MIMO Systems with Degraded Actuators and Sensors

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ABSTRACT

This paper presents a reliable control design technique for linear, time-invariant, multi-input multi-output (MIMO) systems with degraded actuators and sensors. The degradation defined in this paper ranges from normal operational conditions to complete failure of actuators and sensors. We derive linear matrix inequality (LMI) conditions ensuring robust stability of the system using static state feedback. The potential of the proposed technique has been demonstrated by an example of three coupled inverted pendulums.

Keywords—linear matrix inequality; static state feedback; MIMO; degradation.

I. Introduction

Conventional feedback control designs for a MIMO system may result in instability in the event of the degradation of actuators and sensors, even though it may be possible to control the system using only the surviving functions of actuator and sensor. It is worth noting that this condition can occur even if the open loop system is stable. It is therefore of interest to develop feedback control designs which guarantee robust stability despite degradation of actuators and sensors.

There are relatively few methods for design of reliable control. Veillette *et al* [1] develop observer-based reliable centralized and decentralized control systems provided the failure of actuators and sensors occur only within a prespecified subset of the control components. In addition, the actuators and sensors are subject to either failure or normal operation which neglects the possibility of partial degradation. Fujita and Shimemura [2] provide necessary and sufficient conditions by introducing a class of *U*-matrices for examining system stability against arbitrary feedback-loop failures in multivariable control systems with a stable plant and controller. This condition requires an exponential growth in computation as a function of plant dimension. Partial failures are not considered. For a given stabilizing controller of a plant, Vidyasagar and Viswanadham [3] studied the problem of designing the second controller such that either controller acting alone stabilizes the plant and both acting together also stabilize the plant. A computing method for the second stabilizing controller which involves stable coprime factorizations using the plant and the first controller data are established and may result in the second controller of high order. Cho *et al* [4] used the same methodology as [3] except that the second stabilizing controller is of adaptive controller which again ends up with high order.

In this paper, we design a static state feedback control system to tolerate the degradation ranging from normal operational condition to complete failure of actuator and sensor. The degradation is modeled as a multiplicative uncertainty at the plant input or output. We accomplish this via diagonal weighting and norm

bounded matrices. To simplify the derivation we will first develop and prove LMI conditions which tolerate sensor or actuator degradation independently. Our approach is extended to the simultaneous degradation actuators and sensors in the system.

II. Problem Formulation

The purpose of this section is to define the framework on which our approach of reliable control systems is based. The degradation for actuators and sensors will be considered separately to simplify the derivation.

A. Actuator case:

Actuator case represents the degradation of actuators and is shown as follows,

$$\dot{x} = A x + B u$$
$$u = u_k + p_u$$
$$q_u = \varphi_u u_k \tag{2.1}$$
$$p_u^r = \delta_u^r q_u^r, \quad |\delta_u^r(t)| \le 1.$$

where $x \in R^n$ denotes the state of the system, $u_k \in R^m$ is the output of the control gain shown in Figure 1. $u \in R^m$ is the input to the plant. φ_u is the uncertainty weighting of the control input to the plant. Note that p_u and q_u represent the output and input vectors of plant input perturbations. p_u^r and q_u^r correspond to the rth element of vector p_u and q_u respectively.

$$p_u = [\, p_u^1 \,.... \, p_u^r \,... \, p_u^m \,]^T, \qquad p_u^r \in R$$
$$q_u = [\, q_u^1 \,.... \, q_u^r \,... \, q_u^m \,]^T, \qquad q_u^r \in R$$
$$\varphi_u = \text{diag} (\, \varphi_u^1 \,.... \, \varphi_u^r \,... \, \varphi_u^m), \qquad \varphi_u^r \in R$$

$\delta_u^r(t)$ is the real valued unknown bounded uncertainty and it is assumed to be time-varying scalar quantity. The control signal $u(t)$ can be readily computed as

$$u(t) = (\, I + \delta_u(t)\varphi_u)u_k(t)$$

where the I is identity matrix and $\delta_u = \text{diag} (\delta_u^1 \,.... \, \delta_u^r \,... \, \delta_u^m)$, $\delta_u^r \in R$. Since $|\delta_u^r(t)| \le 1$, i.e. $-1 \le \delta_u^r(t) \le 1$, the degradation is modeled by *a priori* weighting φ_u as follows.

