a_i \leq k, i = 1,2, \dots \dots \dots . , n$ --(3)

Also (2) $\Rightarrow \ a_i \leq b_{i-1} \ and \ a_i \leq \ b_i, i = 1,2, \dots \dots \dots . , n$

$\Rightarrow (a_1 \leq b_0, a_1 \leq b_1), (a_2 \leq b_1, a_2 \leq b_2), \dots \dots \dots , (a_n \leq b_{n-1}, a_n \leq b_n)$

$\Rightarrow (a_1 \leq b_0), (a_1, a_2 \leq b_1), (a_2 \leq b_2, a_3 \leq b_2), \dots \dots \dots , (a_{n-1} \leq b_n, a_n \leq b_n), (a_n \leq b_n)$

$\Rightarrow k \leq b_0, a_i \vee a_{i+1} \leq b_i, i = 1,2, \dots \dots \dots, n-1, k \leq b_n.$

$\Rightarrow k \leq b_0, k \leq b_i, i = 1,2, \dots \dots \dots, n-1, k \leq b_n.$

$\Rightarrow k \leq b_0, b_{i-1} \wedge b_i, i = 1,2, \dots \dots \dots, n.$

$\Rightarrow k \leq a_i, for\ i = 1,2, \dots \dots \dots, n$ ---(4)

Equations (3) and (4) $\Rightarrow$, $a_i = k, for\ i = 1,2, \dots \dots \dots, n.$

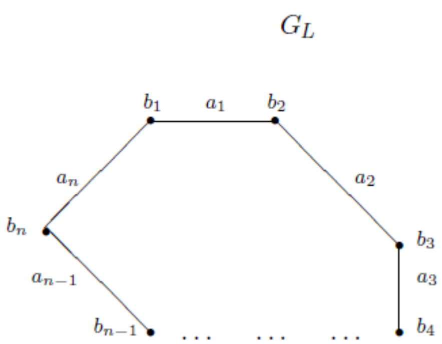
Hence, μ is a constant function.

Theorem 4.7 : For any strong regular L-fuzzy graph $G_L = (V, \sigma, \mu)$ whose crisp graph G_L^* is a cycle, μ is a constant function.

Proof Let $V = \{ v_1, v_2, \dots \dots ., v_n \}. E(G_L^*) = \{ v_1v_2, v_2v_3, \dots \dots \dots .., v_{n-1}v_n, v_nv_1 \}$

$\sigma(v_i) = b_i, for\ i = 1,2, \dots \dots \dots . n$ and $(v_i, v_{i+1}) = a_i$, for $i = 1,2, \dots \dots \dots . n$, where

$$v_{n+1} = v_1\ and\ a_i, b_i \in (L, \leq)$$



Since, G_L is a strong regular L-fuzzy graph, we have

$a_1 = k. a_i \vee a_{i+1} = k, for\ i = 1,2, \dots \dots \dots n-1, where\ k \in L$ -----------------------(1)

$b_i \wedge b_{i+1} = a_i, for\ i = 1,2, \dots \dots \dots n, since\ v_{n+1} = v_1, b_{n+1} = b_1$-------------------(2)

(1) $\Rightarrow a_i \leq k, i = 1,2, \dots \dots \dots ., n$ ---(3)

Also (2) $\Rightarrow a_i \leq b_i\ and\ a_i \leq b_{i+1}, i = 1,2, \dots \dots \dots ., n$

$\Rightarrow (a_1 \leq b_1, a_1 \leq b_2), (a_2 \leq b_2, a_2 \leq b_3), \dots \dots \dots, (a_n \leq b_n, a_n \leq b_{n+1} = b_1)$

$\Rightarrow (a_1 \leq b_1), (a_1, a_2 \leq b_2), (a_2, a_3 \leq b_3), \dots \dots \dots, (a_{n-1}, a_n \leq b_n)$

$\Rightarrow k \leq b_1, a_i \vee a_{i+1} \leq b_{i+1}, i = 1,2, \dots \dots \dots, n-1$

$\Rightarrow k \leq b_1, k \leq b_{i+1}, i = 1,2, \dots \dots \dots, n-1$

$\Rightarrow k \leq b_i \wedge b_{i+1}, i = 1,2, \dots \dots \dots, n.$

$\Rightarrow k \leq a_i, for\ i = 1,2, \dots \dots \dots, n$ ---(4)

Equations (3) and (4) ⇒, $a_i = k, for\ i = 1,2, \ldots \ldots \ldots, n.$

Hence, μ is a constant function.

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