(1) $\varphi_u^r = 1$ indicates the rth actuator may fail fully.

(2) $\varphi_u^r = 0$ indicates there is no possibility that the rth actuator fails partially or fully.

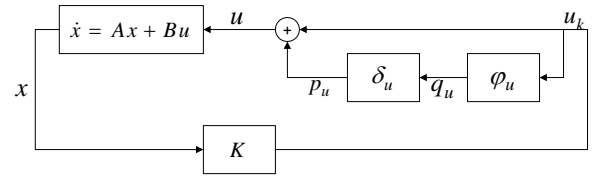(3) $0 < \varphi_u^r < 1$ indicates the rth actuator may fail partially.

Figure 1 Closed Loop System for Actuator Case

B. Sensor case:

Sensor case represents the degradation of sensors and is written as follows,

$$\dot{x} = A x + B u$$
$$x_s = x + p_s \tag{2.2}$$
$$q_s = \psi_s x$$
$$p_s^l = \delta_s^l q_s^l, \quad |\delta_s^l(t)| \le 1.$$

where ψ_s is the uncertainty weighting of the sensor and x_s denotes sensor output. The definition for p_s, q_s, ψ_s, and δ_s are defined in (2.3) in a similar way as in the actuator case and can be seen clearly from Figure 2 where it shows the closed-loop system for sensor degradation.

$$p_s = [\, p_s^1 \,.... \, p_s^l \,... \, p_s^n \,]^T, \qquad p_s^l \in R$$
$$q_s = [\, q_s^1 \,.... \, q_s^l \,... \, q_s^n \,]^T, \qquad q_s^l \in R \tag{2.3}$$
$$\delta_s = \text{diag} (\delta_s^1 \,.... \, \delta_s^l \,... \, \delta_s^n), \qquad \delta_s^l \in R$$
$$\psi_s = \text{diag} (\, \psi_s^1 \,.... \, \psi_s^l \,... \, \psi_s^n), \qquad \psi_s^l \in R$$

The degraded sensor signal $x_s(t)$ can be readily represented as

$$x_s(t) = (\, I + \delta_s(t)\psi_s)x(t)$$

where the possibility of full or partial failure of sensors is defined in the similar way as in the actuator case using variables δ_s and ψ_s in place of δ_u and φ_u, respectively.
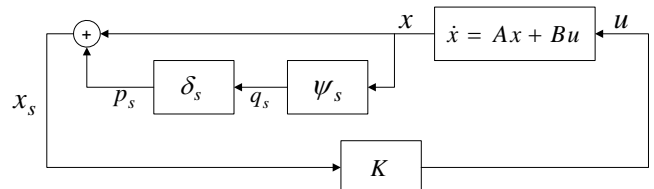

Figure 2 Closed Loop System for Sensor Case

C. Actuator and sensor case

Actuator and sensor case is the case where actuator and sensor are simultaneously degraded shown as follows,

$$\dot{x} = Ax + Bu$$
$$u = u_k + p_u$$
$$x_s = x + p_s$$
$$q_u = \varphi_u u_k \qquad (2.3)$$
$$q_s = \psi_s x$$
$$p_u^r = \delta_u^r q_u^r, \qquad \left|\delta_u^r(t)\right| \le 1$$
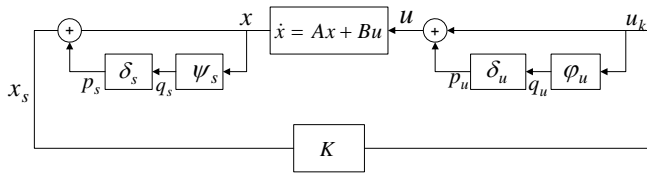$$p_s^l = \delta_s^l q_s^l, \qquad \left|\delta_s^l(t)\right| \le 1.$$

Figure 3 Closed-loop System for Actuator and Sensor Case

The matrices A and B shown in (2.1), (2.2), and (2.3) are assumed to be real constant matrices with appropriate dimensions. φ_u and ψ_s are given *a priori* and represent the uncertainty weighting used to model the degradation for actuators and sensors. We define

$$\Delta_u := \text{diag}\,(\delta_u^r(t))_{r=1,\dots,m}, \qquad \delta_u^r(t) \in R$$

$$\Delta_s := \text{diag}\,(\delta_s^l(t))_{l=1,\dots,n}, \qquad \delta_s^l(t) \in R$$

where $\|\Delta_u\|_\infty \le 1$ and $\|\Delta_s\|_\infty \le 1$.

The control problem is to find, if it exists, static state feedback that stabilizes the system described in (2.1), (2.2), or (2.3).

III. Controller Synthesis

In this section we present the main results. Lyapunov stability theory is used to establish stability criteria. Although there is no trivial method to establish a Lyapunov function, by experience, quadratic Lyapunov functions, i.e. $V(\xi) = \xi^T L \xi$, have been proven to be efficient and easily implemented [7]. We consider static state feedback, i.e. $u_k(t) = Kx(t)$ for the actuator case, $u(t) = Kx_s(t)$ for sensor case and $u_k(t) = Kx_s(t)$ for combination of actuator and sensor case.

Theorem 1. Consider the closed-loop system

$$\dot{x} = (A + BK)x + Bp_u$$
$$q_u = \varphi_u Kx \qquad (3.1)$$
$$p_u^r = \delta_u^r q_u^r, \qquad \left|\delta_u^r(t)\right| \le 1.$$

If there exist matrices Q, Y, and M satisfying

$$Q = Q^T > 0$$
$$\begin{bmatrix} QA^T + AQ + BY + Y^T B^T + BMB^T & Y^T \varphi_u^T \\ \varphi_u Y & -M \end{bmatrix} < 0 \qquad (3.2)$$

for a given weighting matrix φ_u, then the following statements are equivalent,

(1) The closed-loop system (3.1) is asymptotically stable.

(2) $(A + BK)$ is Hurwitz for $K = YQ^{-1}$.

Proof. See Appendix A for proof and notation.

Remark 1. The inequality (3.2) is convex in the matrix variables Q, Y, and M. Thus, (3.2) can be readily solved by convex optimization techniques. We write a feasibility problem

Find $Q, Y,$ and M

Subject to $Q = Q^T > 0$, $M > 0$, and (3.2).

Theorem 2. Consider the closed-loop system

$$\dot{x} = (A + BK)x + BKp_s$$
$$q_s = \psi_s x \qquad (3.3)$$
$$p_s^l = \delta_s^l q_s^l, \qquad \left|\delta_s^l(t)\right| \le 1.$$

If there exist matrices P, W, S and Λ satisfying

$$P = P^T > 0, \qquad S = S^T > 0, \qquad \Lambda > 0,$$

$$-P\Lambda P + W^T S^{-1} W < 0 \qquad (3.4)$$

$$\begin{bmatrix} PA^T + AP + W^T B^T + BW + BSB^T & P\psi_s^T \\ \psi_s P & -\Lambda^{-1} \end{bmatrix} < 0 \qquad (3.5)$$

for a given weighting matrix ψ_s, then the following statements are equivalent.

(1) The closed-loop system (3.3) is asymptotically stable.

(2) $(A + BK)$ is Hurwitz for $K = WP^{-1}$.

Proof. See proof for Theorem 3.

Remark 2. The matrix inequalities (3.4) and (3.5) are inversely coupled through the matrices S and Λ. Moreover the inequality (3.4) is not an LMI. Thus, to solve theorem 2 simultaneously through the use of convex optimization is difficult. However, we may still solve the problem using of sequential method that we solve for (3.5) first and then verify (3.4).

Theorem 3. Consider the following closed-loop system

$$
\begin{aligned}
\dot{x} &= (A + BK)x + BKp_s + Bp_u \\
q_u &= \varphi_u K(x + p_s) \\
q_s &= \psi_s x \\
p_u^r &= \delta_u^r q_u^r, \quad \left|\delta_u^r(t)\right| \le 1 \\
p_s^l &= \delta_s^l q_s^l, \quad \left|\delta_s^l(t)\right| \le 1.
\end{aligned}
\tag{3.6}
$$

If there exist matrices P, W, S_u, S_x, S_s, Λ, and Γ satisfying

$$P = P^T > 0, \quad S_u = S_u^T > 0, \quad S_x = S_x^T > 0, \quad S_s = S_s^T > 0,$$

$$\Lambda > 0, \Gamma > 0$$

$$-\Lambda + B^T S_u^{-1} B < 0 \tag{3.6}$$

$$-P\Lambda P + W^T(S_x^{-1} + S_s^{-1} + \Phi_\Gamma)W < 0 \tag{3.7}$$

$$
\begin{bmatrix}
\begin{pmatrix} PA^T + AP + W^T B^T + BW \\ + BS_x B^T + S_u \end{pmatrix} & W^T & P\psi_s^T \\
W & -\left(\Phi_\Gamma^T S_s \Phi_\Gamma + \Phi_\Gamma\right)^{-1} & 0 \\
\psi_s P & 0 & -\Lambda^{-1}
\end{bmatrix} < 0
\tag{3.8}
$$

where $\Phi_\Gamma = \varphi_u^T \Gamma \varphi_u$, for a given weighting matrix φ_u and ψ_s, then the following statements are equivalent,

(1) The closed-loop system (3.6) is asymptotically stable.

(2) $(A + BK)$ is Hurwitz for $K = WP^{-1}$.

Proof. See Appendix B for proof.

Remark 3. Normally speaking, the matrix inequalities (3.6)-(3.8) should be solved simultaneously. However, it is not possible to solve using convex programming due to inversely coupled terms, S_s, S_u, S_x, and Λ. Thus, we suggest for some given Φ_Γ we solve (3.8) for P, W, S_x, S_s, and S_u and then verify (3.6) and (3.7).

IV. Numerical Example

Consider the system in Figure 3 consisting of three coupled inverted pendulums of point masses m_i, and length l_i. The pendulums interact via three springs and three dampers of stiffness k_{ij} and damping b_{ij}; $i, j = 1,2,3$, and $i \neq j$. The distances from attached point of springs and dampers to the platform baseline are a_i. The system data are shown in Table 1. Using this system, we will demonstrate several examples for actuator and sensor degradation. The system dynamics for pendulums are written in the general form,

$$
\ddot{\theta}_i = \frac{1}{m_i}\left\{ \sum_{j=1, j\neq i}^{3}\left[-\left(b_{ij}\dot{\theta}_i + \frac{a_i^2}{l_i^2}k_{ij}\theta_i\right) + \left(\frac{l_j}{l_i}b_{ij}\dot{\theta}_j + \frac{a_i a_j}{l_i^2}k_{ij}\theta_j\right)\right] + \frac{m_i g}{l_i}\theta_i + \frac{1}{l_i}u_i\right\}
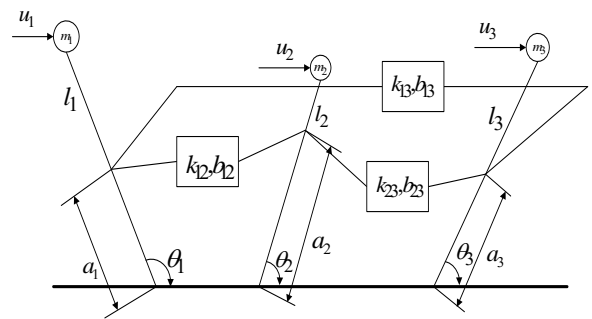\tag{4.1}
$$

Figure 3 Three Inverted Pendulums Systems

Table 1

m_1	m_2	m_3	b_{12}	b_{13}	b_{23}	k_{12}	k_{13}	k_{23}
1	1	1	1	.51	1	1	.51	1

$a_1 = a_2 = a_3 = 0.5$, g=10, $l_1 = 1$, $l_2 = 1.2$, $l_3 = 1.1$.

Initial conditions:

$\theta_1 = -0.1$, $\dot{\theta}_1 = 0.11$, $\theta_2 = 0.34$, $\dot{\theta}_2 = -0.23$, $\theta_3 = -0.29$, $\dot{\theta}_3 = -1.33$.

We will first demonstrate actuator degradation. The weighting is chosen φ_u=diag(1, 0.1, 0.28) which represents the possibility of degradation for each actuator. For instance, actuator for m_1 may fail during its operation, actuators for m_2 and m_3 are subject to 10% and 28% variation of its nominal operation signal. We will demonstrate nominal operation of actuators, i.e. $\varphi_u = 0$, at time $t \le 0.5$. Then, we will show the case where the actuator for m_1 is subject to fail and the actuator for m_2 and m_3 are partial failure. Detail conditions are shown in Table 2.

Table 2

time	m_1	m_2	m_3
$0 \le t < 0.5$	S_1	S_2	S_3

$0.5 \le t < 2.5$	failure	$0.9*S_2$	$0.72*S_3$
$2.5 \le t \le 6$	$2*S_1$	$1.1*S_2$	$1.28*S_3$

where S_i = the nominal operation control signal of actuator associated with each mass. The computed gain followed by Theorem 1 is

$$K = \begin{bmatrix} -44.6423 & -11.3417 & 2.7025 & -1.3565 & 1.2500 & -0.7005 \\ -362.5824 & -93.3584 & -2.9357 & -17.3282 & -4.3807 & -10.9399 \\ -25.4243 & -7.1539 & 4.5474 & -1.6523 & -35.8191 & -10.0032 \end{bmatrix}$$

eigenvalues of $(A+BK)$ = [-45.2643   -3.6514 -1.1316±1.6097i -2.1306±2.1394i]. The simulation results are shown in Figure 4 and 5 for plant outputs and control signals. It is noticed that the control signal for m_1 decays relatively fast at the beginning of operation and then is close to zero as shown in Figure 4. Thus, even though the actuator #1 fails at time $t \ge$ 0.5 sec, it has very little effect to the system. Since the actuator signals for m_1 varies from twice of its nominal value to zero (which is used to represent the failure of the actuator), one possible way to accommodate such a large variation is simply to make the resultant signal as small as possible such that it has the least effect on the system. The jumps shown in Figure 5 are due to the switching at time t = 0.5 sec instantaneously. If there are two actuators in the system subject to the possibilities of failure, e.g. φ_u=diag(1, 1, 0), we are not able to find feasible solutions for the Theorem 1. In other words, it is not possible to maintain stability of the system when two actuators failures in the system.
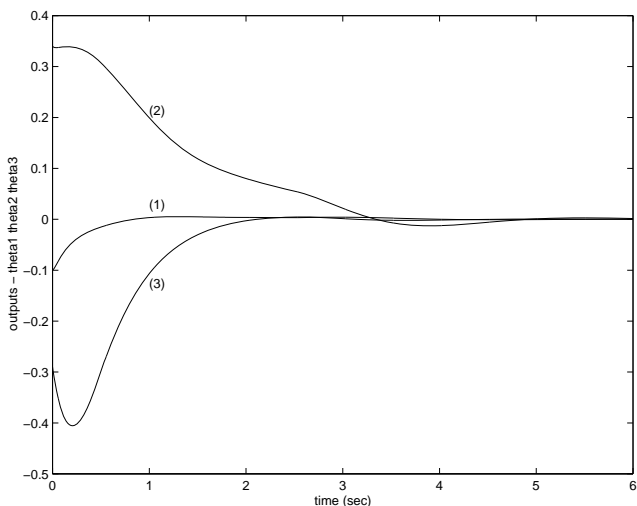

Figure 4 Outputs of Three Pendulums

For the sensor degradation case, the weighting is

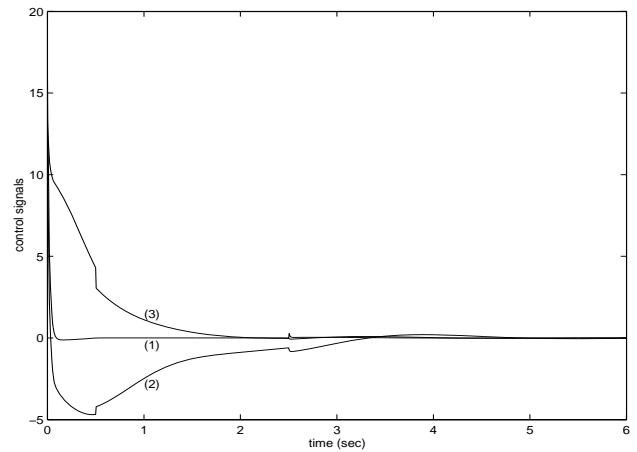chosen ψ_s = diag(0.4, 0.5, 0.4, 1, 0.5, 1) which represents the possibility of

Figure 5 Control Signals

degradation for each sensor. The velocity sensor of m_1 is subject ++to 50% variation, while the position sensor has 40% variation. The velocity sensors of m_2 and m_3 are subject to the possibility of failure, while the corresponding position sensors have 40% and 50% variations. We will demonstrate the extreme case, which is shown in Table 3. The computed gain followed by Theorem 2 is

$$K = \begin{bmatrix} -29.7241 & -8.2556 & 1.0801 & -1.0596 & 1.2811 & 0.3397 \\ 0.5669 & -0.9255 & -29.5401 & -8.0746 & 0.9343 & 0.9577 \\ 1.2812 & -0.2544 & 1.2151 & -0.9838 & -29.3428 & 8.3852 \end{bmatrix}$$

eigenvalues of $(A+BK)$ = [-6.8259   -2.4785 -5.8912 -3.0449 -3.0952 -6.2916]. The simulation results are shown in Figure 6 and 7. The sudden jumps in Figure 7 show the instantaneous switching at time t = 0.5 sec. The following results are observed from simulation. The system will oscillate if we fail three velocity sensors. Since the damping control signals given by velocity sensor are zero, the occurrence of oscillation is not unexpected.

Table 3

time	m_1		m_2		m_3	
	Pos	Vel	Pos	Vel	Pos	Vel
$0 \le t$ $\le .5$	$1.4*$ S_{1p}	$1.5*$ S_{1v}	$1.4*$ S_{2p}	$2*$ S_{2v}	$1.5*$ S_{3p}	$2*$ S_{3v}
$0.5 \le$ $t \le 5$	$0.6*$ S_{1p}	$0.5*$ S_{1v}	$0.6*$ S_{2p}	F	$0.5*$ S_{3p}	F

Pos = position sensor, Vel = velocity sensor, S with all

subscripts = the nominal operational signal associated with each sensor (subscript p = position sensor, subscript v = velocity sensor). F = failure.
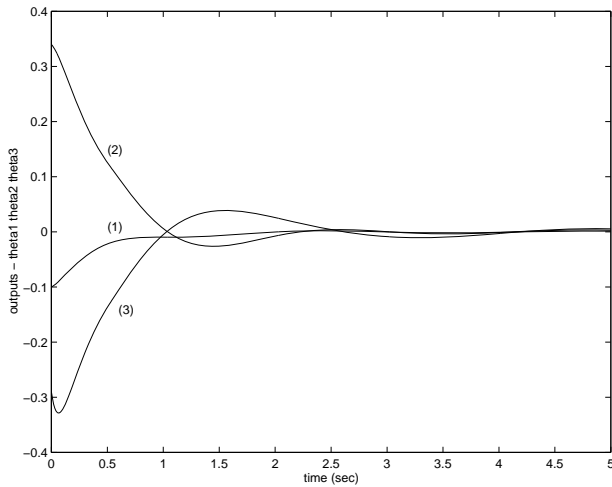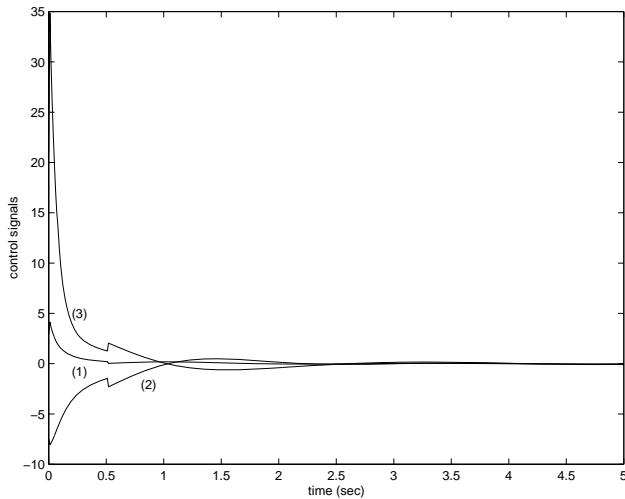

Figure 6 Outputs of Three Pendulums

Figure 7 Control Signals

V.   Conclusions

We ensure the stability of a MIMO system by establishing LMI conditions when the system is subject to degradation of actuators and sensors. The theory developed in Section 3 is demonstrated by three inverted coupled pendulums, which show the system can be stabilized.

VI.   Acknowledgment

Appendix A

Proof.

We consider the quadratic Lyapunov function $V(x) = x^T L x$. The consideration of perturbation will be incorporated in the derivative of Lyapunov function for stabilization of overall system. We have

$$\dot{V}(x) + \sum_{r=1}^{m} \lambda_r (q_u^{r\,T} q_u^r - p_u^{r\,T} p_u^r) < 0 \qquad (A.1)$$

for some $\lambda_r > 0$. Substituting (2.1) into (A.1) and rearranging the expression, we obtain

$$\begin{bmatrix} x \\ p_u \end{bmatrix}^T \begin{bmatrix} \begin{pmatrix} A^T L + LA + LBK + (LBK)^T \\ + K^T \varphi_u^T \Lambda \varphi_u K \end{pmatrix} & LB \\ B^T L & -\Lambda \end{bmatrix} \begin{bmatrix} x \\ p_u \end{bmatrix} < 0 \qquad (A.2)$$

where the $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_m)$. (A.2) is equivalent to

$$\begin{bmatrix} \begin{pmatrix} A^T L + LA + LBK + (LBK)^T \\ + K^T \varphi_u^T \Lambda \varphi_u K \end{pmatrix} & LB \\ B^T L & -\Lambda \end{bmatrix} < 0 \qquad (A.3)$$

Expanding by Schur complement and letting $Q = L^{-1}$, $Y = KQ$, and $M = \Lambda^{-1}$, we may have

$$\begin{bmatrix} QA^T + AQ + BY + Y^T B^T + BMB^T & Y^T \varphi_u^T \\ \varphi_u Y & -M \end{bmatrix} < 0$$

This completes the proof.

Appendix B

Proof.

We need the following lemma used in the proof.

Lemma B ([8]) For any matrices M and N with appropriate dimensions, we have

$$M^T N + N^T M \le M^T S N + N^T S^{-1} M$$

for any positive-definite symmetric matrices S.

We consider the quadratic Lyapunov function $V(x) = x^T L x$. The system perturbation will be incorporated in the derivative of Lyapunov function for stabilization of overall system. We have

$$\dot{V}(x) + \sum_{r=1}^{n} \lambda_r (q_s^{r\,T} q_s^r - p_s^{r\,T} p_s^r) + \sum_{r=1}^{m} \xi_r (q_u^{r\,T} q_u^r - p_u^{r\,T} p_u^r) < 0 \qquad (B.1)$$

for some $\lambda_r > 0$ and $\xi_r > 0$. Substituting (3.6) into (B.1) and rearranging the expression, we obtain

$$\begin{pmatrix} x^T(A_k^T L + LA_k + K\varphi_u^T \Gamma \varphi_u K + \psi_s^T \Lambda \psi_s)x \\ + x^T(LBK + K^T \varphi_u^T \Gamma \varphi_u K)p_s + x^T LBp_u \\ + p_s^T(K^T B^T L + K^T \varphi_u^T \Gamma \varphi_u K)x + p_u^T B^T Lx \\ + p_s^T(-\Lambda + K^T \varphi_u^T \Gamma \varphi_u K)p_s - p_u^T \Gamma p_u \end{pmatrix} < 0 \qquad (B.2)$$

where the $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_m)$ and $\Gamma = diag(\xi_1, \xi_2, ..., \xi_m)$. Use of Lemma B we have

$$x^T LBp_u + p_u^T B^T Lx \le x^T LS_u Lx + p_u^T B^T S_u^{-1} Bp_u$$

$$x^T LBKp_s + p_s^T K^T B^T Lx \le x^T LBS_x B^T Lx + p_s^T K^T S_x^{-1} Kp_s$$

$$x^T K^T \Phi_\Gamma^T Kp_s + p_s^T K^T \Phi_\Gamma^T Kx \le x^T K^T \Phi_\Gamma^T S_s \Phi_\Gamma^T Kx + p_s^T K^T S_s^{-1} Kp_s$$

where $\Phi_\Gamma = \varphi_u^T \Gamma \varphi_u$. Thus,

(B.2) $\le$

$$\begin{pmatrix} x^T \begin{pmatrix} A_k^T L + LA_k + K^T(\Phi_\Gamma + \Phi_\Gamma^T S_s \Phi_\Gamma)K + \psi_s^T \Lambda \psi_s \\ + L(S_u + BS_x B^T)L \end{pmatrix} x \\ + p_s^T\left(-\Lambda + K^T(S_x^{-1} + S_s^{-1} + \Phi_\Gamma)K\right)p_s + p_u^T\left(-\Gamma + B^T S_u^{-1} B\right)p_u \end{pmatrix} < 0$$

Therefore, we require

$$\begin{pmatrix} A_k^T L + LA_k + K^T(\Phi_\Gamma + \Phi_\Gamma^T S_s \Phi_\Gamma)K \\ + \psi_s^T \Lambda \psi_s + L(S_u + BS_x B^T)L \end{pmatrix} < 0 \qquad (B.4)$$

$$-\Lambda + K^T(S_x^{-1} + S_s^{-1} + \Phi_\Gamma)K < 0 \qquad (B.5)$$

$$-\Gamma + B^T S_u^{-1} B < 0 \qquad (B.6)$$

Multiplying $P = L^{-1}$ on both side of (B.4) and applying Schur complement, we obtain

$$\begin{bmatrix} \begin{pmatrix} PA^T + AP + W^T B^T + BW \\ + BS_x B^T + S_u \end{pmatrix} & W^T & P\psi_s^T \\ W & -\left(\Phi_\Gamma^T S_s \Phi_\Gamma + \Phi_\Gamma\right)^{-1} & 0 \\ \psi_s P & 0 & -\Lambda^{-1} \end{bmatrix} < 0 \quad (B.7)$$

where $W = KP$. Multiplying P on both side of (B.5) we have

$$-P\Lambda P + W^T(S_x^{-1} + S_s^{-1} + \Phi_\Gamma)W < 0 \qquad (B.8)$$

This completes the proof.

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